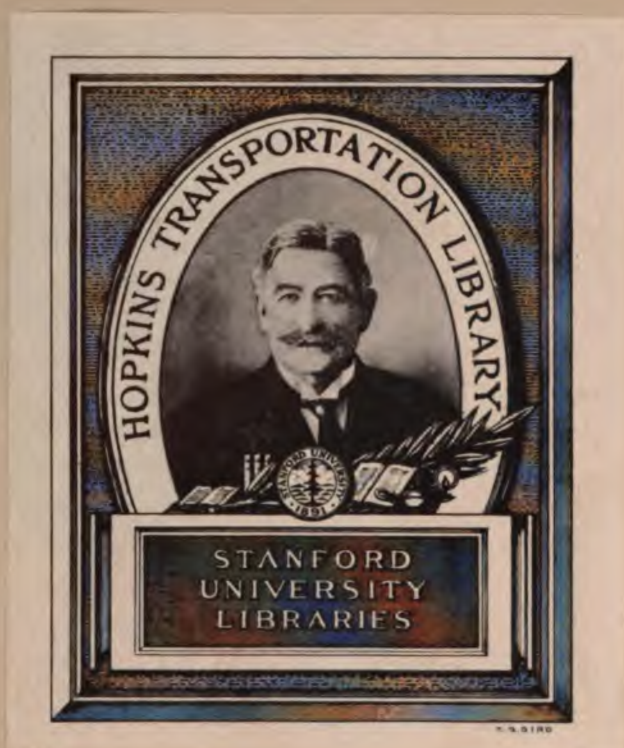
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"**Launhardt** war wohl mit der Erste,
"welcher versucht hat, über die, bei der
"Linienführung von Verkehrswegen in
"Betracht kommenden wirthschaflichen
"Gesetze allgemeine theoretische Unter-
"suchungen auzustellen und diese Gesetze
"mathematisch auszudrücken."

**Franz Kreuter: Linienführung
der Eisenbahnen. 1900.**

The Theory of the Trace:

BEING A DISCUSSION OF

The Principles of Location.

From the German
of
Wilhelm Launhardt,
Privy Councillor, Professor at the Polytechnic,
Hannover.

Premiated
by
The Union of German Railway Administrations.

Part I.

The Commercial Trace.

BY
A. BEWLEY,
FORMERLY
Ex. Eng.,
P. W. D., INDIA.

Madras:
PRINTED AT THE LAWRENCE ASYLUM PRESS, MOUNT ROAD, BY H. PLUMBE, SUPERINTENDENT.

1900.

BY THE SAME TRANSLATOR.

The Wear of Wheel-flanges and Rails in Curves, etc. 1896.—Revised, 1900. 8 Rs.
The Behaviour of Railway Vehicles in their passage through Curves. 1898. 5 Rs.
The Interaction of Rail and Wheel. 1899. 10 Rs.
The Resistance of Trains . . . . . determined experimentally. 1899. 7 Rs. 8 As.

AUTHOR'S PREFACE.

THE "Theory of the Trace" is divided into three parts. The first comprehends the Commercial Location of the Trace; the second, the Technical Location of Railways;[1] the third treats of the Technical Location of Roads.

The present Work treats of the Commercial Location of Lines, viz., the location of the trace in its financial aspect.

It was originally published in 1872 as a reprint of an article that appeared in that year in the "Zeitschrift des hannoverschen Architekten- und Ingenieur-Vereins."

The particular and specialized idea to which I then gave publicity under the expression "Commercial Trace" has found such a ready and wide acceptance that I have not judged it expedient to change this designation; although it is not one that now altogether satisfies me.

The treatment of technical problems from a financial standpoint, which has long been in use more or less definitely and intelligently in the practical execution of work, has only in recent times and for certain particular branches attained a completely scientific character. This enlarged conception and quantitative treatment of technical problems daily gains increased application and importance in all departments of Engineering.

The scope of my earlier work on the Commercial Trace was subsequently extended in Lectures at the Polytechnic, Hannover, and also in various publications. From amongst the latter I may cite an article originally published under the title "Economic Questions in Railway Engineering", in the "Centralblatt der Bau-Verwaltung", 1883: and the Section treating of Goods Transport in my "Mathematical Basis of the Theory of Political Economy", (Leipzig: W. Engelmann, 1885). The contents of this second edition are consequently considerably increased, although by presenting it as concisely as possible I have endeavoured to keep it within reasonable limits.

HANNOVER.
September, 1886.

Wilh. Launhardt.

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[1] Forms the Second Part of this translation of Launhardt's Work—TR.

CONTENTS.

PART I.

THE COMMERCIAL TRACE.

APPENDIX.

1 Millimetre	=	·0394 in. = $\frac{1}{25}$".
1 Centimetre	=	0·3937 in. = $\frac{2}{5}$".
1 Metre	=	3·2809 feet.
1 Square metre	=	10·704 sq. feet.
1 Kilometre	=	0·6214 mile.
1 Kilometre per hour	=	0·6214 M/hr.
„ „	=	0·9114 ft/sec.
1 Kilogramme	=	2·2046 lbs.
1 Tonne (= 1,000 kgs.)	=	0·9842 ton.
1 Kilog. per tonne	=	2·240 lbs. per ton.
1 „ „ sq. m/m	=	1422·31 lbs. sq. in.
1 Kilogramme-metre	=	7·233 foot-lbs.
1 Metre-tonne	=	3·2289 foot-tons.
1 Inch	=	2·54 centimetres.
1 Foot	=	0·3048 metre.
1 Sq. foot	=	0·0929 sq. metre.
1 Mile	=	1·6093 km.
1 M./hr.	=	1·6093 kms./hr.
1 Foot per second	=	1·0972 „ „ „
1 Pound	=	0·4536 kilogramme.
1 Ton	=	1·016 tonnes.
1 lb. per ton	=	0·4464 kilogs. per tonne.
1 lb. per sq. inch	=	0·000703 kg./m.^{2}
1 Foot-pound	=	0·13·825 kilogramme-metre.
1 Foot-ton	=	0·3097 metre-tonne.
1 Are	=	100m^2 = 119.·603 sq. yards.
1 Hectare	=	1000m^2 = 2A. 2280·3 sq. yds.

§ 1.

INTRODUCTION.

In discussing the subject of Transport we distinguish

The Load,
The Motive-power,
The Road.

The **Load** comprises the useful- or Paying-Load which forms the object of the transportation, together with the Weight of the **Vehicle**, of which the function is to lessen the difficulties of, and resistances to motion and to protect the Useful-Load.

Under the **Motive-power** we distinguish the motive-force and the machine, *i.e.*, the contrivance for the production and application of the motive-power.

Finally, as regards the **Road** or track, besides the character of its top surface, there is also its shape longitudinally, to be considered viz., its plan and elevation: this shape longitudinally is termed the **Trace**.

The main subdivisions of Transportation: Paying-Load, Vehicle, Motive-Force, Motor, Track, and Trace, have to be separately considered; and they consequently yield an equal number of bases for the subdivision of the various modes of transport. The Vehicle as regards its nature does not yield any basis for classification of public traffic; and the subdivisions of Power and Motor must be taken together; so that there remain but four bases of classification, as exhibited in the following survey.

I. Classification of the Ways and Means of Communication.

Water-ways.

(*a*) The Sea—characteristic surface—Waves.
(*b*) Rivers— ,, ,, —Running Water.
(*c*) Canals— ,, ,, —Still Water.

Land-ways.

(*a*) Roads—Here there are no special arrangements for guiding the vehicle: they may be surfaced with earth simply, or with stone, wood, or asphalt, etc.

(*b*) Railways—On these the vehicle has a definite path laid down for it; the path, *i.e.*, the rail, formerly of wood or stone, is now almost universally of metal.

II. Classification according to the Trace.

1. In the case of Water-ways we have the same divisions as for the Road, but the bases of the subdivision are different.
   (*a*) The Sea—characterised by horizontality and unlimited extent of surface.
   (*b*) Rivers—characterised by slight inclination, irregular width, and numerous bends.
   (*c*) Canals—present a horizontal surface, with occasional inclined or vertical falls; they are of fixed and limited width, and in direction mainly rectilineal.
2. Roads—have Width, Gradients, and Curvature: these are termed the **Elements of the Trace**, and are of cardinal importance.

§ 2.

The Growth of the Art of Transportation.

The growth of Transportation is of such far-reaching importance for the whole progress of human civilization that all other influences and causes promoting our material and moral progress are scarcely to be compared with it in importance and effect. It is only since the application of steam-power to the annihilation of distance that it has been possible to recognise the full force of this truth, and of the fact that through the rise of the Art of Transportation the vital force and activity of mankind has been so enhanced that, compared with the present, the earlier periods of civilization are merely states of somnolence.

The growth of the art of Transportation began with the search for the best existing roadway, or the natural Trace. This was determined by the directions of valleys, the fords of rivers, a mountain pass, the shape of a coast, the favourable character of the soil; and, in some instances by the necessity for easily obtaining drinking-water and the means of subsistence. Subsequently, the choice of the natural Trace was associated with the Second phase of transport development, viz., the development of modes of carriage, which amongst the oldest civilized peoples had even in pre-historic times reached the stage of wheeled-vehicles and sea-going vessels. Any further real progress in the perfecting of transport could only be attained when a road therefor was developed. This forms the Third stage of development of the Art of Transport; and this stage would be possible only in an economically-ordered or civilised state of Society. The stone-paved roads of the Egyptians from the Nile to the sites of the Pyramids, or those for the processions of the sacrificial cars in the environs of the Greek temples, are of small importance in this connexion, since they were not meant for public traffic. The earliest attempts to build public roads are due to the Assyrians and Persians; but it was the Romans who first possessed a developed and, at the same time, extensive system of roads.*

With the fall of the Roman Empire this network of roads disappeared with the exception of a few still existing remains; and passing over some isolated and feebly maintained attempts at the improvement of roads in the Middle Ages it was not till the commencement of the XVII century that in modern civilised countries attention was again gradually directed to the making of roads. But it was only in the present century that road-building was at all generally and energetically taken up; and metalled roads had only attained a meagre and insufficient development when the introduction of Railways created at once a new system of road-making which thrust the ordinary roads into the background. This new form of road, *i.e.*, the metallic track, had become a necessity from the progress which had been made in the Fourth phase of the art of transport, viz. the growth and development of the Motive-power, due to the application of steam.

With the gradual progress in the improvement of Transport, through the four above-named stages of development, viz., the **Trace,** the **Vehicle,** the **Road,** and the **Motive-power**— the progress already made in any one of these generally called forth further improvements in one or more of the others, so that a continual interaction of one on the other took place. A continual increase in the number of the preparatory works requisite for the different kinds of Transport characterises the collective development of the Art; the object being to diminish the actual labour involved in each individual act of transportation—and consequently, a steady increase in the amount of the construction-capital employed in the reduction of the working-expenses of each act of transportation.

[* See von Curt Merckel: Die Ingenieurtechnik im Alterthum. J. Springer, Berlin: 1899, and a Review of same in the New York "Engineering News": 20th April, 1899: wherein it is stated that the Roman network of roads aggregated 6,569 English miles and extended over Italy, Africa, Spain and England.—TRANS.]

But through the improvement of the art of transportation not only has a greater degree of Cheapness been attained but also a higher degree of Speed, Safety, Regularity, and Convenience, coupled with enormously increased facilities for, and a greater efficiency of, transportation.

Amongst all these advantages conferred by a developed system of transport that of Cheapness is the most generally important; so that under certain circumstances, as for example, in the case of low-priced goods in bulk, all other considerations fall into the background: and consequently, under these particular circumstances, water-transport is the most suitable means of transportation.* But when the water-transport—owing to the low water-level or to the presence of ice—is subject to frequent and long interruptions, it may be surpassed, even for low-priced goods, by the more costly Railway-transport; and this latter for passenger-traffic and for valuable merchandise, owing to its greater rapidity, must be pronounced as being in general the most perfect means of transport. In many cases, however, a simple cart-road best fulfils the requisites of transport, on account of the accessibility which is possible at all times and at all points of it. Accordingly, owing to the variety of the degree in which individual requirements make themselves felt there can be no single method of transport which can be pronounced to be the best for all possible cases. Also, even if we are clear in any given case as to the precise weight to be assigned to the several transport requirements, still we are not able to find a common unit of measure either for their amount, nor for the degree in which they are fulfilled. However, the degree of fulfilment of many of the requisites of transport may be expressed as a money-value. Thus, for example, safety may be represented by the amount of the insurance premium payable. The advantage of great rapidity of travel may be expressed by the saving in interest of the cost of goods, or in the wages of the persons transported by the train; as may be seen from the following example.

Suppose a common cart travels 30 km. on a road in 24 hours, and that a goods-train on a railway does 300 km. in the same time; then in carriage by the railway there is a saving of ·03 day per km. This saving of time, assuming a rate of interest of 5 %, represents a saving of ·0004%. Thus for merchandise costing 1,000 M.† per tonne there is a saving of ·4 pfennig per tonne/km. For passenger-traffic on such a railway there is per km. a saving of time, as compared with the duration of journey on a road, of 6 minutes; which, assuming an average value of 50 pfennig for the working-hour of the individuals travelling, is equivalent to a saving of 5 pfennig per passenger-km.

But by such a calculation as the above the advantage of increased speed of travel is really **insufficiently** estimated. The greater mobility attainable by individuals, and the possibility of being able to rapidly despatch goods immediately when wanted without being compelled to hold stocks for each probable demand—unadvoidably involving the loss of interest, depreciation of stock, and expense of warehousing—and the increased distances to which perishable goods can be transported, are some of the advantages of increased speed of transportation which cannot be precisely represented by any definite figure. The same holds good of the advantages of greater regularity and better preservation of the goods in transport, and as regards passenger travel, of its greater convenience and comfort.

From general considerations, therefore, we find ourselves restricted to estimating the degree of the perfection of a mode of transportation on the basis of the amount of the **cost of transport**, which item alone is determinable with any certainty; and the other advantages due to improved transport must afterwards be taken account of by an approximation.

[* See p. 15 and Appendix, pp. 62-67.]
[† M = Marks.]

§ 3.
The General Problem of Location.

The cost of transport is made up of the direct cost of working, f, per transport-unit (tonne-km., or passenger-km.), plus the interest on the capital cost A, of the installation requisite to carry on the business of transportation per transport-unit. This interest on the capital cost, at an annual rate of interest, i, and for an annual volume of traffic C, amounts to $\frac{Ai}{C}$.

The cost per transport-unit is, consequently,

$$k = f + \frac{Ai}{C}.$$

The direct working-expenses of transportation, f, are dependent on the amount o the capital, so that $f = F(A)$, and therefore

$$k = F(A) + \frac{Ai}{C} \quad \dots \quad \dots \quad \dots \quad (1)$$

The problem to be solved is to make this expression of the cost of transport a minimum: in other words, to evaluate the expression (differentiated with respect to A)

$$F'(A) + \frac{Ai}{C} = 0. \quad \dots \quad \dots \quad \dots \quad (2)$$

If we take various amounts of capital as abscissæ—**Fig. 1**—and the corresponding values of $\frac{Ai}{C}$ as ordinates, then the ends of these ordinates will lie on a straight line OA passing through and slanting upwards from the origin of co-ordinates.

If, further, we lay off as ordinates above OA, the several working-expenses, $f = F(A)$, corresponding to the several amounts of capital, then we obtain the curve BCB, as the *locus* of the ends of these ordinates. The lowest point, C, of this curve for which the working-expenses, $k = DF + FC$, is a minimum, determines the most advantageous outlay, OD, of capital; and consequently expresses graphically the condition $F'(A) + \frac{i}{C} = 0$.

But this solution while most advantageous politico-economically, *i.e.*, best serving the general, public, or communal interests, is not so for the capitalist investing his money as a speculation for gain in the transport business. He desires the maximum dividend, d, on his capital in addition to the current rate of interest, i. If e is the freight or rate per unit of transport, then this dividend is

$$d = \frac{C\left[e - \left(f + \frac{Ai}{C}\right)\right]}{A}$$

or, inserting $f = F(A)$,

$$d = \left[e - \left(F(A) + \frac{Ai}{C}\right)\right]\frac{C}{A} \quad \dots \quad \dots \quad \dots \quad (3)$$

which becomes a maximum for the value of A determined from the equation

$$AF'(A) + [e - F(A)] = 0 \quad \dots \quad \dots \quad \dots \quad (4)$$

If in Fig. 1 we lay off the rate $e = OE$, and draw the line $EKLE$ parallel to the X-axis, then JK, or OL, respectively indicates the gain at the customary rate of interest on amounts of capital per transport-unit which yields the capitalist his dividend. This

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Fig. 1.

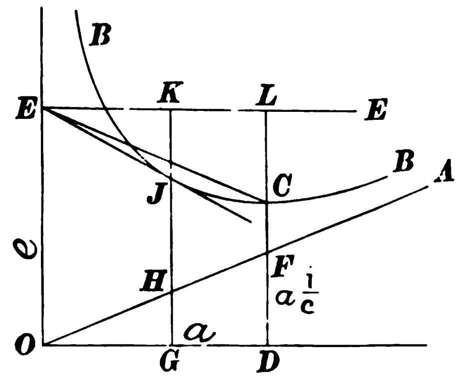


Fig. 2.

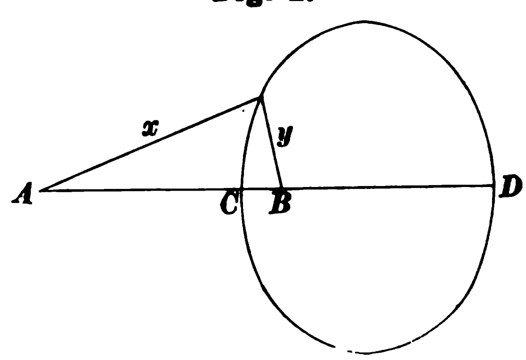
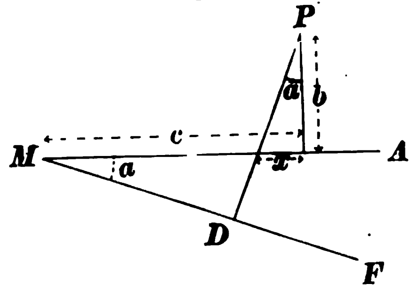


Fig. 3.



dividend may be either $\frac{OL}{EL}$ or $\frac{JK}{EK}$, per unit of capital, and it attains its maximum for a capital $A = OG$, corresponding to the point J in which a tangent drawn from E to the curve BCB of working-expenses touches this latter.

If the capitalist invests his capital to the amount most profitable to himself, *i.e.*, to OG, then the working-expenses GJ incurred are greater than those represented by DC, corresponding to the capital, OD which is invested most advantageously from a politico-economic point of view; and under these circumstances the capital invested in transport does not yield its greatest possible degree of productive benefit. Only the fear that through the competition of other carriers the unit rate might be depressed below the amount represented by OE would lead to the outlay of a larger capital. And only the apprehension that the freight might be so lowered to the point at which it would give only the customary interest on the capital—viz. that the rate or charge per unit might sink to CD—would induce a carrier capitalist to lay out capital as largely as it would be in the public interest to do.

In order to more clearly illustrate the above, let the symbol $F(A)$, representing the relation between working-expenses and the amount of the construction-capital be replaced by some arbitrary and definite quantity. Thus, let the working-expenses per tonne-km. be $f = F'(A) = \left(0{\cdot}015 + \frac{1000}{A}\right)$ M. Then for a traffic of $C = 800{,}000$ tonnes, and with a rate of interest $i = {\cdot}05$, we obtain from Eqn. 2 the amount of capital which would be most profitably or beneficially invested from the point of view of the public interest as $A = 126{,}500$ M. per km. and the cost of transportation per tonne-km. would be $f = {\cdot}0229$.

A private concern charging a freight-rate, $e = {\cdot}04$, would lay out advantageously (Eqn. 4) only a kilometrical capital $A = 80{,}000$ M.; in which case, putting the working-expenses at ·0275, a net profit of

$$800{,}000\ ({\cdot}04 - 0275) = 10{,}000 \text{ M.}$$

would be obtained, thus giving a return of $12\frac{1}{2}$°/₀ on the capital invested. With the politico-economically most advantageous outlay of capital, the net profit would be greater, namely

$$800{,}000\ ({\cdot}04 - {\cdot}0229) = 13{,}680 \text{ M.},$$

but the return on the capital would be only 10·8°/₀. With the capital invested in the manner most advantageous from a politico-economic point of view, at the usual rate of interest of 5°/₀, the net return is 7,355 M.; but in the case where it is invested solely for private benefit the return is only 6,000 M, so that the private enterprise, politico-economically, will produce less profit by $(7{,}315 - 6{,}000) = 1{,}335$ M. annually, per km.

Now in the interests of the public and at the usual rate of interest on capital, as high a *net return* as possible should be aimed at; whereas the private speculator, on the contrary, desires the profit yielded *per unit of the invested capital* to be as large as possible; his object is not the maximum gross profit depending on the absolute amount of capital invested. It is only the apprehension that the freight-charge might be reduced through the competition of some other party that is sufficiently powerful to induce the private promoter to invest a greater capital, and so determine him to introduce a higher standard of efficiency and of type into his undertaking and mode of working. Were it likely that the freight-rate might fall to ·35 he would have to increase his invested capital to 100,000 M.

Now since in railway-working outside competition is practically non-existent and a railway may, up to a certain degree, be worked as a monopoly, it is evident from the above investigation that **the most complete development of the railway machine cannot be looked for from private interest alone.***

In weighing the question whether State Railways or Private Companies are the more advantageous and preferable the above fact is of great importance; however, there are other standpoints from which the question must be regarded; and it cannot be denied that the rapid development of railways has been promoted by being entrusted to private hands.

In the problem of developing transport it is the general benefit and not that of the private individual which should under all circumstances be taken into consideration.

If we knew the form of the function $F(A)$ representing the connexion of the ·enses with the capital cost of a line it would be possible to determine under al· nstances for any given volume of annual traffic the most suitable, *i.e.*, the smalle· ·apital to be invested.

[* See in this connexion § 16, p. 42.—Tr.]

But such generality of the function $F(A)$ is not conceivably attainable; since it depends on the character of the road, of the vehicle, the motive-power, and the location.

In the following investigations the condition of the track, of the vehicles, and of the motive-power, will be assumed to be known; and we shall determine the cost of transport or working-expenses solely from the form of the Trace or centre-line.

These investigations form the Theory of the Trace; and from them are deduced certain rules and principles for the location of the trace.

The financially most advantageous location of the line is determined by the traffic-conditions of the region and the character of the ground.

In order to obtain a clear view of the dependence of the Trace on these two fundamentals it is necessary, temporarily, to leave the actual character of the ground entirely out of sight, and to assume the existence of a **perfectly horizontal and uniform ground.**

The best Trace for the purposes of transport determined on this assumption is termed the **Commercial Trace.** This line when subsequently altered and modified as the actual features of the ground may render necessary is termed the **Technical Trace.**

§ 4.
The Market Area.

The conditions governing the distribution and settling of the population over any area are dependent on the nature of its economic activity: and when this activity is engaged in the cultivation of the surface of the ground and in the husbandry of land and wood and on many kinds of handicrafts and small manufactures this distribution is to be assumed as uniform over the area; although, as a matter of fact, the population usually lives collected together in small hamlets, and the number of the inhabitants per unit of area, or density of population, varies according to the local conditions. Another part of the population, namely that which is engaged in wholesale commerce, the various professions of Art and Science, and that which consists of merchants and officials, lives collected in towns.

According to the Census of 1880, the population of Germany amounted to 45,234,061. Of this number 12,971,554 inhabitants formed town-population, distributed in 612 towns having a population exceeding 5,000; and 32,262,507 formed the country population; so that for the area of 540,522 km. there were on an average 60 inhabitants per square kilometre (qkm.)—excluding the town-population; and 84 inhabitants including it.

The exchange of commodities between the town-population and the country forms the local or Market-traffic, in contrast to the Wholesale or bulk-traffic which is carried on between towns. Each town forms the **market-centre** or centre of sale, and the area surrounding it is its **market-area** or area of sale.

Notwithstanding the apparent irregularity of their grouping the location of these market-centres is not by any means the result of chance. These sites were originally formed on the basis of distance of a day's journey apart, so that it might be possible to make the journey on the same day to and from the most distant points lying on a line half-way between two market-centres and to provide in the case of long-distance merchandise and journeys safe and comfortable places for passing the night. The worse the roads were, the closer together lay these market-centres; so that in mountainous districts or in those having difficult ground they occur in greater proximity to each other than in the plains or in easier ground. Along with these general principles influencing the choice of market-sites there were many influential local conditions which decided the matter, such as a convenient river-crossing, a useful waterfall, the confluence of two valleys, the access to a mountain-pass, a good harbour, strength of position affording protection against attack, a mine, warm or medicinal springs, and so forth.

The limits of the market or sale-area are determined by the price of commodities at the neighbouring market-centres or towns. These prices, in general, are not the same. Also, the commodities despatched from the market-places under the influence of competition have not, per unit of weight, the same intrinsic value.

Consequently, assuming that there is everywhere an equal degree of accessibility, there will not be paid, in general, for the transport of equal volumes of the merchandise of two neighbouring markets mutually competing, the same price per unit of distance.

If in a market A, a commodity is sold at a price p, and transported at a cost f, then at a distance x from the market-place it will cost $p + f x$.

If at a neighbouring market-place B, an equal volume of merchandise is sold at a price p_1 and the transport-cost is f_1, then at a distance y from the market-place it will sell

for $p_1 + f_1 y$. The limits of both market-areas, or areas of sale, will consequently be determined from the equation

$$p + fx = p_1 + f_1 y. \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$$

The *locus* of these limits forms in general a closed curve—**Fig. 2**—about the place of origin of the cheaper merchandise, the unit of weight of which has the lesser money-value, and from which, therefore, an equivalent volume at the higher rate is despatched.

This curve is of the fourth degree—called by Des Cartes an ellipse of the second order. It is the vertical projection on a horizontal plane of the line of penetration of two circular cones having a common vertical axis. The axis—lying in the line joining the two market-places of the curve—is of the length

$$C D = \frac{2 f_1 (p - p_1 + a f)}{f_1{}^2 - f^2}$$

where a is the distance apart of the two markets.

If the two commodities are of equal value so that $f = f_1$ the boundary of the market-area is no longer a closed curve but a hyperbola of which the concave side is turned towards the market at which the higher price prevails.

If the original selling prices at both the market-places are the same then the hyperbola becomes a straight line intersecting at right angles and in its middle the straight line connecting the markets.

The boundary of the market-areas forms a polygon of which the number of sides corresponds to the number of the adjacent market-areas and the sides of which are, according to circumstances, ellipses of the fourth degree, hyperbolas, or simply straight lines.

As we have in **Market traffic** to do with a great number of different goods for each of which, according to the local difference of price, the market-limits are different; and since the difference in price even for each individual commodity undergoes variations with the season, so the limits of the market or sale-area are not to be regarded as sharply defined lines; nevertheless, they will assume a tolerably definite shape for the commoner kinds of merchandise in use, which only in the course of time and through the changing economic importance of the individual market-towns undergo a gradual alteration; whereas by the construction of land and water-ways of communication, or of railways, they frequently undergo very considerable modifications, as will be shown in what follows.

The areas of import and export, or of production and consumption, of **Wholesale traffic** extend far beyond the boundaries of single market-sites, and for many kinds of merchandise are of unlimited extent.

§ 5.

The Area of Influence of Roads.

When a district is everywhere of uniform traversability and there is a uniform intensity of traffic, the necessity arises for better roads, firstly, on the lines leading from one market-town to another; because they accommodate not only the local retail-traffic but also the bulk-business. These lines constitute roads of the **first order,** and form a network of triangles of which the sides radiate, from the market-places. The districts lying within these triangles, are disadvantageously situated as regards accessibility to the three market-places surrounding them; which occasions the establishment of markets of the second order or subordinate markets which attract to themselves in their immediate environs a small local traffic with adjoining market-towns. These market-spots demand a good system of connexions with the neighbouring market-towns, and this demand gives rise to the construction of roads of the **second order.**

If there exist natural or artificial water-ways, or railways, then the loading-up places thereon—or stations in the case of a railway—form the points of origin of roads which may be termed **roads of approach** or access.

With the present extension of railways roads serve almost exclusively the market-traffic; and their financial value as productive works depends on the size and the commercial importance of the district of which they bring the traffic to the market-town.

For a point P lying off the road $M\ A$—**Fig. 3**—the traffic will not go directly to the market along the line $P\ M$, but will first go to the road by the line $P\ B$ and then from B will make use of the road; so that if f is the rate of freight across-country (no made road) along PB, and f_1 the rate on the road $A\ M$, the rate for carriage to the market is

$$k = f.\ PB + f_1\ BM$$

or, introducing the lengths represented in the Fig.

$$k = f\,(x^2 + b^2)^{\frac{1}{2}} + f_1\ (c - x). \qquad \ldots \qquad \ldots \quad (6)$$

Now the new road must deviate from and be inclined to the perpendicular approach at an angle α such that the cost of carriage will be a minimum—the condition for which is given by differentiating with respect to x, etc.; thus

$$\frac{d\,k}{d\,x} = f\,\frac{x}{(b^2 + x^2)^{\frac{1}{2}}} - f_1 = 0$$

whence

$$\sin \alpha = \frac{f_1}{f} \qquad \ldots \qquad \ldots \qquad \ldots \quad (7)$$

If the line $M\ F$ be drawn at the angle α to the road then the traffic from all points in the part of the area served by the road lying on the further side must be directed perpendicularly towards this line $M\ F$; which, consequently, is termed the **Access—,** or **Approach-front,** of the road. The cost of carriage under these circumstances will be

$$k = f.\ P\ B + f_1\ \frac{B\ D}{\sin \alpha}$$

and inserting the value of $\sin \alpha = \frac{f_1}{f}$,

$$k = f.\ P\ D$$

namely, the cost of carriage is precisely the same as if the journey had been made across-country (no road) in a direction perpendicular to the Approach-front of the road.

Consequently, for determining the location of the road the radial line of access to the market-place is no longer the determinant of the direction and cost of the transport; but in the place of the market-town we have the road's Approach-front which is reached perpendicularly.

The boundary between the areas served or controlled by two neighbouring roads radiating from the market-town—assuming a uniform passability of the ground—is a straight line radiating from the market-town and bisecting the angle formed by both roads at the market-town. And since the boundary facing the neighbouring market-town of the area served by the road must be a straight line crossing the road perpendicularly, the road-area of influence, or area served by the road, is consequently divided by the road into two rectangular triangles.

If $M\ A\ B$—**Fig. 4.**—be the half of the area served by the road, $M\ F$ the approach-front of the traffic $M B = l$, the length of the road $A\ B = c$ the breadth of the area, and γ = the volume of traffic arriving annually per unit of area served by the road—which is the definition of the term "**traffic-density**" or "**intensity**"—then the volume of business for despatch is

$$Q = \frac{\gamma c l}{2}$$

and the total transport-cost is the same as if the volume of traffic had to be transported from the centre of gravity, C, of the triangle $M\ A\ B$ along a rough earth-road, $C\ D$, perpendicularly towards the road's approach-front; consequently

$$K = Q. f. CD,$$

or

$$K = \frac{\gamma f c l}{2} \left(\frac{2}{3} l \sin\alpha + \frac{c \cos\alpha}{3}\right)$$

If the road's areas of influence on both sides are equal and if $b = 2c$, *i.e.*, the total width of the area served by the road up to the market-boundary, then the transport-cost for the whole area served by the road will be

$$K = \gamma f b l \left(\frac{l \sin\alpha}{3} + \frac{b \cos\alpha}{12}\right)$$

In order to obtain the total annual outlay for transport within the area served by the road the interest on the capital sunk in the construction of the road plus its cost of maintenance must be added to the actual transport-expenses.

If A is the amount of capital sunk per km.,—*i.e.* the kilometric construction-cost—and i the rate of interest thereon, then the interest on the capital $= A\ i\ l$. The maintenance charges per km. $= B + \beta\ C$, where B is a constant, β a coefficient, and C the mean annual volume of traffic. Since the whole traffic only makes use of the road on the average for two-thirds of its length, C is to be put $= \frac{\gamma\ bl}{3}$.

Accordingly, the total sum of the costs of construction, maintenance, and transport is

$$S = \frac{f\gamma b l^2 \sin\alpha}{3} + \frac{\gamma f b^2 l \cos\alpha}{12} + A i l + B l + \frac{\gamma \beta b l^2}{3}.$$

Whence, after dividing by the volume of traffic $\frac{\gamma\ bl}{2}$—to obtain the cost per unit of weight—we have

$$k = \frac{2}{3} f l \sin\alpha + \frac{f b \cos\alpha}{6} + \frac{2 A i}{\gamma b} + \frac{2 B}{\gamma b} + \frac{2}{3} \beta l \quad \dots \quad \dots \quad (8)$$

In the above, the first term represents the mean cost of the transport by road; the the second term, the mean transport-cost across-country up to the road; the third, the amount of the interest on the capital incident per unit of weight; the fourth, the amount of the constant part of the maintenance-expenses independent of the traffic; and the last term is that part of the maintenance-cost of the road dependent on the amount of the traffic.

Fig. 4.

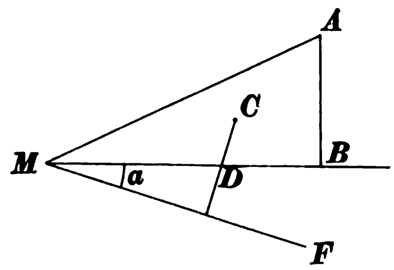


Fig. 5.

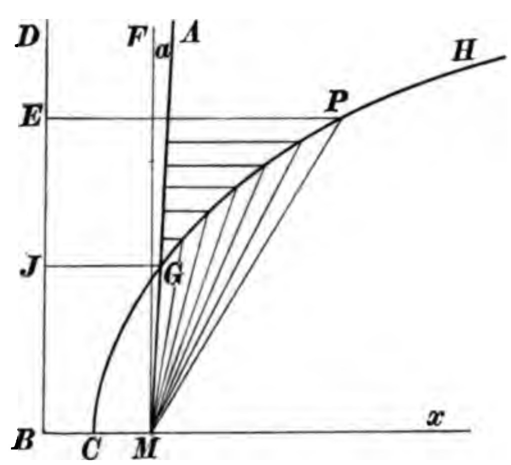
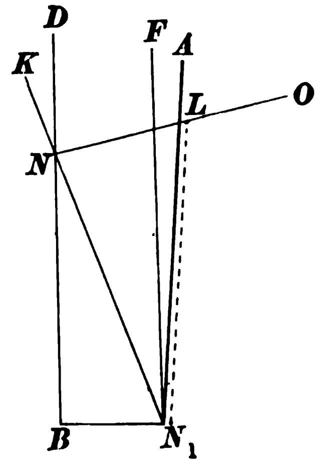


Fig. 6.



In order to determine the most favourable density of the network or system of roads, we have to examine for what value of b the expenses, k, incident per unit of traffic become a minimum. Differentiating with respect to b and equating to zero, we obtain for the most suitable value of b,

$$b = \sqrt{\frac{12\,(Ai + B)}{\gamma f \cos \alpha}} \quad \dots \quad \dots \quad \dots \quad (9)$$

Since there are l kms. of road in the area $\frac{bl}{2}$ served by the road there is, consequently, a length of road $\delta = \frac{2}{b}$ per sq. km. This quantity is defined as the "**density**" of the road system.

The most advantageous density of the road-network is consequently

$$\delta = \sqrt{\frac{\gamma f \cos \alpha}{3\,(Ai + B)}} \quad \dots \quad \dots \quad \dots \quad (10)$$

and is thus proportional to the square-root of the traffic-density, and inversely proportional to the square-root of the cost of constructing the road.

If the average cost of carriage per tonne-km. on roads be taken at 20 pf. and on rough district-roads at 80 pf., then $\alpha = \cdot 25$, and $\cos \alpha = \cdot 94$.

As a matter of fact, the traffic-density is not constant as has been assumed in the above investigation, but diminishes according to some law with the increase in the cost of transport, and therefore, with the distance from the approach-front; so that the C. G. of the traffic is nearer to the market-place than is the C. G. of the area served by the road. Consequently, we shall be more correct if we put, say, $2\,(Ai + B)$ instead of $3\,(Ai + B)$ for the most advantageous density of the road-network in the above formula; so that if we assume $\cos \alpha = 1$ as sufficiently exact the expression becomes

$$\delta = \sqrt{\frac{\gamma f}{2\,(Ai + B)}} \quad \dots \quad \dots \quad \dots \quad (11)$$

If we assume that the traffic-density averages 100 tonnes per sq. km. and take the kilometric cost of construction of the roads at 12,000 M., the rate of interest at 4 %, the maintenance-expenses which are constant and independent of the traffic at 150 M., then the most advantageous density of the road system is

$$\delta = \sqrt{\frac{100 \times 0{\cdot}8}{2\,(12000 \times \cdot 04 + 150)}}$$

or

$$\delta = \frac{1}{4}$$

On this principle the maximum length of the German network of roads ought for the empire's area of 540,022 sq. km. to amount to about 135,000 km. Up to the present this extent of roads has not been reached.

But, manifestly, such an approximate method of calculation as the above, resting as it does on uncertain guesses at the average traffic-density and on the postulate of a uniform horizontal surface of the ground has little practical value. The important result arrived at is the proof of the law that **the most advangeous density of the road-network is a function of the square-root of the traffic-density and the construction-cost**; and the prominence of the fact that **there is a limit to road-making which for financial reasons should not be exceeded**: and that in the construction of roads we must not allow ourselves to be carried away by catch-cries such as "Every village should have its own connecting road."

§ 6.

Water-ways of Communication.

When merchandise is trasported by water it is conveyed up to the ship as a rule by cart-roads; and usually by the same means from the unloading wharf to the destination of the merchandise. The cost of this loading and unloading we shall call u per tonne, and the cost of carriage per tonne-km. on the water-way, f_2. Since this rate f_2 may be taken at, say, 2 pf. the approach-front for access by rough roads only deviates by a very small angle from the line of direction of the water-way, since for this angle we have

$$\sin \alpha = \frac{2}{80}$$

If we draw—as in **Fig. 5**—the line $B\,D$ parallel to the approach-front $M\,F$, at a distance $M\,B = \frac{u}{f}$ then the transport-cost from any point P to the market-town by the water-way $M\,A$ is fixed by what would be the actual cost of transport on a rough road along the line $P\,R$, perpendicular to the parallelwise displaced approach-front $B\,D$.

The boundary line between the part of the market-area, from which the traffic must arrive directly without assistance of the water-way to the market-town, and the area of influence which the water-way will create for itself is consequently a parabola $C\,G\,P\,H$ of which the parameter is $B\,M = \frac{u}{f}$ and of which the axis $C\,M\,X$ cuts the approach-front $M\,F$ perpendicularly in the point M, and of which the apex C lies in the middle of $B\,M$. The shortest distance for which the water-way is used is

$$G\,M = G\,J = \frac{\frac{u}{f}}{1 - \sin \alpha} = \frac{u}{f - f_2}.$$

Thus if the cost of the loading and unloading per tonne is assumed to be $u = 150$ pf. this distace is $\frac{150}{80 - 2} =$ about 2 km.

If a road $N_1\,L$ runs parallel to the water-way $M\,A$* —**Fig. 6**—of which the access-front is $M\,K$, then the boundary of the area from within which the water-way or the road is advantageously used is a straight line $N\,L\,O$, bisecting the angle $M\,N\,D$ which the approach-front of the road $M\,K$ makes with the displaced approach-front, $B\,D$, of the water-way. The smallest distance $M\,L$ for which the water-way can be advantageously employed is given by the equation

$$f_1\,M\,L = u + f_2\,M\,L$$

whence

$$M\,L = \frac{u}{f_1 - f_2}$$

and consequently, for $u = 150$, $f_1 = 20$, and $f_2 = 2$, it is $8\frac{1}{3}$ km.

Thus water-ways in competition with roads have the advantage only for the more remote lying portions of the market-area.**

[* In this Fig. the letter M has been omitted—See Fig. 5.—Tr.]

[** We have frequently pointed out to our readers how the steady growth in the power of locomotives and the weight of trains has reduced the cost of moving freight by rail. The first triumph of the locomotive was as a carrier of passenger traffic. There were railway managers in the early days of the railway era who contended that only high-class freight could be economically hauled by rail. To this day, in England and on the continent of Europe the inland waterways are able to carry freight as cheaply or more cheaply than competing railway lines. American railways, however, have developed far beyond those of any other country, and American railway managers have proved to the world that with the steel rail for a roadway, and steam for a motive power, freight can be moved far more cheaply than in any artificial waterway or river channel. The modern American freight train is in fact excelled only by the vessels on the Great Lakes and on the ocean as an economical machine for the transport of bulk freight.

Unfortunately this fact is not as yet clearly understood, save by the few who are conversant with recent progress in the railway field. It will probably surprise many engineers even to learn that this new Illinois Central locomotive is more than twice as powerful as the largest locomotives of fifteen years ago. *The fact is that railroading back in the '70's and European railroad practice even at the present day is toy railroading in comparison with the work now being done on American railways of heavy traffic.**

We have recently presented facts as to the very low rates which have been charged for bulk freights on some American railways. In our issue of Aug. 17, we showed, for example, that the average rate of the Chesapeake & Ohio R. R. for all coal shipped to the seaboard in the year ending June 30, 1899, averaged only 2·21 mills per ton-mile. It is sometimes argued, however, that these rates really represent business carried at a loss. It is said that railways carry at less than cost traffic of this class, and make both ends meet by charging exorbitant rates on high-class merchandise. It is our belief that this is far from being the case, and that the most profitable business which the railway can handle, provided only that it is obtainable in sufficiently great volume, is the movement of through bulk freights in the heaviest possible train loads. The cost of handling traffic of this class can be very closely ascertained, and the profit upon it can, therefore, be accurately known, while the cost of handling small shipments of high-class goods is made up of such a multiplicity of items that only an approximate idea of its total amount can ever be reached.

We have shown above that the railway can carry freight at less cost than any inland waterway, river or canal. The attempts to revive water transportation in the face of rail competition are doomed, therefore, to only a temporary success at best. In the end the law of the "survival of the fittest" will prevail, and it will never be the steel roadway that will be given up. On the contrary, we have directly before us in our old time canals, the last of which are now being abandoned, practical evidence of the railway's economic superiority.

The last argument of the defender of the waterways, however, is that these must be kept open in order that their competition may keep railway charges down to a reasonable figure. We are told again and again of the influence which the Erie Canal has exerted in the past as a regulator of railway tariffs, and it is said that it would be profitable for the public to keep natural and artificial waterways open, even if not a pound of freight were carried upon them, merely for the sake of their influence in lowering railway rates.

Such a policy, however, would be extremely short-sighted. The railways have passed the point where the competition of any artificial waterway is of serious moment to them; and that an abandoned waterway, with no boats moving upon it, could have any serious influence upon railway rates is a proposition absurd upon its face.

Besides this, however, it is, as a matter of fact, against the public interest that traffic should be diverted from the railways. The right of the Government to fix and regulate railway rates has been fully established, and the principle that such rates should not be in excess of what is necessary to pay the operating expenses, and a fair return on the capital invested, has also been plainly laid down in court decisions. The proper method to regulate railway rates, therefore, is not by trying to create competition, either by building parallel railway lines or inferior water routes, but by the direct exercise of Government control. With this effected it will be for the direct interest of the public, as well as the railway owner, to preserve and increase the volume of traffic moved by rail. Diversion of traffic to waterways under these circumstances, means higher rates for freight moved by rail, since the cost of maintenance of way and fixed charges has to be borne by a smaller volume of traffic.—(New York) *Eng. News: 26, October 1899.* [For another view, however, see Appendix, p. 57.]

[See also "Chemins de fer: notions générales et économiques:" Léon Leygue. Baudry: 1892.—Chap. IV: Comparaison entre les chemins de fer, les routes et les voies de navigation intérieure.—Tr.]

[* The italics are inserted.—Tr.]

§ 7.

The Railway Area of Influence.

The accessibility of railways is distinguished from that of roads and water-ways—which latter are, in general, accessible at every point in their length—by the circumstance that the former only permit the arrival and departure of traffic at fixed individual points in their length, termed stations.

Suppose a railroad MAB, on which there are stations A, B, C, etc., radiates from a market-centre M—**Fig. 7**. If the rate of carriage on the railway be f_2 per tonne-km. and the cost of loading and unloading per tonne is u then the cost of transport to the market-centre from any point P—the merchandise meeting the railway at A—is

$$u + f_2 a + f . AP.$$

And for a direct journey to the market by common road it is

$$f . MP.$$

For any point P in the boundary of the area served by the station A we have, therefore, the condition

$$f . MP = u + f_2 a + f . AP$$

or

$$MP - AP = \frac{u + f_2 a}{f}$$

The boundary line, DPE, of the station-area—or area feeding the station—is therefore a hyperbola: and similarly, the boundary-line, FGH, of the station-areas A and B.

If the charge for carriage on railways is 4 pf. per tonne-km., and the distance of the first station, A, from the market is 6 km., and if the cost of the loading and unloading is put at 150 pf. then we obtain the distance, MD, up to which the railway is unused from the equation

$$150 + 6 \times 4 + (6 - MD)\, 80 = MD . 80$$

whence

$$MD = 4 \text{ kms.}$$

If, however, a cart-road ran parallel to the railway then the first station A would not be used at all for goods traffic: F would be the point lying between the first station A and second station B— the latter 12 km. distant from the market-centre M— at which the traffic would first turn to this latter station and take to the railway instead of going direct as heretofore to the market-centre by the parallel cart-road.

For the distance, MF, of this point we have

$$150 + 12 \times 4 + (12 - MF)\, 20 = 20\, MF.$$

whence $$MF = 11 \text{ kms.}$$

Thus railways and canals cannot replace the country-roads for small market-traffic; they are only of utility for the market-business of large towns of which the feeder area is large.

For every 1,000 of the 12,971,554 inhabitants of the 612 towns in Germany having having a population exceeding 5,000 there is, on an average for the entire area of Germany, 41·7 sq. km. So that a town of 10,000 inhabitants will have on an average a business-producing area of 12¼ km. radius; whence it follows that, in general, only at towns of this size is a railway of any utility for the petty or market-traffic.

The greater economic value of canals and railways is based on their handling of bulk-traffic; and this the gradual reduction of the cost of carriage has carried to an extraordinary degree of development.*

[* See § 6.—Tr.]

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Fig. 7.

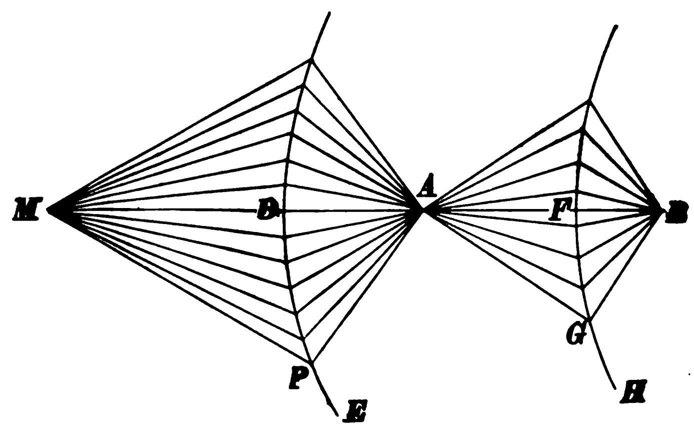


Fig. 8.

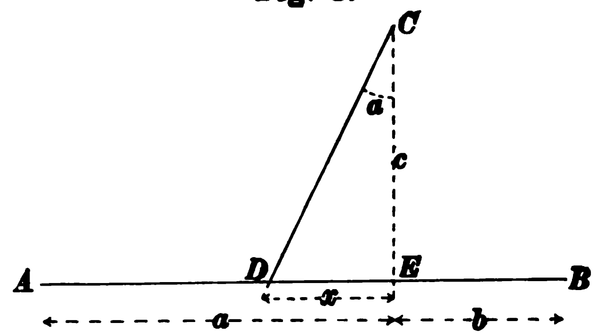
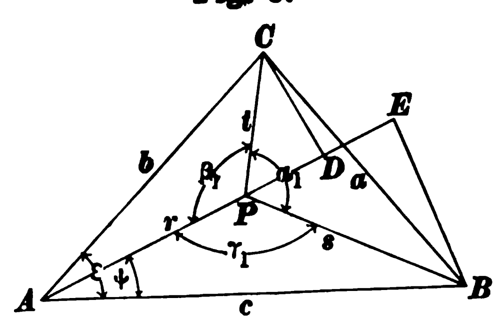


Fig. 9.



If a commodity will bear an outlay on transportation of v, then for a rate f it can be despatched, radially, up to a distance $\frac{v}{f}$, and consequently possesses a distribution area $=\frac{v^2\pi}{f^2}$. As the volume of traffic transported is proportional to the area of distribution it follows that the volume of the goods-traffic increases inversely as the square of the rate of carriage, and the number of tonne-kms. increases inversely as its cube.

The introduction of railways, which now (1886) carry goods at an average cost of 4 pf. per tonne-km., should accordingly increase the volume of merchandise transported 25-fold relatively to the road cart-traffic working at a rate of 20 pf; and for certain descriptions of bulky goods for which the railway charges little more than 2 pf. per tonne-km. the volume of traffic might even be increased a hundred-fold. Certainly for all classes of goods the increase of traffic would not be in this proportion either because there were not enough of it or because its transport-distance were diminished through the competition of neighbouring *foci* of production, or owing to the levying of octrois or tolls, etc.

The assumption of a uniformly distributed density of traffic or of one distributed according to some definite law over the land-area, which has been thus far made use of, is only permissible so long as it is merely a question of determining the general laws of traffic. For definite problems of location the above assumption can only be employed for the locating of forest- and field-roads. In the more complex and far-reaching problems of the location of streets, canals, and railways, the fact is to be reckoned with that the inhabitants and their consequent traffic requirements are concentrated in groups in the individual localities.

The Commercial Trace must therefore be determined according to the position of the various localities under consideration and to the importance of their traffic.

§ 7a.

The Principle of Junctions.

The simplest problem in Commercial Location is the determination of the best line of connexion of a locality with an already existing route of traffic.

If C—**Fig. 8**—be such a locality and $A\ B$ the pre-existing route of traffic, the problem is to fix the point D in which the connecting-line or road, $C\ D$, shall most advantageously debouch in $A\ B$; or, in order words, to determine the angle α at which this connecting line should deviate from the perpendicular $C\ E$.

If the probable volume of merchandise coming to and going from C annually is C tonnes, and the rate of carriage on the junction-line, $C\ D$, is f; and if of this volume C a part A goes in the direction from D to A and another part, B, of the same from D to B; and if the rate of carriage on $A\ B = f_1$ per tonne-km., then the annual traffic cost is

$$Cf\ (c^2 + x^2)^{\frac{1}{2}} + A\,f_1\ (a - x) + Bf_1\ (b + x).$$

If the kilometric construction-cost of the line $C\ D$ be K, the cost of its maintenance per km. $U + \beta\ C$, then, at a rate of interest i, the total construction- and maintenance-expenses per annum are,

$$(Ki + U + \beta\ C)\ (c^2 + x^2)^{\frac{1}{2}} + \beta_1\ A\ (a - x) + \beta_1\ B\ (b + x).$$

The sum of the traffic- and construction-expenses is therefore

$$S = [Ki + U + (\beta + f)\ C]\ (c^2 + x^2)^{\frac{1}{2}} + (\beta_1 + f_1)\ A\ (a - x) + (\beta_1 + f_1)\,B\,(b + x) \quad \ldots(12)$$

This is a minimum when

$$[Ki + U + (\beta + f)\ C]\ \frac{x}{(c^2 + x^2)}\ ^{\frac{1}{2}} + (B - A)\ (\beta_1 + f_1) = 0.$$

And since

$$\frac{x}{(c^2 + x^2)}\ ^{\frac{1}{2}} = \sin\alpha$$

$$\therefore \quad \sin\alpha = \frac{(A - B)\ (\beta_1 + f_1)}{Ki + U + (\beta + f)\ C} \quad \ldots \quad \ldots(13)$$

To illustrate the above, let a numerical example be taken.

At a perpendicular distance, $c = 2$ km. from a road $A\ B$ there is a village C of 1,000 inhabitants from which a road is to be made connecting the different districts with the main road. Let K be $= 5000$, $U = 100$, $\beta = {\cdot}02$, $f = {\cdot}5$. It is assumed that 1,500 tonnes will be annually transported on the road, of which 1,300 moves in the direction A, and 200 in the direction B. For the road $A\ B$, $\beta_1 = {\cdot}03$ and $f_1 = {\cdot}20$, so that for a rate of interest at 4°/₀, *i.e.*, $i = {\cdot}04$, we obtain

$$\sin\alpha = \frac{(1300 - 200) \times {\cdot}23}{5000 \times {\cdot}04 + 100 + 1500 \times {\cdot}52} = {\cdot}234.$$

Accordingly, the distance x from the point of rectangular junction will figure out to ·482 km. and the length of the road itself to 2058m.

Omitting the amount $(\beta_1 + f_1)\,.(A\,a + B\,b)$—constant for any direction of the junction-road—the total expenses for this position of it is

$$S = (5000 \times {\cdot}04 + 100 + 1500 \times {\cdot}52)\ 2{\cdot}058 - (1300 - 200) \times {\cdot}23 \times {\cdot}482 = 2{,}101 \text{ M.}$$

If the junction were made perpendicularly to the road the cost would be 2,160 M. or 50 M. greater than the best or theoretic position.

If the junction-road be metalled at an outlay of 14,000 M. per km. then the most advantageous direction is given by

$$\sin\alpha = \frac{(1300 - 200) \times {\cdot}23}{14000 \times {\cdot}04 + 150 + 1500 \times {\cdot}23} = {\cdot}24$$

and is consequently, considerably greater than for an unmetalled connecting-road.

In certain cases two connecting-roads, may be more advantageous than one, the one going towards A and the other towards B.

If in the present instance, the road towards A were paved, and the other towards B unpaved, then the distance of the junction from the foot of the perpendicular would be

$$\sin\alpha = \frac{1300 \times \cdot 23}{14000 \times \cdot 04 + 150 + 1300 \times \cdot 23} = \cdot 296$$

and

$$\sin\alpha_1 = \frac{200 \times \cdot 23}{5000 \times \cdot 04 + 100 + 200 \times \cdot 52} = \cdot 114.$$

But such a duplex arrangement would be dearer than a single road, the traffic being so small.

In the case of an approach-road to a railway the distance between the theoretic best junction and the perpendicular one represented above by x is always so small that its calculation is objectless.

Much more important is the direction of the connecting-road in the case of the junction of a railway branch-line with a pre-existing line.

For example, suppose the cost of construction of the branch-line is 60,000 M. per km.; the annual maintenance and supervision expense per km. 2,000 M.; the working-expenses per tonne-km. ·02 M. Then with a traffic of 80,000 tonnes, of which 70,000 tonnes moves towards A, and the remainder towards B, we have for determinating the commercially most suitable junction-point

$$\sin\alpha = \frac{(70000 - 10000)\cdot 02}{60000 \times \cdot 04 + 2000 + 80000 \times \cdot 02} = \cdot 2.$$

In most cases it is preferable, in order to obviate the expense of a new station, or to avoid the debouching of the branch-line into the main-line between stations, to take the branch-line to the station lying nearest to the theoretically most favourable junction-point.

§ 8.

The Principle of Nodes.

If the line connecting two places A and B—**Fig. 9**—with which points a third place, C, is to be connected—is not as yet built it is preferable to make the route from A to B not rectilinear but as $A\,P\,B$, thereby joining the line C to P. The position of the node P is fixed from the consideration that the sum of the construction- and working-costs shall be a minimum.

If on the three rays $A\,P$, $B\,P$, $C\,P$, respectively, the annual traffic be A, B, and C tonnes, carried at kilometric-rates of f, f_1, f_2, M.; and if, further, the kilometric construction-cost for the three rays be K, K_1, K_2, and the kilometric maintenance-expenses be U, U_1, U_2, then the sum of the kilometric construction- and working-expenses along the line $A\,P$ is

$$A_1 = K\,i + U + f\,A$$

and on the line $B\,P$

$$B_1 = K_1\,i + U_1 + f_1\,B$$

and, similarly, on $C\,P$

$$C_1 = K_2\,i + U_2 + f_2\,C.$$

According to the figuring shown in the Fig. the sum of the construction- and working-expenses on all three lines is therefore

$$S = A_1\,r + B_1\,s + C_1\,t$$

or,

$$S = A_1\,r + B_1\,(r^2 + c^2 - 2\,r\,c\cos\phi)^{\frac{1}{2}} + C_1\,(r^2 + b^2 - 2\,r\,b\cos(\epsilon - \phi))^{\frac{1}{2}} \quad .. \quad (14)$$

The position of the node P is determined by the two variables r and ϕ in this equation. To obtain the conditions for the most advantageous position of P we differentiate the equation with respect to each of these variables and equate to zero.

Differentiating first with respect to r and equating to zero we obtain

$$A_1 + B_1\,\frac{r - c\cos\phi}{(r^2 + c^2 - 2\,r\,c\cos\phi)^{\frac{1}{2}}} + C_1\,\frac{r - b\cos(\epsilon - \phi)}{(r^2 + b^2 - 2\,r\,b\cos(\epsilon - \phi))^{\frac{1}{2}}} = 0,$$

and putting

$$(r^2 + c^2 - 2\,r\,c\cos\phi)^{\frac{1}{2}} = s,$$

$$(r^2 + b^2 - 2\,r\,b\cos(\epsilon - \phi))^{\frac{1}{2}} = t,$$

and noting that

$$c\cos\phi - r = P\,E, \quad \text{and} \quad b\cos(\epsilon - \phi) - r = P\,D,$$

then

$$A_1 - B_1\,\frac{P\,E}{r} - C_1\,\frac{P\,D}{t} = 0,$$

or

$$A_1 + B_1\cos\gamma_1 + C_1\cos\beta_1 = 0. \quad \ldots \quad \ldots \quad (15)$$

A second equation between the expenses A_1, B_1, and C_1, and the node-angles α_1, β_1, γ_1, is obtained by differentiating the equation for S with respect to ϕ and equating it to zero; we then obtain

$$B_1\,\frac{r\,c\sin\phi}{(r^2 + c^2 - 2\,r\,c\cos\phi)^{\frac{1}{2}}} - C_1\,\frac{r\,b\sin(\epsilon - \phi)}{(r^2 + b^2 - 2\,r\,b\cos(\epsilon - \phi))^{\frac{1}{2}}} = 0$$

or

$$B_1\,\frac{E\,B}{s} = C_1\,\frac{C\,D}{t}$$

viz.

$$B_1\sin\gamma_1 = C_1\sin\beta_1 \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

Fig. 10.

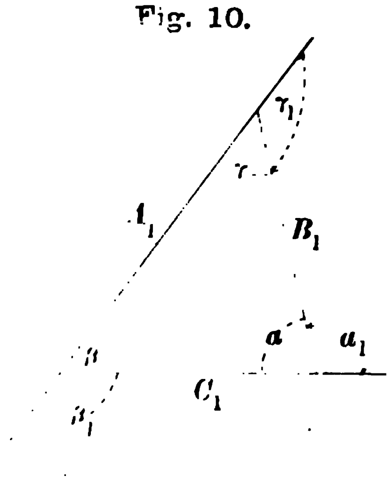


Fig. 11.

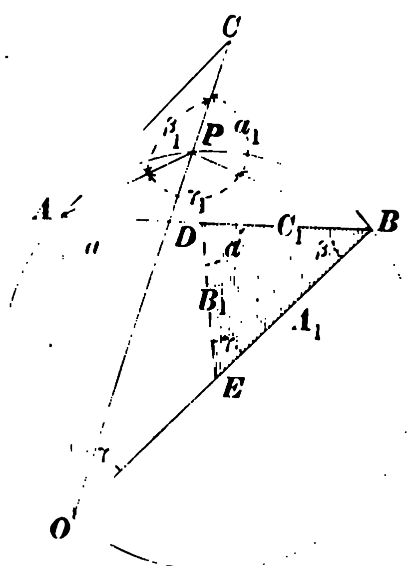
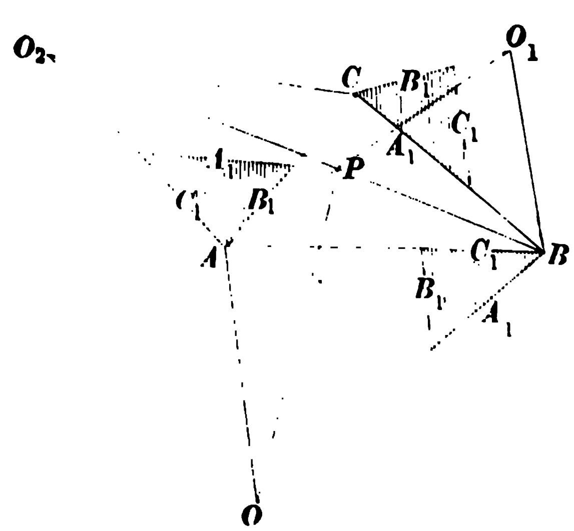


Fig. 12.



The above two relations between the kilometric-costs and the node-angles which must be satisfied for the best position of the node are precisely those which hold between the sides of a triangle and its exterior angles. Accordingly, if we form a triangle of the kilometric construction- and working-expenses A_1, B_1, C_1,—**Fig. 10**—its exterior angles are the node-angles required.

The condition for the best position of the node can accordingly be expressed thus:— **The sines of the angles at the node must be proportional to the kilometric construction- and working-expenses represented by the three rays from the node**: or: At the node there must be equilibrium between the three forces of which the magnitudes are proportional to the kilometric construction- and working-expenses, and acting in the direction of the three rays.

Mechanically—neglecting friction—the best position of the node would be obtained if the position of the three places A, B, C, were marked on a horizontal sheet—say, of thin metal, holes bored at these points and threads passed through the holes and knotted together on the upper-side of the sheet, and on the under-side, loaded with weights proportional to the kilometric construction- and working-expenses. The node or knot would then be brought into the correct position under the action simply of the three weights.

Geometrically, the best position of the node may be determined as follows:—

On any one of the three sides of the triangle $A\,B\,C$—**Fig. 11**—describe a triangle of the kilometric traffic-expenses B, D, E, in the manner shown in the Fig. so that the kilometric-costs lie opposite to the angular points to which they correspond. Draw $A\,O$ parallel to $D\,E$, produce $B\,E$, and we obtain, on a larger scale, a similar triangle $A\,B\,O$ of the kilometric-costs, of which *the apex* O is termed **the Pole.** Draw from the pole O towards C, the line $O\,C$; then the point of intersection P of this line with the circle circumscribed about the triangle $A\,B\,O$ gives the position of the node.

For the traffic proceeding from C the pole O replaces the two points A and B according to the direction; so that this traffic must follow the line $C\,O$, but only up to the node-circle $A\,P\,B\,O$, beyond which the traffic must divide towards A and B.

The position of the pole determines not only the direction but also the amount of the expenses. The total cost of working the traffic between the three places A, B, C, is

$$S = A\,P.\,A_1 + B\,P.\,B_1 + C\,P.\,C_1$$

Taking the unit in which these costs are represented in the triangle $A\,B\,O$ as the unit of kilometric-cost we may put

$$S = A\,P.\,B\,O + B\,P.\,A\,O + C\,P.\,A\,B$$

or by the theorem of Ptolemy, according to which the sum of the products of the opposite sides of a quadrilateral inscribed in a circle is equal to the products of the diagonals,

$$S = P\,O.\,A\,B + C\,P.\,A\,B$$

or

$$S = C\,O.\,A\,B = C\,O.\,C_1$$

That is, **the sum of the construction- and working-expenses for the traffic between the three places A, B, C, is the same as if the out-going traffic from one place were carried in a straight line up to the pole of the other two places.**

The lines drawn from each of the three places to the pole which replaces, cyclically, the other two remaining places, must intersect in the node P—**Fig. 12**: so that this latter may be determined by the intersection of two such pole-lines. To determine the pole the triangle of the kilometric traffic-expenses must be so placed on, or applied to, the triangle formed by the three places that each place lies opposite to that side of the triangle of

kilometric-costs corresponding to the traffic issuing from that place; and the kilometric-costs triangle must be drawn to such a scale that its side, lying on a side of the triangle formed by the places A, B, C, is of the same length as, or is coincident with, this latter side.

So long as the kilometric traffic-expenses remain constant the pole of any two of the three places remains unchanged in position; as, for example, the pole O of A and B, wherever the place C may happen to lie, if it lies at all within the angle $A\,O\,B$; so that—as represented in **Fig. 13**—the traffic from the point C—variable according to the position of C within this boundary—will always proceed towards O.

If C lies on the other side of the line $A\,B$, then we have a symmetrically-lying pole O_1 to which the traffic from all points lying within the angle $A\,O\,B$ will proceed.

If the point C lies within the angle $A_1\,A\,A_2$ there is no node: the traffic then goes from this place direct to A, and thence to B. Similarly, the traffic from all points lying inside the angle $B_1\,B\,B_2$ will proceed directly to B, and thence to A. Also, if the point C lies within the two arcs $A\,P\,B$ there will be no node, and the traffic will proceed by the line $A\,C\,B$.

In illustration of the foregoing principle take the following.

There is an annual traffic between two places, A and B, amounting to 320,000 tonnes; between A and C one of 30,000 tonnes; and between B and C one of 10,000 tonnes. Then on the introduction of the node P there will be in the ray $A\,P$, a traffic of 350,000 tonnes, in the ray $B\,P$ one of 330,000 tonnes, and in the ray $C\,P$ one of 40,000 tonnes.

If a railway is constructed on the line $A\,P\,B$, of which the kilometric construction-cost is 100,000 M., and of which the annual maintenance and supervision per km. amount to 2,300 M., and on which the working-expenses are ·02 M. per tonne-km., then the kilometric construction- and working-expenses are

$$A_1 = 100{,}000 \times \cdot 04 + 2{,}300 + 350{,}000 \times \cdot 02 = 13{,}300 \text{ M.}$$
$$B_1 = 100{,}000 \times \cdot 04 + 2{,}300 + 330{,}000 \times \cdot 02 = 12{,}900 \text{ M.}$$

If the ray $C\,P$ is a cart-road built at a cost per km. of 15,000 M., and if its maintenance-expenses per km. are $150 + \cdot 03C$ with a freight-rate of ·2 M. per tonne-km., then

$$C_1 = 15{,}000 \times \cdot 04 + 150 + 40{,}000\ (\cdot 03 + \cdot 20) = 9{,}950 \text{ M.}$$

Let the distance apart of the three places be $A\,B = 15$ km., $A\,C = 9$ km., $B\,C = 10$ km.; then, after making the geometrical construction above described, the ray $A\,P$ will be found to be 7·56 km., $B\,P = 8{\cdot}67$ km., $C\,P = 2{\cdot}72$ km.; and the node P has a perpendicular distance = 3·08 km., from the rectilinear line of connexion $A\,B$.

The total traffic-expenses on the three rays are accordingly,

$$\begin{aligned} 7{\cdot}56 \times 13{,}300 &= 100{,}548 \\ +\,8{\cdot}67 \times 12{,}900 &= 111{,}843 \\ +\,2{\cdot}72 \times 9{,}950 &= 27{,}064 \\ \hline \text{Total} &= 239{,}455 \text{ M.} \end{aligned}$$

The pole O is distant 24·07 km. from C; whence the total traffic-expenses amount to $24{\cdot}07 \times 9950 = 239{,}497$ M.

It is worthy of note, that the most advantageous position of the node determined in this way may vary *very considerably* without the total traffic-expenses becoming thereby notably increased. Thus, for example, if the node were fixed at 2 km. instead of 3·08 km. from the line $A\,B$, so that the lengths of the three rays $A\,P$, $B\,P$, $C\,P$, were respectively 7·28, 8·25, and 3·93 km. respectively, then the whole traffic-expenses would amount to

$$\begin{aligned} 7{\cdot}28 \times 13{,}300 &= 96{,}824 \\ +\,8{\cdot}25 \times 12{,}900 &= 106{,}424 \\ +\,3{\cdot}93 \times 9{,}950 &= 39{,}104 \\ \hline \text{Total} &= 242{,}352 \text{ M.} \end{aligned}$$

or about $1\frac{1}{4}$ °/₀ greater than for the best possible position.

In the case of a very heavy traffic and very low construction-cost it is possible that the node arrangement even for the most favourable location of a node may be more disadvantageous financially than the building and working of a line on the sides of the triangle; since the *existence* of a node increases, under all circumstances, the working-expenses.

To put this matter more clearly, let it be assumed that the three places A, B, C, lie at the ends of the sides of an equilateral triangle of a side unity; and that the construction-

cost per km. = A, and that the traffic between every two of these places is in each case Q. Supposing the three sides of the triangle were railroads then the total working-expenses for a rate of interest i, and a transport-unit rate, are

$$S = 3\,(A\,i + f\,Q)$$

By the establishment of a node the three rays from the places to the node have a total length of 1·732, and there is a traffic $2\,Q$ on each ray; so that the total traffic-expenses are

$$S_1 = 1{\cdot}732\,(A\,i + 2\,f\,Q).$$

Now the establishment of a node is only advantageous so long as

$$A\,i > {\cdot}37\,f\,Q.$$

For a kilometric outlay of 140,000 M. a rate of interest of ·05, and working-expenses $f = {\cdot}02$ per km., the *volume* of traffic Q must be not less than 946,000 tonnes, if the existence of the node is to justify itself.

If the railroad be constructed on the sides of the triangle the outlay on a junction station at the node is saved; nevertheless, we see that only in the case of a very considerable volume of traffic would it be worth while to construct the road on the sides of the triangle in place of the node-arrangement.*

* The Principle of the Node can be also employed to determine the best position of an industrial location—for example, of an iron smelting furnace, when the iron ore is obtained from A, the coal from B, and the pig-iron has to be despatched to C. On this point see Launhardt: "Zeitschrift des Vereins deutscher Ingenieure," 1882.

[For further illustration, see the numerical example of the practical application of Commercial Tracing, Appendix, p. 78—Tr.]

§ 9.
The Commercial Trace.

If a line of communication is to be laid down between two points A and B, whether it be a road, a canal, or a railway, by which a traffic of any given amount is to be distributed laterally to the localities laying on either side of it, then its Commercial Trace will form a chain of straight lines of varying directions from the nodes of which the connecting lines will proceed to the laterally out-lying localities.

For each individual node—**Fig. 14**—the conditions laid down by the Principle of Nodes must be fulfilled, namely, that the kilometric traffic-expenses on each of the three lines meeting in a node, conceived of as forces proportionate in magnitude to their amounts acting in their respective directions, must produce equilibrium at the node. Accordingly, we obtain at once a simple mechanical procedure for the determination of the Commercial Trace. Plot the locations or points in their respective positions on a thin plate, say, of metal, perforate the plate in these points, pass the ends of a thread through the holes in the end-points of the line, and load them with weights equivalent to the kilometric traffic-cost of the end-sections. At the same time, through all the holes representing the intermediate localities pass other threads, and load their lower ends with weights proportionate to the kilometric traffic-cost of the branch-lines, and attach the other ends of the threads to rings capable of sliding along the above-mentioned thread stretched from end-point to end-point. The final position assumed by the threads due to the action of the various attached weights will represent the Commercial Trace.

Geometrically, the Commercial Trace may be determined as follows.

The kilometric traffic-expenses are calculated for all the branch-lines, and for all the individual intermediate sections of the main-line, from node to node, and therewith a polygon of the kilometric traffic-expenses is constructed—**Fig. 15**—in which the rays meeting in any point represent the kilometric-expenses of the individual sections of the main-line: the lines joining the ends of the rays represent the kilometric-expenses of the branch-lines from the intermediate points. The angle of each individual adjacent triangle is the supplement of the required angle at each node.

We begin then by determining the pole O for the terminal A and the next-lying intermediate place C, which then takes the place of the two localities A and C, so that the total number of the localities to be considered has been reduced by one. Then we again determine a new pole O, for the first pole and the next intermediate place D; thus again diminishing the total number of localities by one.

We continue in this way until only the end-point and one pole remain; or, more convenienly, we begin with the substitution of two places by a pole, starting from both ends of the Trace; so that step by step a pole is determined from both ends of the Trace. If then the appropriate node-circles be drawn for each of the poles—as in **Fig. 16**—and also the line connecting the two poles O_1 and O_2, we obtain the length P_1 P_2 between the node-circles as a part of the Trace. From P_2 we draw lines up to the node-circle in E and B, and from P_1 towards the pole O up to the junction with the node-circle P, from which we then draw the lines P C and PA. In this manner we obtain the Trace A P P_1 P_2 B, along with the branch-lines P C, P_1 D, and P_2 E.

If we are dealing with the Trace of a railway we have to determine beforehand which of the intermediate localities is to be connected with the main-line, either by a road or a branch-line; since on the determination of this point the whole position of the Trace depends.*

[* See Appendix, p. 78, for a numerical illustration of the determination of the Commercial Trace.—Tr.]

To face

Fig. 13.

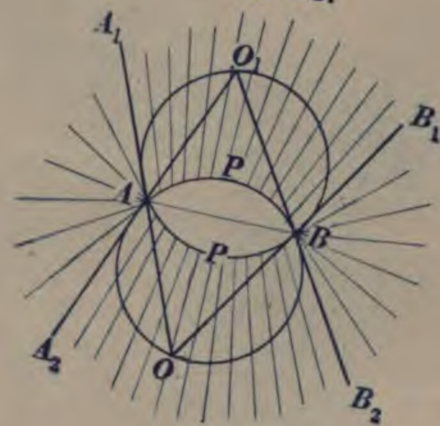


Fig. 14.

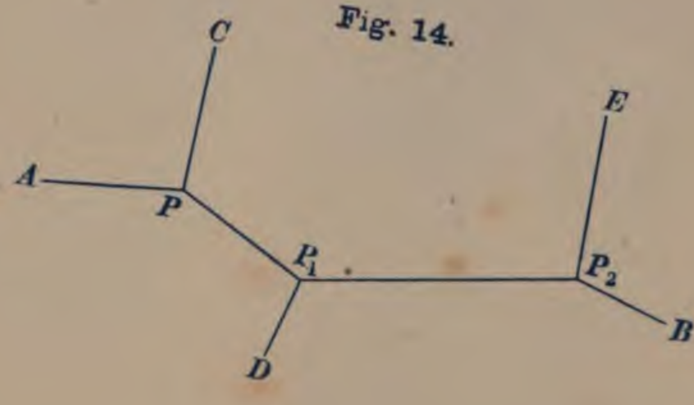


Fig. 15.

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Fig. 16.

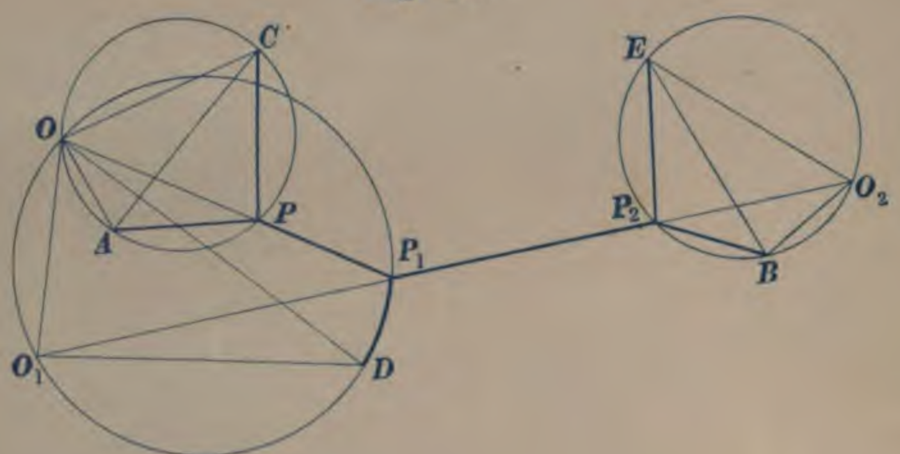
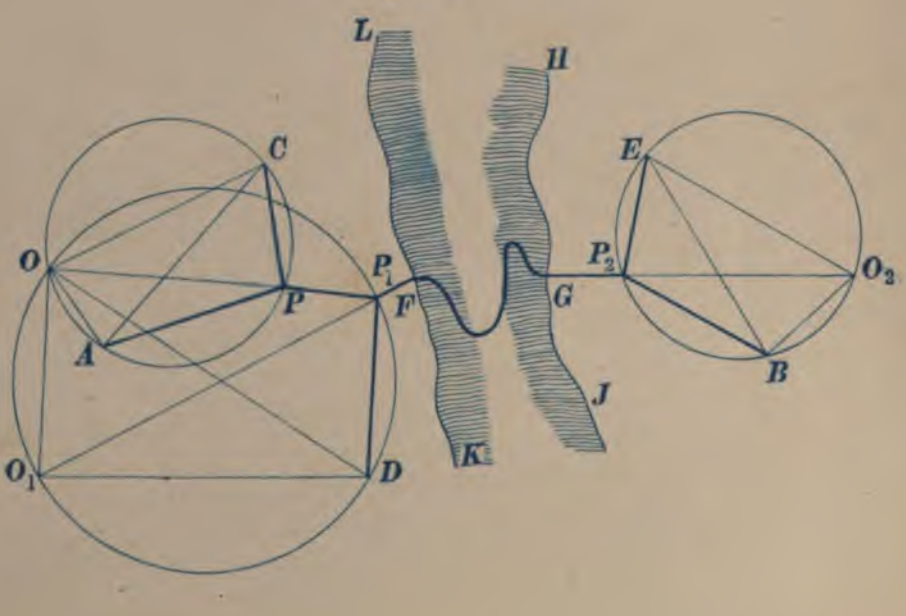


Fig. 17.



§ 10.
Technical "Fixed Points." Development-Area.

It is but rarely necessary to investigate the position of the Commercial Trace for any great distance nor for any large number of intermediate localities, since most usually in addition to the terminal points there will be intermediate points through which the Trace under any circumstance must of necessity pass. Such an initially fixed and obligatory intermediate point may be either a large town, a station on a line to be crossed by the Trace, an advantageous river-crossing, a mountain-pass, or the entrance to a mountain valley leading to such a pass, the shores of lakes, and so forth. The commercial trace then becomes divided up by such intermediate points—termed technical "**fixed**" or obligatory **points**—into sections for each of which the Trace has to be separately determined.

Of particular and especial importance for the position of the commercial trace,—for which, of course, a horizontal datum is postulated, as already explained—are mountains which have to be surmounted by means of an artificial lengthening of the line. Here the length of the trace within the limits of the **Development-Area** is dependent on the height to be surmounted and on the gradient on which this is to be done. The Commercial Trace in these circumstances will then be only sought for in the sections lying outside the development-area and will be fixed in such a way that the development-area is reached by the shortest route, viz., the Trace must be carried perpendicularly up to the boundary of the area in which development is to take place.

Suppose the five localities, for which the Commercial Trace in Fig. 16 has been already determined, to be divided between C and D, by a chain of hills of which the boundaries of the area within which the linear development of the line is possible are represented in **Fig. 17** by the lines $K\ F\ L$ and $J\ G\ H$. Then the Commercial Trace from the pole O_1, which latter has been determined for the three localities A, B, and C, must be drawn perpendicularly to the development-area boundary $K\ F\ L$, in the direction $O_1\ F$. From the intersection-point P_1 of $O_1\ F$ with the node-circle, the branch-route $C\ P_1$ must proceed in the direction $P_1\ O$ and be carried on up to its intersection, P, with the node-circle of the localities A and C, from where it will bifurcate towards A and C. Similarly, from the pole O_2 the line must be drawn perpendicular to the development boundary $J\ G\ H$, in the direction $O_2\ G$; and the trace then proceeds from G to P_2—the intersection with the node-circle of the localities D and E—and then must bifurcate towards these two places.

Comparing the trace thus obtained and represented in Fig. 17 with that in Fig. 16 we see what great changes in form the Commercial Trace may undergo through the occurrence of an intermediate chain of hills.

It is perhaps scarcely necessary to remark that a trace determined as above is rarely suitable for construction without further alteration. The development-area boundaries for a mountain chain can hardly be determined with any high degree of definiteness or precision: further, the given points of departure, F and G are not always the most advantageous for the purpose of development. Accordingly, after carrying out the above construction we shall have to examine how far, on **technical**, *i.e.*, constructional, grounds some alteration of these starting-points F and G, for the development would be advantageous; and then how far the altered points can still be regarded as technical fixed-points to which the trace is to be run from the pole replacing all the localities lying to one side of the development-area.

Also, when there is a branch-line to the sea-coast, or to the shore of a lake, at which merchandise has to be loaded on to ships or ferried across, the pole has to be determined for all the localities coming into consideration; and the trace then drawn from the pole perpendicularly to the shore-line. It must then be determined whether the coast-point thus found is serviceable as a haven, or whether another place in the vicinity would not make a better one; in which case the pole-line of the trace is to be drawn to it.

§ 11.

Crossings.

If between the points A and B,—**Fig. 18**—each of which may possibly be the pole for a large number of places—the trace has to cross an already existing line of communication then the Commercial Trace is to be drawn from A to E and from B to F according to the principle of junctions developed in § 7, so that the piece $E\ F$ of the already existent line forms a portion of the trace to be projected.

If the line of communication $C\ D$ be a road, then the Commercial Trace is at once determined.

But if the line $C\ D$ be a railway, only in rare instances will it be advisable to build new stations at the junction points E and F. Most usually, the junction will be made at the next-adjoining station; or it may be even preferable to unite the two points E and F in a single point of crossing.

The best position of the point of crossing is to be determined by repeated graphical trials. For this purpose the traffic-expenses on the line $E\ F$ per km. are to be taken somewhat greater than they are in reality, thus causing the points of junction E and F to move closer to each other.

This arbitrary increasing of the kilometric traffic-expenses is to be continued until the two crossing-points finally coalesce and form one.

These crossing-points can also be determined mechanically. Thus, lay down the points A and B on a thin sheet, say, of metal, and perforate it in these points; fix a thin wire on which a ring works, to represent the line $C\ D$, and to this ring attach four threads and thread them through the four holes A, B, C, D, and attach a weight to each end corresponding to the kilometric traffic-expenses, etc.

The problem of the determination of a crossing-point may also occur in another form. It may, for instance, be required to draw the Commercial Trace between four places A, B, C, D, between which as yet no road-connexion of any kind exists, to a single crossing-point, K, of all four places together.

Mechanically, the solution of this problem is similarly very simply arrived at; the four threads are knotted together into one knot, and then at the sites of the four localities the threads are weighted, each with its corresponding kilometric traffic-costs.

By graphical construction, the point of crossing can only be fixed after repeated trials. The Commercial Trace with the two nodes E and F is first determined, and then the construction repeated with arbitrary increasings of the kilometric traffic-expenses for the connexion-line $E\ F$, thus producing a mutual approach of these nodes.

This experimental procedure is to be repeated, gradually increasing the kilometric costs, represented by the connexion-line $E\ F$, until the two nodes E and F coalesce in one.

§ 12.

Normal Working-Year: Normal Working-Expenses.

The mode of procedure laid down for the determination of the Commercial Trace is based on the assumption that the kilometric cost of traffic is known. This cost is composed of the interest on the capital invested, and the annual maintenance- and working-expenses. The construction-cost is determinable from an estimate with a sufficient degree of accuracy; but such is not the case with the maintenance-expenses; and as to the working-expenses, this involves an estimate of the traffic to be expected, to find a correct basis for which is very much more difficult. Further, the traffic does not remain the same for all time; but, apart from temporary variations, is, in general, gradually on the increase. When determining the Trace, which, humanly speaking, once fixed is fixed for ever, we must reckon with this gradual expansion of traffic and consequently, on a gradual increase of maintenance- and working-expenses which will be brought about by the probable rise in prices, if by nothing else. Neither the working-expenses of the first year nor those in the remoter future are the ones to be made use of. **A Normal traffic year,** and the **Normal working-expenses** of this Normal year must be determined; and it is this latter, which is a quantity invariable throughout all time, that is to be taken for the purpose of calculation as the equivalent of the actual and variable working-expenses.

It is easily seen that here we can only get at an approximation, since **the law of the increase of traffic is not known.** Only by means of the fictions of a Normal working-year and of Normal working-expenses can we obtain a foundation on which to base our otherwise entirely uncertain estimate, as the following investigation will more clearly show.

Let the maintenance- and working-expenses in the first and succeeding years be B, B_1, B_2, etc.; then since the working-expenses of the first year are paid for out of capital, the sum which at compound interest will defray the working-expenses for all time is given by

$$S = \frac{B_1}{1+i} + \frac{B_2}{(1+i)^2} + \dots\dots\dots$$

If the working-expenses are constant per annum, and $= B$, then their capitalized value is

$$S_0 = B\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \dots\dots\dots\right)$$
$$= \frac{B}{i}$$

On the other hand, if the working-expenses increase annually by a constant amount b, then we should have

$$S = \frac{B_0 + b}{1+i} + \frac{B_0 + 2b}{(1+i)^2} + \frac{B_0 + 3b}{(1+i)^3} + \dots\dots\dots$$
$$= \frac{B_0}{i} + \frac{b}{i^2}$$

Equating this sum to that for constant working-expenses B,

then

$$\frac{B}{i} = \frac{B_0}{i} + \frac{b}{i^2}$$

and we obtain for the **Normal working-expenses**

$$B = B_0 + \frac{b}{i} \qquad \dots \quad \dots \quad \dots \quad (17)$$

which actually occur in the n^{th} or Normal working-year

$$n = 1 + \frac{1}{i} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

Thus taking the rate of interest at 5%, the Normal year would be the 21st year, and for the rate of interest of 4% it would be the 26th year.

But if the working-expenses increase annually by a constant percentage $(1 + a)$, then the capitalized sum of the working-expenses would be

$$S = B_0 \left[\frac{1 + a}{1 + i} = \left(\frac{1 + a}{1 + i}\right)^2 + \left(\frac{1 + a}{1 + i}\right)^3 + \ldots\ldots\ldots\right]$$

$$= B_0 \left(\frac{1 + a}{1 + i}\right)$$

Equating this capitalized sum to that for constant working-expenses B_0

then $$\frac{B}{i} = B_0 \left(\frac{1 + a}{1 - i}\right)$$

Whence the normal working-expenses are

$$B = \frac{i\,(1 + a)}{1 - a}\,B_0 \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

Equating this sum of the normal working-expenses to that in the nth year, then

$$B_0\,(1 + a)^{n - 1} = B_0\,\frac{i\,(1 + a)}{1 - a}$$

or $$(1 + a)^{n - 2} = \frac{i}{i - a}$$

Whence the normal working-year is

$$n = 2 + \frac{\operatorname{logn} i - \operatorname{logn}\,(1 - a)}{\operatorname{logn}\,(1 + a)} \quad \ldots \quad \ldots \quad (20)$$

Taking the rate of interest i as ·05, then for $a = ·01$, we obtain the 24th year as the Normal year: while for $a = ·02$ it is the 28th: and for $a = ·03$, the 33rd, etc.

Manifestly, the assumption that the working-expenses increase from year to year by a fixed amount is a more probable one than the first one—according to which the annual increase of the working-expenses is constant.

However, a determination of the law of the increase of the working-expenses dependent as they are on the volume of the traffic and the heights of prices is quite impossible. The value of the preceding investigtion is consequently limited to exhibiting the fact that in locating the Trace neither the working-expenses of the first year nor those in even the more distant years can form the real basis for the calculation of the working-expenses.

We may *fairly assume,* however, that the Normal working-expenses occur somewhere between the 20th and 30th year. In addition to the uncertainty of the estimate is to be added the fact that we cannot reckon on an invariable rate of interest. But uncertainties of this kind enter into all economic problems in which the future has to be taken into account.

§ 13.

Railway Rates.

In location problems it is the actual cost of working, and not the charge to be levied for transport, which is a quite different thing, that is to be taken into consideration. Nevertheless, the rates *are* to a certain degree dependent on the amount of the working-expenses; and consequently, on the character of the location.

In the transport of merchandise by land or water-ways of communication, both of which are at the service of everybody, the freight-charges are brought down through the competition of carriers to a figure—only slightly exceeding the working-expenses—at which the financially weakest party still finds sufficient inducement to carry on the business.

On railways this automatic regulation of rates is not possible; because each line can only be practicably worked by a single Administration. Although there may be several lines available for the despatch of goods to great distances yet even when the lines available belong to several companies the field of competition is still very limited, and the competition will ultimately terminate in the mutual combination of the competitors. The lines which might have competed with each other become by such combination monopolists as regards the exploitation of the traffic.

The most favourable rates for such lines under these circumstances are determined in the following manner.

A commodity which is produced at a cost p, and is sold at a price m, has a **transport-value** $v = m - p$. The maximum **transport-distance** of this commodity at a charge f, per unit of distance and weight, namely per tonne-km. is

$$r = \frac{v}{f}.$$

Thus at the place of consumption this commodity can be supplied by a **market-district** of the area

$$\pi r^2 = \frac{\pi v^2}{f^2}$$

and if γ units of goods are produced per unit of this market-area, the supply will amount to

$$\gamma \pi \frac{v^2}{f^2}.$$

The number of tonne-kms. performed in this supply is therefore

$$V = 2\gamma\pi \int_0^r x^2\, dx = \frac{2}{3}\gamma\pi r^3 \qquad \ldots \qquad \ldots \qquad \ldots \quad (21)$$

If the working-expenses be f_0 per tonne-km. and a freight f is charged to the public there will then be a net-profit of

$$U = (f - f_0)\, V$$

or inserting the values of V and r,

$$U = \frac{2}{3}\gamma\pi v^3 \; \frac{f - f_0}{f^3} \quad \ldots \qquad \ldots \qquad \ldots \quad (22)$$

We see at once that this is a maximum for $f = \frac{3}{2} f_0$

and thus

$$U_1 = \frac{8}{81}\frac{\gamma\pi v^3}{f_0^2} \qquad \ldots \qquad \ldots \qquad \ldots \quad (23)$$

or, for brevity, putting

$$\frac{\gamma\pi v^3}{f_0^2} = W,$$

the maximum gain or profit is

$$U_1 = \frac{8}{81} W \qquad \ldots \qquad \ldots \qquad \ldots \quad (24)$$

If instead of a feeder area or district of production from which an article of merchandise is despatched to a place of sale we consider an area of production from which the distribution of a commodity over an area of sale takes place we arrive at the same result, viz., that the maximum net gain is derived from transport when and if $f = \frac{3f_0}{2}$.

The very important principle disclosed by the above simple discussion may be stated thus:

For goods of which the transport-distance is limited solely by the amount of the rate of carriage, the maximum net return is reached when the rate charged is 1½ times the amount of the working-expenses.

Thus it is to the advantage of the Administration to raise the freight-charge to $\frac{3}{2} f_0$ and to work only two-thirds of the maximum transport-distance $\frac{v}{f_0}$ which would be possible if the rate were fixed at the working-expenses.

In that case the working-expenses are to be determined excluding the interest on the capital outlay when this latter is independent of the volume of the traffic.

In the foregoing investigation an equal density of traffic, γ, has been assumed for the whole extent of the traffic-area; whereas, in reality, this density as a rule must diminish with the increasing distance of transport. This fact does not, however, in the least affect the correctness of the above principle. In proof of this, take **Fig. 19**, where the transport-distances are measured on the axis of abscissœ, and the densities of traffic corresponding to the freight-charge f, whatever it may be, are plotted as ordinates of which the upper ends are bounded by a curve $A\ G\ E$: so that the traffic-density, $\gamma = F\ G$, corresponds to a transport-distance, $x = O\ F$, and consequently to a charge $p + fx$. If the freight-charge be diminished then the transport-area becomes enlarged: the same rate which was formerly levied for a distance OF now only is so for the distance $O\ D$; and therefore, the traffic-density $\gamma = F\,G$ now moves forward through the larger distance $O\,D$ until $\gamma = D\,B$. Equal ordinates of the curves $A\,G\,E$ and $A\,B\,C$, representing the traffic-densities for various freight-charges, correspond to abscissœ which are inversely proportional to the freight-charges. We can therefore assume instead of a commodity with variable traffic-density γ, a great number of different commodities having constant traffic-density $d\gamma$, but with different transport-values; as is represented by the horizontal lines in Fig. 19. For each of these various commodities, since they have constant traffic-densities, the principle just stated holds; *i.e.*, that the maximum profit will be attained by fixing the freight-charges at $\frac{3}{2}$ times the working-expenses. Consequently, this principle is equally true for the sum of all these commodities, which is thus the same as a single commodity with variable traffic-density.

But as all the directions radiating from the market-place are not equally traversable with the same facility and consequently, the working-expenses, and as a result thereof, the rates charged, are not the same in each case so also the traffic-density, depending on the rates charged, will not be the same at all points which are equally distant from the market-place. This circumstance does not affect the general validity of the above-stated principle, since the investigation leads to the same conclusion if it be applied to any selected sector of the circle instead of to the whole circular area.

The correctness of the principle of the maximum gain—viz., that the maximal net-profit is obtained when the rate is $\frac{3}{2}$ times the actual working-expenses—is based on the

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Fig. 18.

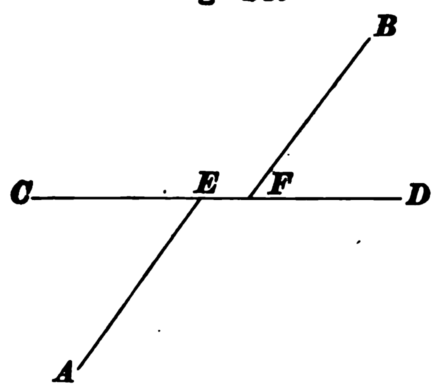
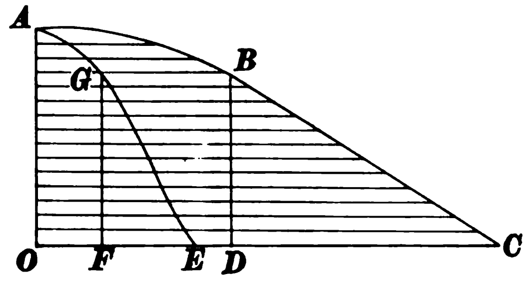


Fig. 19.



postulate that the maximum transport-distance of the commodity is not greater than the limits of the railway system of the individual Administration.

If the system of the Administration does not extend up to the extreme limit, r, of the transport-distance but only up to a distance r_1, then in that case the net-profit for a freight, f, would be

$$U = 2\gamma\pi(f - f_0)\left(\int_0^{r_1} x^2\,dx + r_1\int_{r_1}^{r} x.\,dx\right).$$

Consequently, $$U = \gamma\pi(f - f_0)\left(r^2 r_1 - \frac{r_1^3}{3}\right) \quad \ldots \quad \ldots \quad (25)$$

If we put in the above $r = \dfrac{v}{f}$, and differentiate with respect to f we obtain for the most advantageous freight-charge the condition

$$f^3 + 3\left(\frac{v}{r_1}\right)^2 f - 6\left(\frac{v}{r_1}\right)^2 f_0 = 0.$$

Now the extreme transport-distance is $r = \dfrac{v}{f}$, or $v = rf$; inserting this value, we obtain

$$f = \frac{6}{3 + \left(\frac{r_1}{r}\right)^2} f_0 \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

The extreme transport-distance r_1 on the same network may be anything between 0 and r, so that **the most favourable unit rate of carriage lies between** $f = 2f_0$ **and** $f = \frac{3}{2}f_0$, **according as the powers of the carrier extends over a small or a large system of lines.**

The fact here demonstrated, that **great railway Administrations have in their own interest to charge lower rates than smaller corporations,** is of considerable importance in its bearing on the solution of the question of State Ownership *versus* Private Ownership of Railways.

Of further interest in this connexion is the consideration of the best or most profitable freight-rates for Branch-Lines.

To take the simplest case, when the branch-line has but one station on it so that all goods are hauled over its whole length; then if its length be z, and the freight-charge f_2 there is a remainder $(v - f_2\,z)$ from the transport-value, v, of the goods on reaching the main-line available for the further transport of the merchandise thereon.

The volume of goods passing on to the main-line system is therefore

$$Q = \pi\gamma\left(\frac{v - f_2\,z}{f}\right)^2$$

where f is the freight-rate charged on the main-line system.

There is on the branch-line a gain of $(f_2 - f_0)$ per unit of length of the network of the main-line; so that the total net-profit of the branch-line for the goods in question is

$$U = Q\,(f_2 - f_0)\,z$$

or $$U = \frac{\gamma\,\pi\,z}{f^2}(f_2 - f_0)\,(v - f_2\,z)^2 \quad \ldots \quad \ldots \quad (27)$$

which becomes a maximum for

$$f_2 = \frac{2}{3}f_0 + \frac{v}{3z} \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

Consequently, the freight to be charged for the whole length of the branch-line is

$$f_2\,z = \frac{2}{3}f_0\,z + \frac{v}{3}.$$

Such a high rate as the above amounting to more than a third of the transport-value of the goods could not in practice be levied; since it would probably in most cases be greater than the cost of carriage by cart-road.

If the branch-line is not worked as a separate concern, but in common with the main-line network, then it will be will advantageous to the Administration to draw a large traffic from the branch-line on to its main-line.

The volume of traffic passing from the branch to the main-line network in tonne-kms. is

$$V = 2\gamma\pi \int_0^r x^2\,dx = \frac{2}{3}\gamma\pi r^3$$

wherein

$$r = \frac{v - f_2 z}{f}.$$

Adding the profit arising from this branch-line traffic to that on the branch itself, we have

$$U = \frac{\gamma\pi z}{f^2}(f_2 - f_0)\,(v - f_2 z)^2 + \frac{2}{3}\frac{\gamma\pi}{f^3}(f - f_0)\,(v - f_2 z)^3 \quad \dots \quad (29)$$

Differentiating with respect to f_2 we find that this is a maximum for

$$f_2 = \frac{2 f_0 v - f v + 2 f f_0 z}{z\,(f + 2 f_0)} \quad \dots \quad \dots \quad (30)$$

whence, for $f = \frac{3}{2}\,f_0$, we obtain

$$f_2 = \frac{6}{7} f_0 + \frac{v}{7z} \quad \dots \quad \dots \quad \dots \quad (31)$$

A comparison of this with the most profitable rate (Eqn. 28) for a branch worked as a separate concern brings out the notable fact that the freight-charge for a branch-line which is worked at the cost of the owners of the main-line must be fixed lower than when it is worked as a separate concern.

A great railway system can often afford to build branch-lines which although if worked for themselves do not yield a sufficient return yet pay by bringing additional business to the main-line. Consequently, the construction of branch-lines and the increase of the density of the system is promoted and facilitated by the unification of the whole system under a single Administration—as is the case with State-owned lines.

Finally, the case remains to be considered where the sale-area of commodiaty is lessened by the competition of neighbouring production-areas; as for example, the sale-area of the Westphalian coal by the coal from the Saar coal-fields; and also, where the demand-area of a mineral is contracted through the competition of neighbouring places of consumption. In these cases a general reduction of the freight-rate will *only slightly* alter the size of the traffic-area; and will only increase the volume of traffic in tonne-kms. in proportion as the reduction of the commodity-price induces an increased demand and, consequently, a greater traffic-density.

A calculation of the freight-charge, which shall produce the greatest net-profit, is in such cases only possible when and if we know the law governing the connexion between the commodity-price and the traffic-intensity.

If we arbitrarily assume this law, which it would be very difficult to determine exactly, to be expressed by

$$\gamma = \gamma_0\,(v - f x) \quad \dots \quad \dots \quad \dots \quad (32)$$

then for a market-area of constant radius r_0 the profit arising is

$$U = 2\,\pi\,\gamma_0\,(f - f_0)\int_0^{r_0}(v - f x)\,x^2\,d\,x$$

or

$$U = \pi\,\gamma_0\,r_0^{3}\,(f - f_0)\left(\frac{2}{3}\,v - \frac{f\,r_0}{2}\right) \qquad \dots \qquad \dots \quad (33)$$

This attains a maximum for a rate

$$f = \frac{f_0}{2} + \frac{2}{3}\,\frac{v}{r_0} \qquad \dots \qquad \dots \qquad \dots \quad (34)$$

The smaller the market-area the higher must be the unit-cost of carriage.

A market-area of radius $r_0 = \frac{2}{3}\frac{v}{f}$ corresponds to a freight-charge of $f = \frac{3 f_0}{2}$: and this area should not be exceeded if the maximum profit is to be attained, unless there occurred a contraction due to the presence of an adjacent market-place.

Although the law connecting the traffic-intensity with the price of commodities has been arbitrarily assumed nevertheless the preceding calculation shows that in traffic-areas restricted by the presence of neighbouring market-places, or areas of sale, the rate of carriage must always be fixed at a figure greater than $\frac{3 f_0}{2}$ in order to realise the maximum profit.*

[* For further remarks on the Formation of Rates—see Appendix, p. 74.—Tr.]

§ 14.

Differential Tariffs and Terminal Charges.

The preceding discussion shows that the most advantageous unit charge for carriage, *i.e.* the rate per unit of traffic from which the maximum net-profit is derived when the traffic-area is unlimited, is $\frac{3}{2}f_0$. Under all other circumstances, viz. when the extent of the area of a railway system is limited, or when the traffic-area is restricted by the presence of a neighbouring place of sale, or for branch-lines, the rate charged must be greater than $\frac{3f_0}{2}$.

When there is no attempt to make a profit, namely when the freight-rate is fixed at the working-expenses, then only two-thirds of the extreme transport-distance which would be thus possible would, under the most favourable circumstances, be exploited. It is then an easy matter to push the traffic beyond this limit by slightly lowering the rate for the extra distance beyond. These considerations, along with others which cannot here be discussed, lead up to the idea of the so-called **Differential Tariffs.**

If we abandon the principle of making the rate increase in proportion to the kilometric distance, *i.e.*, "mileage" rates, then the cost of carriage might conceivably be a charge entirely independent of the distance, as is the case with letter-postage. However, this idea cannot here be further developed.

With a logically carried-out differential tariff for a distance x the charge would be fixed from the expression $f\,x - f_1\,x^2$. The net-profit then would be

$$U = 2\,\pi\,\gamma \int_0^r (f - f_0 - f_1\,x)\,x^2\,d\,x \qquad \ldots \qquad \ldots \quad (35)$$

or

$$U = \gamma\,\pi\,r^3 \left(\frac{2}{3}\,(f - f_0) - \frac{f_1 r}{2}\right)$$

For the extreme transportation-distance r the rate is $(f r - f_1\,r^2)$: and if for this limit the working-expenses are $f_0\,r$, then the following equation holds

$$f_0\,r = f r - f_1\,r^2$$

whence we have

$$r = \frac{f - f_0}{f_1}$$

Giving r this value within the above brackets, we obtain

$$U = \frac{\gamma\,\pi\,r^3}{6}\,(f - f_0).$$

Since in this expression all the quantities are constant except f the net-profit will vary only with this latter. But since the maximum rate for the extreme transportation-distance occurs only when $r = \frac{f - f_0}{f_1}$, the quantity $x_1 = \frac{f}{2 f_1}$ for which the freight $f x - f_1\,x^2$ is a maximum, can at most $= r$: so that we obtain the equation

$$x_1 = \frac{f}{2 f_1} = r = \frac{f - f_0}{f_1}$$

whence

$$f = 2\,f_0$$

is the maximum value which f can attain.

If $f = 2\,f_0$ then $f_1 = \frac{f_0^{\,2}}{v}$: consequently, the expression for the rate is

$$2\,f_0\,x - \frac{f_0^{\,2}}{v}\,x^2 \qquad \ldots \qquad \ldots \qquad \ldots \quad (36)$$

The net-profit for the whole traffic-area consequently is

$$U_1 = 2\gamma\pi \int_0^{\frac{v}{f_0}} (2f_0 - \frac{f_0^2}{v}x - f_0)\, x^2\, dx$$

$$= \frac{\gamma\pi v^3}{6 f_0^2}$$

or
$$U_1 = \frac{W}{6} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (37)$$

A comparison of this with the profit represented by Eqn. 24, which is obtained when the freight-charge $f = \frac{3}{2} f_0$ is uniform for all distances, shows that **the above parabolic differential tariff yields a $1\frac{11}{16}$ times greater net-profit.**

Although the demand for goods will be somewhat diminished for the smaller distances and in consequence the net-profit will not quite attain the calculated amount, nevertheless the calculation shows so very distinctly **the advantage of a rate diminishing as the distance increases,** that the introduction of such a basis for fixing rates should be seriously considered.

If the law (expressed by Eqn. 32) of the intensity or volume of traffic diminishing as the price of commodities increases be taken as a basis for rates,
namely

$$\gamma = \gamma_0 (v + fx) = \gamma_0 \left((v - 2f_0 x + \frac{f_0^2}{v} x^2 \right)$$

then the profit for the whole extent of the traffic-area would be

$$U_1 = 2\pi\gamma_0 \int_0^{\frac{v}{f_0}} \left(v - 2f_0 x + \frac{f_0^2}{v} x^2 \right) \left(f_0 - \frac{f_0^2}{v} x \right) x\, dx$$

whence
$$U_1 = \frac{1}{30} \frac{\pi \gamma_0 v^4}{f_0^2}$$

or putting
$$W_1 = \frac{\pi\gamma v^4}{f_0^2}$$

then

$$U_1 = \frac{W_1}{30} \quad \dots \quad \dots \quad \dots \quad (38)$$

In all investigations as to the most profitable rate of carriage it has so far been tacitly assumed that the cost of the collection and delivery of the traffic—the so-called **terminal-charge**—is levied along with the kilometric charge in the form of a terminal-fee.

It now remains to investigate whether instead of fixing the terminal-charge at the actual cost of the collection and delivery of the traffic it would not be more profitable to fix it at some other figure, either larger or smaller.

In investigating this question we may not neglect the consideration of the effect of the amount of the items of the rate on the volume of the traffic; for otherwise—if this were not done—it is evident that the most profitable course to pursue would be to levy a terminal-charge equal to the full transport-value of the commodity and to fix the kilometric rate at *nil.*

If the cost of the collection and distribution of goods per unit be A_0, and a terminal-charge be levied of the amount A, then for a kilometric charge f, a kilometric working-expense f_0, and a transport-distance x, there is a net-profit of

$$A - A_0 + (f - f_0)\, x.$$

If we assume that the volume of traffic is represented by Eqn. 32 which was previously taken arbitrarily, but not improbably, as a basis by way of example, namely that

$$\gamma = \gamma_0 [v - (A - A_0) - f x]$$

then we obtain the net-profit for the volume of traffic to be despatched a distance x, as

$$d\,U = 2\,\gamma_0\,\pi\,[v - (A - A_0) - f x]\,[A - A_0 + (f - f_0)\,x]\,x\,d\,x\,.$$

On differentiating this with respect to A, and to f, we find that this expression has a maximum value for

$$A = A_0 + \frac{v}{2}$$

and

$$f = \frac{f_0}{2}$$

and that this max. value is

$$d\,U_1 = \frac{\gamma_0\,\pi}{2}\,(v - f_0\,x)^2\,x\,d\,x.$$

The total net-profit is therefore

$$U_1 = \frac{\gamma_0\,\pi}{2}\int_0^r (v - f_0\,x)^2\,x\,d\,x$$

wherein $r = \frac{v}{f_0}$ is to be substituted; so that finally

$$U_1 = \frac{\gamma_0\,\pi\,v^4}{24 f_0^{\,2}}$$

or, inserting the previously employed abbreviation,

$$U_1 = \frac{W_1}{24} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

If the above-assumed law of the traffic-intensity be employed in the case also where the terminal-charge is fixed at the amount of the actual cost and the kilometric rate is $f = \frac{3 f_0}{2}$, then

$$U_1 = \gamma_0\,\pi f_0 \int_0^r \left(v - \frac{3 f_0}{2}\,x\right) x^2\,d\,x$$

or

$$U_1 = \frac{2}{81}\;\frac{\gamma_0\;\pi\,v^4}{f_0^{\,2}}$$

$$= \frac{2}{81}\;W_1 \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

On comparing this with Eqns. 38 and 39, we see that **it is advantageous to make the terminal-charge greater than the actual amount of the expense of collection and delivery at the same time diminishing the kilometric charge.**

But, and this it is hardly necessary to point out, this process cannot be carried out farther and the terminal-charge raised by half of the transportation-value above the actual cost of the collection and delivery; because, firstly, the result of the above calculation is based on the arbitrarily assumed law of the traffic-intensity; and, secondly, also

because the transport-value of goods varies with time and place, and cannot be precisely determined; and finally, because if such a high charge were made for transportation the traffic would largely seek the cart-roads in place of the railway.

The determination of the exact point at which the terminal-charge would be most advantageously fixed is a very difficult matter. But in fixing it the truth here demonstrated—that the terminal-charge should not be fixed much higher than the actual cost of the collection and delivery of the traffic if a high degree of profit is aimed at—should not be lost right of.

Particularly as regards passenger-fares the question merits serious consideration whether it would not be a good policy to make passenger-fares include an additional or terminal-charge for collection and delivery, viz., a booking fee, and at the same time to reduce the kilometric-unit charge. According to the present almost universal practice of fixing passenger-fares simply in proportion to the length of the journey, short journeys are favoured at the expense of the longer ones, and the amount of profit is thus diminished. This could be only justified if other compensating economic advantages were at the same time obtained. But the favouring of the short distance travel which the increase of large towns so promotes, is certainly not an advantage which can be considered as counterbalanced by the disadvantages of diminished profits.

§ 15.

The Communally best Railway Rates.

When the (unit) charge for carriage is reduced, the profit to the carrier arising from the transport of a *unit-volume* of goods is diminished by the amount of the freight-reduction: and at the same time there accrues to the consumer, to the producer, or to a middle-man, or to all these several individuals together in some proportion varying with the conditions of distribution, a pecuniary advantage of an equal amount per unit-volume of the merchandise in question. And conversely, an increase of the freight-charge increases the profit to the carrier, *per goods-unit*, by the same amount as is lost by the parties—despatcher and consignee—concerned in the traffic of this commodity.

Consequently, if the volume of goods offered for transport to a given distance is **constant**, a reduction of, or an increase in, the freight-charge simply results in a different distribution of the economic gain amongst the parties concerned; the attainable politico-economic or communal gain remains on the whole unchanged.

But the assumption that when the freight-charge alters the volume of goods offered for transport to a given distance is constant is only true in exceptional cases. As a rule, a reduction in the freight-rate reduces the price of the goods to the consumer, although by only a fraction of the amount of the decrease in freight; this decrease in freight increases the demand for the goods, and the price paid to the producer of the commodity rises. This induces the latter to offer greater quantities for transport; so that the volume of goods transported increases in consequence of the increased demand and increased production.

If a volume of goods is transported at a rate of carriage f for a distance x, *i.e.*, for a freight-charge fx, then on a reduction of the freight-charge by $d\,f$ the net-profit to the railway on this volume of goods is diminished by $\gamma\, x\, d\,f$; and at the place of sale an advantage of an equal degree will be enjoyed by the dealers. Thus the originally-existing goods-volume is, economically, neither increased nor decreased. The reduction of the freight-charge results in an increase of the price offered to the producer to such a degree that he is induced to increase his selling-price by $d\gamma$; whereas, for the consumer, the price falls to such a degree that he is induced to increase his purchases by the amount of $d\,\gamma$. Neither the consumer nor the producer profit by this increase $d\,\gamma$ of the goods-volume, since they buy and sell neither more nor less than seems good to them after the change of price. Whereas from any increase in the goods-volume there results to the owners of the railway a new profit of the amount,

$$d\,g = d\gamma\,(f - df - f_0)\,x$$

or, neglecting magnitudes of the second order,

$$d\,g = d\,\gamma\,(f - f_0)\,x. \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

Since all other advantages and disadvantages arising from the decrease in freight-charge equalize each other the last-named amount is the communal benefit or profit arising from this decrease. This gain for a continued decrease of the freight-rate will continue to increase until f becomes $= f_0$; namely, until the freight-rate, f, has attained the amount of the actual cost of the working, f_0.

From the above we glean the important truth that, **communally, the maximum advantage from railways is reached when the rate of freight is fixed at the actual cost of the working;** and when, as is the case with the common highway roads, there is no payment of interest on the capital from the profits of the working; this payment being provided for, as in the case of cart-roads, from the general State-revenues.

For the purpose of illustrating the principle here developed, let the admittedly arbitrary assumption be made regarding the law connecting the intensity of traffic and the price of commodities which has already previously been adopted for the sake of greater facility of explanation, viz. that

$$\gamma = \gamma_0 (v - f x)$$

wherein v is the transport-value of the merchandise.

For a decrease of the rate of freight by $d f$ there arises, accordingly, an increase of the traffic-volume $d \gamma = \gamma_0 \, d f x$: and, according to Eqn. 41, a communal or public gain of

$$d g = \gamma_0 \, x^2 (f - f_0) \, d f \quad \dots \quad \dots \quad \dots \quad (42)$$

For a freight-rate $f_1 = \frac{v}{x}$ the traffic-intensity $= 0$.

If from this rate, which is that at which the first stimulus to traffic occurs, we revert to a rate f then we obtain for a transportation to a distance x a communal or public gain of

$$g = \gamma_0 \, x^2 \int_f^{\frac{v}{x}} (f - f_0) \, d f$$

and therefore

$$g = \gamma_0 \, x^2 \left(\frac{v^2}{2x^2} - \frac{v f_0}{x} - \frac{f^2}{2} + f f_0 \right)$$

For a ring of width $d x$ of the traffic-area and for a freight-rate f there is a communal gain of

$$d G = 2 \gamma_0 \, \pi \, x^2 \, d x \left(\frac{v^2}{2x^2} - \frac{v f_0}{x} - \frac{f^2}{2} + f f_0 \right)$$

The traffic-area is determined by the transport-distance $x = \frac{v}{f}$, so that the communal gain for the whole transport-area for a rate f is therefore

$$G = \pi \gamma_0 \int_0^{\frac{v}{f}} \left(v^2 x - 2 v f_0 \, x^2 - f^2 x^3 + 2 f f_0 \, x^3 \right) d x$$

whence

$$G = \frac{\pi \, \gamma_0 \, v^4}{f^3} \cdot \left(\frac{f}{4} - \frac{f_0}{6} \right) \quad \dots \quad \dots \quad \dots \quad (43)$$

Differentiating this with respect to f we see that the above attains a maximal value for $f = f_0$; and this max. value is

$$G_1 = \frac{\pi \, \gamma_0 \, v^4}{12 f_0^{\,2}}$$

or

$$G_1 = \frac{W_1}{12} \quad \dots \quad \dots \quad \dots \quad (44)$$

Were the rate to be fixed at $f = \frac{3}{2} f_0$, *i.e.*, at the most favourable rate for the railway carrier, there would be obtained a communal gain—substituting $f = \frac{3}{2} f_0$ in Eqn. 43—of

$$G_2 = \frac{5}{81} \, \frac{\pi \, \gamma_0 \, v^4}{f_0^{\,2}}$$

$$= \frac{5}{81} W_1 \quad \dots \quad \dots \quad \dots \quad (45)$$

i.e. 20/27ths, or, say, $\frac{3}{4}$ ths of the maximum possible communal gain.

For the parabolic differential tariff, viz., for the freight-rate $f = 2f_0 - \frac{f_0^2 x}{v}$, the communal gain for a transport-distance x, is

$$g = \gamma_0 x^2 \left(\frac{v^2}{2x^2} - \frac{v f_0}{x} - \frac{f_0^4 x^2}{2v^2} + \frac{f_0^3 x}{v} \right)$$

and for the whole transport-area it is

$$G_1 = 2\pi \int_0^{\frac{v}{f_0}} g\, x\, d x$$

and consequently,

$$G_1 = \frac{\pi \gamma_0 v^4}{15 f_0^2}$$

$$= \frac{W_1}{15} \qquad \ldots \qquad \ldots \qquad \ldots \quad (46)$$

which is 8% greater than it is for the constant freight-rate $f = 3/2\, f_0$ which has been shewn to be the most profitable for the private owner. **The parabolic differential tariff is thus better than an invariable kilometric freight-rate; not only from the standpoint of private interest but also from that of the public advantage.**

For the assumed law of the traffic-intensity the gain for a constant kilometric rate would be

$$U = 2\pi \gamma_0 \int_0^{\frac{v}{f}} (v - f x)\,(f - f_0)\, x^2\, d x$$

$$= \frac{\pi \gamma_0}{6} \frac{v^4}{f^3} (f - f_0)$$

Consequently, for $f = \frac{3}{2} f_0$,

$$U_1 = \frac{2}{81} \frac{\pi \gamma_0 v^4}{f_0^3}$$

$$= \frac{2}{81} W_1 \qquad \ldots \qquad \ldots \qquad \ldots \quad (47)$$

If the rate most advantageous to the private proprietor were levied—assuming that the transport-value of the merchandise be fully exploited and that the law of the intensity of traffic be correct which was assumed for the purpose of example—the public benefit from the railway would then be 2½ times the amount of the net operating gain—as a comparison of Eqn. 45 with Eqn. 47 shows.

If the traffic-area be restricted by the proximity of neighbouring places of sale, then not only the total net-profit arising from the transport of merchandise but also the communal gain arising therefrom diminishes; but the former does so in a greater degree than the latter, so that the amount by which the communal gain exceeds the net operating profit increases.

But any numerical determination of the public benefit arising from railways must always be untrustworthy—apart from the uncertainty as to the degree to which the transport-value of the merchandise is exploited—so long as the law connecting the intensity of traffic with the rate of freight is unknown.

Nevertheless the fact is of the highest importance that the truth of the above law—viz., that the maximal communal gain is attained when the rate of freight is fixed at the actual cost of the working—is entirely independent of the law of the traffic-intensity, just as it is of the degree to which the transport-value of the merchandise can be exploited. The correctness of this principle rests simply on the fact that with a decrease of the rate of freight the volume of traffic does increase, though the precise law of this increase is unknown.

This fact, however, is undeniable; for even in the case in which the volume of a commodity is unalterably fixed as, for example, a petroleum well, it is still true that by a decrease in the rate of freight the volume of traffic will increase in tonne-kms.

The exploitation of a commodity of unalterable volume may be worked as a monopoly by fixing the price at the place of production so high that with the existing freight-rate the extreme distance of transport requisite for the sale of the whole volume will be obtained.

By a reduction of the rate of railway freight—provided the price at the place of production remained unaltered—the price in the entire area of distribution or sale would fall and hence the demand for it would increase. This increased demand would be greatest at the boundaries of the original area of sale, and a demand would arise even beyond those limits. This will occasion an increase of price at the place of production whereby the price of the commodity rises up to a certain distance from the place of production in spite of the reduction of rate of freight; and the demand within this circle diminishes, while beyond it the demand increases with the decreased price; and the sale becomes extended beyond the boundaries of the original area of sale. Consequently, although the gross-weight remains constant the number of tonne-kms. increases. It will be easily intelligible without a somewhat long demonstration that in this exceptional case also the reduction of the freight-rate increases the public gain up to the point at which the rate of freight is lowered to the actual cost of the working of the traffic.

§ 16.

The Fundamental Principle of the Formation of Railway Rates.

The **business of transport** on high-roads and canals is advantageously left to private enterprise, because on these particular ways of communication a free and unrestrained competition is possible which acts as an effective incitement to the introduction of improvements and of the best and cheapest method of transport. But it is undeniable that the **construction and maintenance** of roads and water-ways is a matter which concerns the common or public welfare: and in well-regulated polities these duties would be performed by the State as a whole or by some department of it; because herein it is not possible to have that free and unrestricted competition by which alone private enterprise is controlled so as to conduce with sufficient certainty to the common benefit and advantage. State supervision and legal limitations can certainly prevent many abuses and mitigate many dangers incident to a private enterprise worked as a monopoly, but it is impossible to compel private enterprise to work exclusively in the common interest.

But in the case of a railway not only is the construction and the maintenance a monopoly, but its working is so also. Experience has proved most conclusively that where there are several lines belonging to different proprietors available for the traffic between two places the competition or war of rates which occurs from time to time is very speedily brought to a conclusion by a combination of the competitors.[1] The monopoly of a railway is really effectively injured only when roads and canals are alongside of it, since on these the competitive traffic cannot be crushed by combination.

The undeniably communal character of railways is recognisable from § 3, according to which the correct locating of the Trace and the investing of the proper amount of construction capital can only be made on a communal apprehension of all the requirements of traffic as regards communications. It is also evident from the considerations brought forward in §§ 14, 15; where it is shown that the maximum communal or public benefit arising from railways is only attained when and if the freight-charge is fixed at the actual cost of the working, and no repayment of the interest on the capital expended is attempted; which thus excludes all possibility of private enterprise.

The erroneous idea that railways are fit and proper subjects of private enterprise is based mainly on this: that in many instances the construction of railways has in the past been left to private enterprise, and that undeniably we have to thank private enterprise for the rapid growth of our German railway system. This has been due to the fact that the prospect of gain acted as a powerful inducement to a strenuous activity which the State could not have displayed in an equal degree: finally, also to the fact that private enterprise in railways has frequently worked very beneficially, and does so still. However, all this should not

[1] The theory that railway rates should be left for competition to regulate has been fully tried in the anthracite coal business. Eight different railway companies have lines reaching from the anthracite fields to tidewater, a sufficient number to insure competition if it can ever be insured in the railway service. Legislation has been tried to foster competition, and state commissions and the Interstate Commerce Commission have lent their aid to keep it alive. To say that there has been no competition among the anthracite carrying companies would be untrue; but we believe it is safe to say that this competition has never secured the "reasonable rates" for the carriage of anthracite, to which producers and consumers are fairly entitled. Such competition as has taken place has resulted chiefly in the giving of secret rates and rebates, and in benefiting favored shippers and corporation officers "on the inside." The fruit of adherence to the theory of free competition in fixing railway rates on anthracite coal, then, has been the transfer of many million dollars a year from the pockets of coal users, and to some extent of coal producers, to the pockets of those interested in the railways which carry this traffic.—*Engineering News*: 25 *Jan.* 1900.

obscure the fact that **the full advantage of railways can only be attained when they are in the hands of the State, and are worked as enterprises by order and for the account of the public.**[1]

According to Sax,[2] who fully recognises the communal character of railways, it is quite permissible for the State to make over railways as "Public Undertakings" to private enterprise, but burdened with the mandate to run them in the interests of the public.

Ulrich,[3] however, opposes this view in a concise and striking work. He is of opinion that "it is vain to attempt to compel private railways against their inclination by laws and "regulations to place the public interest before their own in the matter of rates and general "management."

Amongst the various principles of administration which are usually distinguished in the science of Economics the only one which can possibly govern private ownership of railways is that of self-interest, viz. that of getting the greatest possible net return. There is no special "Principle of Management of Public Enterprises," such as several writers, regarding private railways as "Public enterprises with delegated popular functions" have sought to establish. It certainly cannot be assumed that private enterprise will be deterred—whether by legal restrictions imposed on it as to the mode of levying rates and fixing their limits, or by State Regulations as to the mode of working the railway—from endeavouring to obtain as large a profit as is possible within the limits of these restrictions; nor that it will go at all beyond its bond at the risk of injury to itself simply to benefit the public. The idea that the management of private railways can be so regulated by legal prescriptions and supervision that while following their own interests they shall at the same time promote the general good is no longer tenable so soon as the fact becomes recognised that in the public interest there should be no payment of interest on the construction-capital: to such procedure private interest will never consent.

In the administration of Private Railways, there can be only one principle of working, viz. that of Self-interest, in obedience to which the maximum net-profit will be striven-after which is attainable within the State-imposed limitations under which they may work.

In the administration of State Railways, on the other hand, **all the three principles of management that we are accustomed to distinguish can be employed namely, 1°. The working gratuitously, i.e., without profit, for the general advantage. 2°. The system of tolls. 3°. The working as a private concern aiming at the maximum net-gain possible.**

If the maximum communal benefit and advantage is to be derived from railways then, as has already been shown, they must be worked as unremunerative objects of public use. But it will be shewn presently that the State does not possess sufficient administrative power to be able to satisfactorily carry out this task; and that as a rule it will have to rest satisfied with less than the maximum profit; and that it is compelled to work according to the second, or even, to the third principle above stated.

As examples of unremunerative objects of general use and enjoyment administered by the State or its departments we have the public monuments, museums of Arts and Sciences, public gardens, the preservation of person and property, common roads, and partly, also, canals and harbours. From the application of this principle to the management of railways it in nowise logically follows that the State should undertake the gratuitous preservation of person and property; but only that by the application of this principle there should be no payment of interest on, and no redemption of, the expenditure on the

[[1] Scientific socialism is strongly represented in the German Universities and seats of learning. For a recent example, see Appendix, p. 110.—Tr.]

[2] Sax: "Die Verkehrsmittel in Volk-und Staatswirthschaft." Wien. 1878.

[3] Ulrich: "Das Eisenbahn-Tarifwesen." Berlin und Leipzig. 1886.

initial installation requisite for each individual enterprise; only the interest on the necessary expenditure for its direct performance should be paid.

Whatever the unit-rate of carriage may be fixed at, and however large the traffic as a result of this rate may be, there is always a sum of money to be provided-for independent of the rate for interest on capital, maintenance of way, and certain items of working-expenses independent of the volume of the traffic, for which an amount A has to be subtracted from the communal gain, G, previously obtained by calculation, and which varies with the freight-rate. In determining the profit G only that part of the expenses which increases proportionately with the number of units transported has to be taken account of, *i.e.*, the rate f_0 per unit, which arises from in the actual working of the enterprise.

In the investigation in § 15 it was proved that the public gain G and consequently the clear profit $G - A$, remaining after deducting the constant quantity A, is a maximum when the freight-rate f is the actual net working-cost f_0 which arises directly from the working of the enterprise.

Thus in the case of railways, it is only the road fully equipped and ready for working that is to be treated as an unremunerative object of general property or use; consequently, the rate of freight is to be fixed at the cost of transport of the loads plus the cost of maintenance of the line.

In the case of common highway-roads beside this necessary charge the community bears also the expense of the maintenance necessitated by the movement of the traffic thereon. This part of the highway road-maintenance expense may be assumed in general to be about 1/10th the working-expenses; so that the traffic only pays $\cdot 9 f_0$. Substituting $f = \cdot 9 f_0$ in Formula 43 for the communal gain we obtain

$$G = \frac{175}{2187} \frac{\pi \gamma_0 v^4}{f_0^3}$$

which as compared with the maximum attainable amount of $\frac{1}{12} \frac{\pi \gamma_0 v^4}{f_0^3}$ for $f = f_0$ is about 4% less.

If it were desired to recoup by a road-toll, as was formerly the custom, the expenses of the maintenance and repair of roads due to wear by the traffic, it would be necessary to raise the cost to the public of the roads by 4%. But the levying of road-tolls would occasion new expenses; and as peculation easily enters therein, and the collecting is felt to be a great nuisance, it is quite reasonable that the public should defray gratuitously the expenses of the road-maintenance also.

The administration of roads, as national property of general use and enjoyment, is for the above reasons carried beyond the degree most advantageous from a communal point of view. In the case of railways this has not hitherto been realisable, because the State is not financially strong enough to do so. As a rule the State is unable to defray the payment of interest and the repayment of the capital sunk in railways out of other sources of revenue, and is therefore compelled to devote itself to the earning of profits for the payment of these charges. Where there is a possibility of increasing the net-profit beyond the degree requisite for the payment of interest on the capital and for its redemption it will usually be impossible to avoid doing so; since there is no more convenient, more reasonable, and less oppressive way of indirect taxation than that through railway rates.

The State will therefore as a general rule work railways for a profit and on the same self-regarding principles that guide private companies. But with this important difference; that the amount of the profit above the amount requisite for the payment of interest shall become the property of the public; whereas in the case of private concerns this goes to enrich the individual at the expense of the public.

That railways, so long as the rates of freight are not brought down to the mere working-expenses but are so fixed as to produce the maximum net-profit, do not yield the

maximum advantage to the public is a defect which enters into every form of taxation by tolls; and could only be got rid of if it were demonstrable that there were some other equally productive mode of taxation which worked less injuriously to the public.

For the formation of rates on the basis of private interest we have considered three several principles in §§ 13 and 14: namely,

(1) Imposition of a high terminal-charge, greater than the actual expense of collection and delivery, plus a constant kilometric freight-charge.

If on this principle the terminal-charge were raised by half the transport-value above the actual cost of the collection and delivery of the goods, and the kilometric freight-rate were fixed at the half of the actual costs, then the maximum profit would be—from Eqn. 39—

$$U_1 = \frac{W_1}{24}$$

But as already stated, there can never be any question of introducing such a mode of forming rates, because for short and even for moderate distances, the cart-road rate would be less.

(2) Fixing the terminal-charge at the amount of the actual cost of the collection and delivery and levying a freight-rate diminishing as the distance increases.

According to Eqn. 38, the maximum gain obtained is then

$$U_1 = \frac{W_1}{30}$$

the rate being assumed to gradually diminish from $f = 2f_0$ to $f = f_0$, according to the law

$$f = 2f_0 - \frac{f_0^2}{v} x$$

Rates framed on this basis—the so-called "Parabolic Differential Tariff" are open to the objection that to correctly determine the decrease of the kilometric freight-rate the transport-value, v, of the merchandise must be known beforehand; and that for each individual merchandise the decrease must be determined differently, *i.e.* in exact proportion to its transport-value.

(3) The terminal-charge is fixed at the amount of the actual cost of the service of collection and delivery and the kilometric rate of freight is made a constant for all distances. In this case, for the maximum gain, the rate of freight is to be fixed at 1½ times the actual working cost: whence from Eqn. 40 we have

$$U_1 = \frac{2}{81} W_1$$

This method of framing rates is very simple practically, because in this case it is not necessary to know beforehand the transport-value of goods.

But remembering that the net-profit on the above principle amounts to only 20/27ths. of that of the Parabolic Differential Tariff, and considering further that the public advantage arising from railways (Conf. § 15) with the Parabolic Differential Tariff is some 8% greater than with the constant rate of freight it certainly seems desirable on public grounds to take the introduction of the Parabolic Differential Tariff into the most serious consideration.

It is to be noted that the above numerical values rest on the assumption that the intensity of tariff $\gamma = \gamma_0 \, (v - f x)$: in other words, that it is proportional to the transport-value of the commodity remaining after the subtraction of the amount of freight. We may, however, have doubts as to the correctness of this law of the intensity of the traffic and, consequently, of that of the results arrived at without the general truth being thereby invalidated, viz., that the Parabolic Differential Tariff yields a higher profit than the tariff of constant rate of freight. For the necessity of determining the law of the volume of traffic may be

entirely evaded by substituting in place of a merchandise with variable traffic-intensity any number of different kinds of goods each of which has, for a constant traffic-density, a varying transport-value. For each of these individual classes of goods with constant traffic-density the net-profit on the basis of the parabolic differential tariff is—Eqn. 27,

$$U_1 = \frac{W}{6}$$

and for a constant freight-charge—Eqn. 24,

$$U_1 = \frac{8\,W}{81}$$

In both the methods of basing freight-charges here compared, the rates of freight are $2f_0 - \frac{f_0^2}{v}x$, and $\frac{3f_0}{2}$ respectively; they are the most favourable ones only when the transport can be pushed to its utmost limit so as to completely exhaust the transport-value.

If the mileage of the railway is insufficient to fulfil this condition, or if the extreme transport-distance is restricted by the presence of adjacent places of sale, or if there is the traffic of a branch-line to be taken into account, then for the attainment of the maximum profit with either method of forming rates the rate must be increased. This circumstance makes the correct determination of the rates that should be levied in order to obtain the maximum profit, when $f = 2f_0 - \frac{f_0^2 x}{v}$, and $f = \frac{3}{2} f_1$, respectively, are the lower limits, very difficult and in most cases impossible.

If the rate be fixed too low then the profit is diminished, but the gain in public advantage is greater than the loss; whereas if the rates are fixed too high, not only is the gain diminished but the public advantage derived from the railway is likewise diminished in a still higher degree. Accordingly, the fixing of a too high rate is to be most carefully guarded against. This could be done once for all by taking the lower limit of the freight-rates. In this way, there would be a transition from the principle of purely private interest to that of tolls.

If the parabolic differential tariff were to be chosen, the difficulty of fixing the transport-value of goods could be got over, practically, by diminishing the rate of freight all round by the introduction of Zones; say, by fixing $2f_0$ for the first 50 km.: $1{\cdot}8\,f_0$ for the second 50 km.: and so on, until finally, for all distances over 250 m., the kilometric or "mileage" rate f_0 comes into play.

In that case there would be only as many classes of rates as there were varieties in the actual cost of working.

However, the realisation of the theoretical investigation here developed in the practical fixing of freight-charges is not the proper subject of the Theory of Location; and still less, the solution of the question how far the economic power of a State permits it to pass from the formation of tariffs on the basis of private interest to the principle of tolls; nor is it concerned with the administration of railways for the general or communal advantage.*

[* For further remarks on the Formation of Rates, see Appendix, p. 74.—Tr.]

§ 17.

The Earning Capacity of a Projected Railway.

Having now in the foregoing investigations determined both the public benefit and also the profit of the private owner derived from railways we can now proceed to determine the commercial or financial value of any projected line; and this forms an important part of the Theory of Location.

In the solution of this problem we have to deal with the cost of construction and working, and have therefore to form an estimate of the **probable future traffic.**

The cost of the construction and of the working of the line are known on the completion of the preliminary studies and estimates with a sufficient degree of exactness; whereas the estimate of the traffic, even when made with the greatest care and guided by correct methods of procedure, cannot always be framed with the degree of exactitude desirable. Not infrequently even at the present day the existing traffic on the roads is ascertained and a certain multiple of it assumed as the probable future railway traffic. **But any estimate so based is nothing more than uncertain and helpless guessing.**

The first writer who laid down a correct and at the same time simple procedure to attain this object was, as is well known, the French engineer **Michel.** His work on this subject was published in 1868.*

Michel makes the volume of the probable future traffic proportional to the population of the area served by the station, adding thereto the population resident in the suburbs of this area; and he further distinguishes different degrees or grades of economic importance which a district may have. In this way Michel found that for the whole railway system of France there was annually 6·5 passengers and 2·1 tonnes of merchandise carried per head of the population participating in or forming the railway traffic. In purely agricultural districts this figure diminishes to two-thirds, and in industrial ones increases to four-thirds.

If we take the net-profit per passenger-kilometre as the same as that of the goods-tonne-km., *i.e.* at 2 pf.—which for the average rates and working-expenses of the German Railways is fairly accurate—then on the basis of the French average traffic of 13 passengers, **coming and going,** and 4·2 tonnes as the sum of the **received and despatched** merchandise, would result per head of the population making use of railways, and per km. over which this traffic is hauled, a net-profit of

$$c = (13 + 4{\cdot}2)\ {\cdot}02 = 1/3 \text{ M., say.}$$

Michel next investigates the net-profit on branch-lines on the assumption—certainly permissible—that the total traffic of all the stations on the branch-line extends up to the main-line. If then the population of the stations distant x_1, x_2, &c., from the main-line is ϵ_1, ϵ_2, &c., then the gain is

$$U = c\,\Sigma\,(E\,x)\,.$$

This simple method has been widely employed in Italy and Austria, mainly with the object of determining the value of the traffic-coefficient, c, for the local economic conditions.

An exact determination of the average traffic-coefficient for the German Railways, based on the Traffic Returns and on the Census of 1880, is given in the following investigation.

In the year 1880, there were 4,502·7 stations, including the frontier stations used in common with foreign railways; which latter stations are introduced into the calculation by a corresponding fractional number. These stations serve 4,450 localities. The number

* Études sur le traffic probable des chemins de fer d'interet local—par Louis Jules Michel. *Annales des ponts et chaussées*: 1868. p. 145.

of the inhabitants of these localities and of 754 additional stopping-places (flag stations) amounts, according to carefully-made estimates, to about 18¼ millions. In order to arrive at the total population producing railway traffic we must further add a quantity to represent the degree of participation in the traffic of the population of the hinterlands of the stations. Since in the year 1880 the total population of Germany amounted to 45,234,061, after deducting the population resident in the station-areas, there remains an average of 6,112 persons as the population of the hinterland of each station-area. Taking the total area of the German Empire at 540,522 square kms. (q. km.), the average area served by a station is 121·5 q.km.: so that subtracting an area of 3 to 4 q.km. for the station itself there remains 52 inhabitants for each q.km. of hinterland.

The mean distance apart of the stations is obtained by dividing the total length of the German railway system, viz. 33,430 km. by the L. C. M. of half the number of the lines radiating from each station. This gives 6·96 km. or sufficiently accurately, 7 km. The area served by a station has thus on an average a length along the line of 7 km. and a breadth on either side of it of 8¾ km.

The hinterland traffic of the stations to and from the railway is carried-on mainly on metalled roads; partly, also, on rough earth-roads and only exceptionally on canals. As the freight per tonne-km. on metalled roads is on an average 5 times as great as that on railways, and on unsurfaced roads about 20 times, a considerable fraction of the transport-value of goods will thus be already used up before the merchandise reaches the railway.

The mean transport-distance of goods in 1880 on the German railways was 81 kilometres. As the short-distance business of the station hinterlands is included in this mean figure the average transport-distance of the traffic occurring between the stations must be assumed somewhat greater, say, at 85 km. If the traffic of the station-hinterland has to travel on an average for a distance of 6 km. on a metalled road to or from the railway, which is equivalent in cost to a transport of 30 km. on the railway, the hinterland traffic will then only travel over the railway for a distance of $(85-30) = 55$ km.

Thus the hinterland traffic makes use of railways on an average for only 55/85ths $= \cdot 64$ of that distance which the out-going traffic from the station-area itself does.

Since we have found that the number of the tonne-kms. carried on railways is proportional to the fourth power of the transport-value, the hinterland goods traffic will form only the $(0{\cdot}64)^4$ part of the tonne-kms. carried on the railway; namely, about only 1/6th of that volume of traffic yielded by the same number of the inhabitants resident in the station-area.

In a similar way the same ratio-figure will be obtained for the share of the hinterland in the *passenger* traffic of the railway.

The investigations of Sonne, published in the "Deutsche Bauzeitung", 1881, p. 216, and which are based on statistical data furnished by Köpcke, likewise lead to the result that with the present density of the German railway system the population of the hinterland of a station yields only the 6th part of the railway traffic furnished by the inhabitants of the station-area. Sonne gives a curve of which the abscissa x represents the distance from the station-area in kms. while the ordinates represent the share which the inhabitants residing at this distance x take in the railway traffic.

Expressing the share of the station-area population in railway traffic per head as unity then, according to Sonne's curve, the share of the population living at a distance x km. from the station-area per head is, approximately,

$$y = \left(1 - \frac{x}{12}\right)^4$$

For the whole extent of the station-area this formula gives on an average about 1/6th.

The method of Michel in which the influence of the hinterland is taken into account by adding the population resident in its environs (Banmeile) to the number of the inhabitants of the station-area will lead to a similar result.

Manifestly, the method pursued above to determine the influence of the hinterland on the traffic of the railway cannot lay claim to a completely satisfactory degree of trustworthiness. But in the absence of exact statistical data we must be satisfied with this mode of estimating. And this we can do all the more readily since any error that there may be will have no influence that need be taken into account on the final results of the calculation which we are about to make.

In the year 1880 there were about 27 million of inhabitants resident outside the railway-station areas; so that to the 18¼ millions of station-population there is further to be added $1/6 \times 27 = 4\frac{1}{2}$ millions to give the population producing railway traffic—or a total of 22¾ millions.

Now in this year—1880—there were carried on the German railways 215 millions passengers, representing 6,479 million passenger-kms.; and 165 million tonnes of goods, corresponding to 13,487 million tonne-kms.: so that per head of the population producing railway traffic there was on an average 9·5 passengers, representing 285 passenger-kms., and 7¼ tonnes of merchandise corresponding to 595 tonne-kms. The sum of the passengers and goods going and coming is thus annually per head of the railway population 19 passengers and 14½ tonnes of goods. Accordingly, in Germany the goods traffic was some 3½ times greater than that in France in the year 1866 as given by Michel's figures. This is, of course explicable from the circumstances that Michel's figures refer to a year in which the French, railway system had less density than had the German in the year 1880; and also by the fact that a considerable share of the goods-transport in France is carried on its fine system of internal water-ways, and by coasting-service: and finally, by the difference in the industrial activity of both countries: in Germany the goods-traffic consists to a greater degree in heavy merchandise than in France.

For Germany, accordingly, the fundamental or basic unit of the net-profit—on the previously-made assumption of an operating gain of 2 pf. per tonne-km. or per passenger-km.—is

$$c = (19 + 14\tfrac{1}{2})\ \cdot 02 = \frac{2}{3}\text{ M.}$$

On a railway of l km. in length which introduces a new population of E individuals into the volume of railway traffic it may reasonably be expected, on the assumption that the traffic travelled the whole length of the line, that there would be an annual operating gain of $\frac{2}{3} El$; and if the traffic moved on an average over only ⅔rds of the whole length of line, an operating gain of $\frac{4}{9} El$. Reckoning the kilometric construction-cost of the line at only 80,000 M. then in order to obtain a return of 5% on this outlay a new population of from 6,000 to 9,000 individuals would have to be drawn into the traffic. But since in Germany in 1880 there were only 1,078 places of above 2,000 inhabitants having an average population of 3,400 which were without the advantage of railway facilities, it would only be in quite exceptional circumstances that a new railway would be possible which out of its own net-profit from operating would be able to pay interest on its cost.

Moreover, the method of Michel requires another modification. In the determination of the earning capability of a projected railway we may not limit ourselves to the calculation of the net-operating gain alone; we must also take into account **the increase of traffic which the new line will bring to the already existing system.** If the projected line worked as an independent financial concern would not yield a sufficient return it might nevertheless be a sufficiently profitable undertaking for the owners of the adjacent systems, or for the particular Administration owning it. From this point of view **it is not the number of passengers and tonnes which is to be regarded, but the number of passenger- and tonne-kms.**

per head of the population which is thus newly brought into the sphere of railway traffic. In this way, we obtain the net-operating profit for the whole railway system per head of the population newly included in the traffic as

$$(285 + 595) \cdot 02 = 17 \cdot 6 \text{ M.}$$

Almost exactly the same figure is obtained if the net-working gain of the whole German railways—which in 1880 amounted to about 401 million M.—be divided by the 22¼ millions of population producing railway traffic, thus giving 17·5 M. per head.

If the interest on the total construction-cost of a new railway be $A\,i$, then the traffic-producing population, E, requisite to afford sufficient business to cover it, is given by the equation

$$17 \cdot 6\, E = A\, i.$$

And assuming a rate of interest (i) of 5%,

$$E = \frac{A}{352}.$$

For example; supposing a line were 10 km. in length, and that its construction-cost = 1,000,000 M.: it would be well worth constructing if thereby only 2,800 persons were secured as additional business-contributors to the existing traffic.

It is assumed when employing the fundamental figure of 17·6 M. as the gain from railway-working per head of the population, that the economic importance of this population is a mean or average one. According to the figures of Michel, this economic importance in purely agricultural districts may sink to ⅔rds of the mean figure: but it is prudent to assume for the economic importance of a purely agricultural population only the half of the mean value; so that in this case the financial value of the line can only be considered as assured when and if

$$E = \frac{A}{700}$$

A being the total construction-capital in M.

For highly industrial districts, the traffic-value of the population rises, according to Michel, to 1⅓ times the mean value. Consequently, for such cases the value of the line as a financial undertaking is assured even for a population of

$$E = \frac{A}{260}.$$

In order to determine the number of residents yielding the required traffic it is necessary, as shown above to raise the number of inhabitants of the station-area by a certain figure which shall bring in the hinterland as a participator into the calculation. But with an increasing density of the railway network the influence of the hinterland on the traffic diminishes; since not only does the station-area become smaller, but also the population residing on the unit of area diminishes. It must also be borne in mind that every new line withdraws from the older lines a portion of their hinterlands; so that a number of individuals corresponding to this loss to the older lines must be brought proportionally into the estimate of the population newly drawn by the new line into the traffic. These considerations render the determination of the influence of the hinterland, already in other respects very uncertain, still more difficult.

Fortunately, the errors possible in the estimation of this influence are not of much importance for the final results of the determination of the value of the line. After carefully weighing all the conditions, this additional hinterland influence on the population of the station-area may be put down at $\frac{2}{3}\, d\, e$, where d is the half-width of the station-area in km., *i.e.* the greatest distance which the hinterland traffic has to travel to reach the railway-station, and e is the population per q.km. resident in the station-area.

If we now take into account the circumstance that for many reasons traffic is worked on branch-lines less favourably than on main-lines—if for no other reason than on account of the smaller number of trains and their smaller velocity—it seems reasonable that the figure found for all the German railways of 17·6 M. should be diminished to 15 M. The increase in operating gain which a branch having n stations and a station-area population of E individuals yields is then

$$N = 15\,E + 10\,n\,d\,e. \quad \ldots \quad \ldots \quad \ldots \qquad (48)$$

Of this mean value we may take the half for purely agricultural districts, and for highly industrial or manufacturing ones, the four-thirds.

A railway has two stations of which the populations are 800 and 2,000. It handles the traffic of a district averaging 7 km. on each side of the line and having a population of 40 per q.km. Such a line will accordingly bring to the system, assuming a mean economic value for the district, an operating-gain of

$$U = 15\,(800 + 2000) + 10 \times 2 \times 7 \times 40 = 47{,}600 \text{ M.}$$

It is perhaps scarcely necessary to mention that the general formula is not valid where extra-ordinary and unusual conditions and circumstances of traffic obtain; as for example, for watering-places, and places of pilgrimage, or for places much visited on account of unusual scenic attractions or for other reasons, or for points of contact with canals, or for places having a heavy bulk-traffic. It must be borne in mind that for such exceptional cases the financial value of such a line is not to be estimated, as Michel has done, and as has been done hitherto almost universally, solely from the gain to be expected from the new line itself; but that the increase in traffic and the resulting gain which the new line will bring to the **previously existing ones** is also to be taken into consideration.

The gain to the community arising from railways considerably exceeds the maximum attainable operating net-profit. The former was found—p. 40—to be, for a definite volume of the traffic, $2\frac{1}{2}$ times the net-operating profit, and will probably not be far below this amount.

But considering the uncertainty of the assumptions which are unavoidable in the calculation of the public economic advantage, the determination of the financial value of a line from the point of view of private-interest just made must suffice. An examination, based on these calculations, of the number of additional railways still possible in Germany shows that the most advantageous density of the railway system is still far from being yet attained; and that an increase of 2 or $2\frac{1}{2}$ times the existing state of things would be possible while obtaining at the same time a sufficient return on the capital invested.*

[* For an account of Michel's method with Cossmann's modifications, see Léon Leygue: **Chemins de fer; notions générales et économiques**: Chap. VII—Evaluation des Recettes probables des Lignes nouvelles. See also the Appendix to this pamphlet.—Tr.]

[For a numerical example of the calculation of prospective revenue, see Appendix, p. 100.—Tr.]

§ 18.

The Effects of improved Means of Communication.

The economic effects so often discussed of good means of communication are most thoroughly and completely analysed in Karl Knies' *"Railways and their Effects"*, published in 1853; and, in more recent times, in Emil Sax's *"Means of Communication"*. These effects, influencing deeply and decisively the economic side of existence and the various sections of society, and indeed the whole development of human civilization, are not easily focussed into a single picture, because not only are they very manifold and ramified but they also often lead-up to quite contrary phenomena.

Although, generally speaking, improved means of communication lower the price of commodities by cheapening the cost of transport, there are, nevertheless, many commodities of which the volume produced is dependent on local conditions and cannot be increased at will; and of these the price is raised.

Increased facilities of communication, by nullifying their lingual differences irresistibly leads the divided branches of a race to the attainment of a national unity; it imparts new vigour to peoples which before appeared almost to be in a process of absorption by more numerous or more predominant populations; and it revivifies languages which before were on the point of extinction.

Adequate means of transportation renders accessible new sources of enjoyment to the masses of the population and conduces to an improvement in the material conditions of their daily life; but at the same time it directly and ruthlessly destroys the comfortable independence of the craftsman and the peasant. Railways and steamships while increasing the safety, convenience, and pleasure of travelling bring about accidents of a degree of frightfulness previously undreamt of. Again, improvements in the means of locomotion strengthen the governing powers in a very high degree; but, concurrently, these improvements also favour the anti-sociological forces and facilitate the escape of the criminal.

All the manifold effects of improved means of communication which frequently manifest themselves as contradictory phenomena may nevertheless be referred and correlated to the same simple fact, namely, **the lessening of the importance of mere distance.** The command over space is extended, and thereby every manifestation of activity which was formerly limited in its development by spatial barriers is enlarged and promoted; on the other hand, every activity which needed the protection of seclusion is weakened and restricted.

One immediate consequence of the improvement in the existing means of communication is **the increased ability to enjoy life, since the efforts are lessened which were formerly requisite to obtain luxuries and to enjoy them**: and the same amount of work produces a greater volume of general enjoyment than was formerly the case

Another result of yet greater influence on human well-being is the lessening of the intermittent variation in the price of commodities. The price of a commodity at a given locality cannot be greater than the cost incurred in procuring it from a distance, *i.e.* than the cost of importation, nor less than the figure at which it is sold outside, *i.e.* the export-price. The limits of this variation in price, *i.e.* the difference between the interior and exterior prices, is equal to twice the amount of the cost of transport between the interior and the exterior; and must accordingly decrease as the means of transport improve. But the less the difference between the import- and export-price the greater must be the increase in the number and variety of the agricultural and industrial productions which are permanently exported or permanently exposed to the competition of import irrespective of the various harvest-outturns, or of

the variations in the cost of production. For such commodities there is no longer a difference between the export- and the import-prices; the price is subject only either to the variations of the import-price, or to those of the export-price. The greater stability of prices diminishes the many evils arising from their fluctuation and lessens the danger of great scarcities or even of famines.

This beneficial lessening of the intermittent variations of prices, however, makes the the income of the Agriculturist very inconstant and dependent on the harvests. But the improvement in means of communication only injures his average annual income as derived from the cultivation of crops which are also articles of import; and this either because in his country these products are not produced to a sufficiently great extent, or because they are not produced under sufficiently favourable conditions. Although any improvement in traffic-conditions facilitating imports may tend to injure the agriculturist yet, on the other hand, the cultivation of such crops as are exported will be benefited in an equal degree by every improvement in transport facilitating their despatch to foreign markets.

Consequently, the progressive improvement in means of transport will compel the agriculturist to devote himself to the production of such crops which are or may be exportable; and to abandon every kind of cultivation which has to struggle against a foreign import. **Agriculture increasingly must adapt itself in the presence of improved means of communication to the production of such articles for which the peculiarities and character of the soil and the climate offer the most favourable conditions; and must abandon the cultivation of those crops for which the local conditions are less favourable.** When the means of communication are undeveloped all the necessaries of life must, so far as the climate and the nature of the land permit, be obtained by a country from within its own area; whereas with improved communication each special excellence of the ground can be exploited to its fullest extent. In other words, the aims and objects of agricultural labour must become more and more specialised to suit the local conditions of the land.

In an equal, or, perhaps, in a still higher degree, the action above described will work itself out in the industrial domain. When the production of commodities takes place under favouring circumstances the improvement of the means of transport increases the area of their sale at the expense of neighbouring localities which offer fewer facilities therefor. The producer situated in the more unfavourable localities only maintains himself so long as he merely enjoys the protection arising from inferior means of communication. So soon as improved communications place him within the reach of a more powerful competition he is at once crushed and annihilated; just as a fortress loses all its importance as soon as artillery attains a range which renders possible an effective bombardment from neighbouring heights.

Mercantile profit is increased through the enlarged area of sale which an advantageously-situated industry obtains by improved communications; and thereby an impulse is given to the formation of new business enterprises, mutually competing, in the same locality. With this increase of the number of businesses excessive profits and the prices of commodities diminish, which leads to the further lessening of foreign competition.

Thus **for the different branches of industrial commerce special foci are formed which offer the most advantageous conditions for carrying on the particular business concerned.** This local or territorial grouping of classes of industry offers such decided advantages for the improvement of business operations, for the growth of an expert class of workers, and for the transaction of business, etc., that once formed it becomes speedily organised in an increasingly distinct manner.

In this way the division of labour also is distinctly promoted. A separation of industrial activity into a series of independent and graded lines of business takes place. Thus, for example, instead of a woollen-goods manufactory undertaking all the manufacturing operations from the cleaning of wool up to its exportation, we have now separate combing, spinning, dyeing, weaving, and despatching, etc., businesses. In the neighbourhood of the site of

any specific industry we find the auxiliary businesses grouped alongside; so that the entire economic character of a district is a consequence of the predominant branches of industry. Thus in the case of an industry, just as in that of agriculture, the perfecting of communications brings out into stronger relief the local characteristics.

The enlargement of the area of sale greatly increases the exploitation, more especially, of mineral wealth: in former times, owing to its great weight this could be sent to only moderate distances. Much of this mineral wealth which nowadays conduces so greatly to our material well-being has been only possible since the means of transport have been improved.

Just as in the case of the mining industry so the locally grouped branches of manufacturing industry come in time to assume the features of the larger and wholesale ones. The old trade-traditions become untenable; the shackles of guild-membership are ruptured; workshops expand into factories.

The regulation of labour by classes was formerly firmly interconnected on the craft-guild system, but it rested on a narrow basis. This is dissolved, and with it the rigidly-ordered membership of human society, without its being possible to introduce new and satisfactory substitutes. Such a powerful and far-reaching transformation affecting all economic and social conditions cannot be carried out all at once, but only after long and indecisive struggles. On one side there are unreasonable claims—as is unavoidable: on the other there is a too rigid adherence to established institutions. But the hardships and dangers of the struggle should not delude us into over-estimating the comfortable peacefulness of the "good old times"; neither should we be blind to the recognition of the immense economic progress and the advance of civilization which **the growth of large-scale and wholesale trade** has brought about. This progress which in all its consequences can hardly be exaggerated and which has been possible only through the growth of wholesale trade, consists in **the substitution of manual labour by machinery** operated by the forces of Nature as motive-power.

By this substitution the human race is, if not entirely relieved, at least considerably lightened of the torment and burden of labour under which it has groaned for ages. Many useful things which formerly owing to their high price could only be obtained by the favoured few have been rendered accessible to the generality by the cheaper labour of machines. **For numberless sections of human society the enjoyment of life is increased as much by its release from the martyrdom of bodily toil as by the creation of new sources of enjoyment.**

In the struggle of competition in the production of commodities sharpened as this struggle is by the effects of improved means of transport victory can only be gained by so perfecting the methods and organisation of labour that either an improvement of the quality of the article or its cheaper production is effected. But the manifold and continuous exertions of mental activity directed to this end can find prizes worthy of the magnitude of these exertions only in large industrial undertakings; and from this fact again other and further advantages result. The improvement in the methods of work arrived at for conducting large businesses becomes equally advantageous when applied to those industries which from their nature can only be worked continuously on a small scale, either because their productions will not keep, or because these are only produced to individual orders, or because their production demands an artistic endowment and temperament.

But this wholesale production is not by any means to be regarded as an unqualified benefit. The war relentlessly waged against small producers and the struggles of the large producers amongst themselves destroys the property and happiness of many individuals, and often sensibly injures the public welfare. This restriction of and injury to the economic independence of great masses of the population carries with it serious dangers for the general civic existence, for society, for morals and for the family life. **The economic and social polity should not regard these dangerous to themselves with indifference; but should, as far as possible, intervene as protectors, exercising a certain degree of supervision.**

Just as in the case of the mechanical industries so also in Agriculture a transition to large-scale business due to the improvement of communications must take place, although not under the same conditions nor with the same degree of compulsion. Agricultural business can only maintain itself in the face of facilitated imports by reducing its working expenses to the lowest degree. The introduction of machinery thus becomes imperative; this, however, is only practicable with success in large-scale business. And in addition there is the fact already mentioned, viz., that the price of natural products is regulated with improved means of transport by the general or world-market, and only varies within narrow limits; consequently, the net-profits of agriculture fluctuate to a considerable degree with the harvest-outturn. **This variability in the annual outturn can only be successfully withstood by landowners sufficiently provided either with Capital or Credit:** whereas the small agriculturists fall at once into the hands of the money-lender and are ruined. Here, also, as in the mechanical industries, a healthy economic policy carried out energetically and with a clear view of the object to be attained must employ every means to preserve the undeniable blessings of large-scale business and at the same time to minimize the pernicious results to human society and welfare that unavoidably arise therefrom.

There is a further danger which threatens Agriculture, viz., that arising from **the transformation of the residential conditions of the populations** brought about by facility of communications. When communications were as yet undeveloped the populations of towns often suffered severely during extreme variations in price of the necessaries of existence, due to the variability of the outturn of the local harvests: whereas, the country populations were exposed to only comparatively slight variations in their incomes, since prices rose with bad harvests and fell in the same degree with good ones. The towns, in order to protect themselves in some measure against these evils, were compelled as far as possible to take to agriculture and cattle-rearing; with the result that the increase of population was kept within reasonable limits.

In addition, the very limited area of sale for the products of mechanical industry hindered a further increase of the towns. In former times towns could increase and prosper only when they possessed good natural means of transport by water.

With the improvement in the means of communication the relations between Town and Country have become *completely inverted*. The prices of the natural productions of the soil now no longer depend on the outturn of local harvests but are identical with the more uniform ones obtaining in the open or world-market: The towns are thus assured a regulated subsistence; whereas the welfare of the agricultural population is dependent on the variable outturn of the harvests. As a consequence of this we have the wholesale emigration of the country populations into the towns. This influx, coupled with the extension of industrial activity, promotes the creation of new working classes. The consequent increasing thinning of the country population aided by emigration, will render the introduction of machinery increasingly necessary, and will accelerate the extinction of small properties.

The increase of towns leads undoubtedly to a rapid development of civilization: it induces the development of large and comprehensive measures for the promotion of health, comfort, and agreeableness; and conduces energetically to the advancement of all branches of Science and Art. But the increasing severity of the struggle for existence associated with the increasing density of population, the unrest arising from the haste to become rich, and the desire to enjoy leading to vice and crime, are certainly not to be reckoned amongst the beneficial consequences of the improvement of communications. These phenomena should not however discourage; for concurrently evil has lost the protection of concealment and exclusiveness and is thus brought within the power of the Law without a chance of escape.

In purely **political** matters the improvement in communications exhibits itself as promoting the formation of large States. But it is only when masses of peoples thus brought

together are originally related that the building up of a great State is in this way strengthened and promoted; whereas the foreign race-elements present gain new powers of resistance against absorbtion by the dominant population. Gunpowder when distributed in single grains through mass of sand is innocuous: but if the individual grains are brought into contact one with the other—thus rendering possible a simultaneous ignition—these same grains can occasion violent disruptive effects. Similarly, sections of society now inimical and dangerous which before the improvement in communications were unable to combine in action and which were accordingly repressed without difficulty, now become powerful by the facilities of travel and dangerous from their greater capability of united action. Every foreign element in a people which formerly had almost escaped notice and had been regarded with indifference becomes after the improvement of communications a veritable thorn in the flesh; just as salt distributed in a food is a pleasant relish, but the reverse when it occurs in a lump. Different peoples who formerly lived together in mutual peace, now with the advent of improved means of communication fall to blows and strife which can end only with the suppression of one or the other. And similar results are seen, although not with the same distinctness, in the living side by side of the adherents of different religious professions.*

Thus in the distribution of peoples, just as in the domain of economics, the improvement of communications leads to distinctive groupings.

The peculiar character of **locality** thus becomes more distinctly marked and acquires in every way a vast importance for precisely the same reasons which weaken the effects of distance and liberate mankind from attachment to the soil. The facility now attained of change of place gives men an increased power over Nature and leads them to loftier aims in life. All the beauties of Nature, the treasures of art and learning, formerly only accessible to the favoured few, have thus become the common property of mankind. Buds of spiritual life which formerly mostly withered where they grew now find in enlarged space the condition most favourable to their development.

[* All these are allusions to the political conditions of the new German Empire.—Tr.]

APPENDIX.

APPENDIX A.

CLASSIFICATION OF RAILWAYS.

In Germany Railways are, for technical purposes, classified as :—

(I) Main-lines, (Vollbahnen) :

(II) Subsidiary, (Nebenbahnen,) or Secondary, or Vicinal lines :

(III) Local, (Kleinbahnen,) Minor, or Tertiary, lines.

The exact extent of meaning of the terms, "Main" and "Subsidiary" is in Germany defined by law : so also in Prussia is that of the III Class, or Local lines.

The distinction drawn between the above classes of lines does not depend on the gauge —as might be imagined—for all of them may be either Normal-gauge ($1{\cdot}435^{m}$.) or narrow; and the first two may also be broad-gauge. The distinction depends on the nature of the traffic, actual, or possible—in the case of new projects—on the manner of working, and on the particular regulations in force thereon—whether those fixed by law, or those in force by the joint mutual agreement of the Railway Administrations—viz. "the Technische Vereinbarungen"—or "Standard Conventions" answering, in a way, to the Indian "Standard Dimensions."

I. To **Main** lines of the First Class belong especially those lines which mainly carry through-traffic over great distances, viz., international business, connecting two terminals by the shortest possible route. Local interests are quite subordinate. Lateral deviations are to be avoided, even when this involves leaving aside the larger towns. In such lines the great object to be attained is cheapness of working, in comparison wherewith cost of construction is quite a secondary consideration.

Main lines of the Second Class also include those lines whose object is to join-up the more important centres of traffic of a country. Détours to important towns are not only permissible but desirable, together with severer curvature and gradients. They may also under these circumstances possibly come in for a share, more or less, of the through-traffic.

There are many lines of characters lying between these two classes, and a precise discrimination in every case is not possible.

II. **Subsidiary** lines are those of which the object is to cheaply connect out-of-the-way, remote, and unimportant districts with Main lines; and to confer the advantages of improved communications on populations of less favoured districts; at the same time these lines often act as feeders to Main lines (Local traffic). They are usually of moderate length: of normal-gauge, similar in all essentials to Main lines of which they are frequently only branches. The maximum speed is limited to 25 miles, the gradients to $\frac{1}{40}$ and the minimum radius of curves to 180^{m}. In Austria a more general extension of the term includes,

III. **Tertiary** or Local lines, viz., Industrial lines—usually private undertakings—connecting factories —often lying in valleys or on plateaux—with Main lines, etc. They may be of either normal- or narrow-gauge, 1^{m} to $\cdot 75^{m}$; they have small passenger-traffic, and a goods-traffic of raw materials: are of short length and have stiff grades $\left(\frac{1}{25}\right)$ and curves [1^{m} gauge: $R=60^{m}$: $\cdot 75^{m}$ gauge, $R = 40^{m}$: $\cdot 50^{m}$ gauge, $R = 25^{m}$] and the maximum speed thereon is 30 km/hr. They differ amongst themselves both in construction and in their economic importance, and are usually only worked during day-light.

"Kleinbahnen" in Prussia constitute a class apart from the above. They comprise suburban tramways and street-railways, whether worked by horse, steam, rope, gas, or electricity. They are not subject to the Railway Law of 1838.—[Tr.]

APPENDIX B.

Note on the "Verein."

The growth and development of Railways in Central Europe has been greatly promoted by the existence of the organisation known as the "Verein Deutscher Eisenbahn-Verwaltungen", or "Union of German Railway Administrations."

From its earliest beginnings the Verein had recognised that its most important function was the introduction of uniformity in all Engineering and traffic matters; and to this end it instituted **Technical Conferences** at which the representatives of all the Railway Administrations assembled for general discussion. It is perhaps hardly necessary to point out what great importance and value such an institution must have where the many-sided experiences of its members drawn from the most varied circumstances are, as it were, focussed to a single point of view and discussed by eminent specialists of different schools.

These Technical Conferences produced a mass of material of the highest value, the publication of which in the Supplement Volumes of the "Organ für die Fortschritte des Eisenbahnwesens in technischer Beziehung" constitutes a mine of wealth of engineering science.

The earliest of these Conferences was held at Berlin (February 1850), since when they have been continued at odd intervals of years down to the present time. The meetings at Hamburg, Constanz, and Gratz are notable for the production and publication of revised "Grundzüge für die Gestaltung der Eisenbahnen Deutschlands" (Principles for the Formation of German Railways) which, since the Dresden Meeting, have become celebrated under the title of "Technische Vereinbarungen des Vereins Deutscher Eisenbahnverwaltungen" (Technical Decisions of the Union of the German Railway Administrations), standardising and regulating the construction and traffic arrangements of main lines.

These "Decisions" are practically the expression of the sublimated opinions of the most eminent Railway Engineers in the German Empire on disputed points of Railway practice. Although devoid of any official governmental character, they possess an authoritative effect in themselves, indicating as they do the high water mark of existing railway knowledge; and they accordingly form the standard of technical practice in the German railway-world.—[Tr.]

APPENDIX C.

Collections of the Rules and Regulations in force.

In connexion with the preparatory technical studies of a project of a new railway there are a number of Rules and Regulations to be observed which have grown-up in the course of time and become standard.

Such Regulations have been drawn up by the Verein, or Union of German Railway Administrations; some of them are unconditionally binding on all the railways in membership with the Verein, while others are only more or less so.

In regard to the different characters of the lines, there is a distinction drawn between Main lines and Secondary lines on the one hand, and Local lines on the other; and corresponding to this classification there are two independent collections of Rules and Regulations. Under the term Local Lines, is included also the so-called small railways coming under the (Kleinbahm) law of 28th July 1892, when worked by steam.

The difference between Main and Secondary lines is defined in two collections which regulate railway practice within the domain of the Verein. These non-official collections of Regulations are :—

(1) The Technische Vereinbarungen (briefly referred to in technical literature as T.V.) or Technical Resolutions agreed upon for the standardizing of the construction and working of **Main** and **Secondary** railways. (Berlin Technical Congress: 28th, 29th, and 30th July 1896).

(2) The Grundzüge(= Grz. f. L.)—or Principles for the construction and working arrangements of **Local** lines. (Berlin: July 1896).

In addition there are two Codes of Regulations issued in 1892 and binding by law in the German Empire. These although differing in points are closely modelled on the above unofficial Regulations of the Verein. The Regulations relating to Secondary and Local lines are comprised in one collection, and not the Main and Secondary, as might be expected; and Kleinbahnen as defined by Prussian law, *i.e.* trams and street railways, are not noticed at all.

These Codes are the following :—

(1) Standards for the building and equipment of German Main lines (Briefly referred to in technical literature as N. F. H).

(2) Traffic Regulations for German Main lines. (= Bt.*O*)

(3) Regulations for German Secondary lines (= Bhn. *O.*)

The above have lately been slightly modified by the Imperial Chancellor on the 24th March 1897.

[**Handbuch der Ingenieurwissenschaften**: Vol I, Part I, p. 33].

APPENDIX D.

Rail vs. Water Transportation; a Defence of the Latter.

To

THE EDITOR,

"ENGINEERING NEWS,—"

Sir: In your issue of Oct. 26* you announce that a railroad has a new locomotive; which phenomenon inspires an editorial entitled "The Revolution in Railway Transportation." The motive of which seems to be the theory that under ideal conditions and "neglecting terminal charges" a railway can move freight in 2,000-ton train loads, and for a mill per ton-mile.

(1) Wherein does the "revolution" consist? This is certainly no news to those who keep reasonably well posted on transportation matters. But it does not mean that many men now living will get freight transported by rail at any such rate. Between that dream and its fulfilment interpose two facts, viz., "terminal charges" and the percentage of efficiency of the railway as a machine.

(2) The same editorial indicates very clearly (assuming your figures as unital) that this efficiency is about 25%, and that on a railway approximating your ideal conditions the mill per mile vision materializes as a 4-mill rate. I allude to your statement that in 1898 this rate ruled on the Pittsburg & Bessemer Ry. The builders of this much-talked-of road were advised (free and at a price) by all the doctors and announced that it should be the latest and best thing in railroading and give the lowest freight rates possible by rail transportation. The outcome is the 4-mill rate—certainly not revolutionary, and probably disappointing.

This is not good stuff to make dreams of; and its force is attempted to be broken by the statement "The traffic" (on the P. & B.) "is almost wholly in one direction and the length of haul comparatively short, making the terminal expenses a large proportion of the total."

Now the return traffic is there in abundance: and other roads (as the Pennsylvania) haul it, presumably because they can haul it cheaper. The terminal expenses on the P. & B. ought to be less than on any trunk line because one terminal is an ore dock where the cars are loaded with a steam shovel, and the other is a stock-pile where the cars are dumped, and which costs the railway not one cent.

(3) It is erroneous to assume that through trunk-line traffic can be handled at a lower rate than local bulk freights. The volume of the latter is more regular: and such freights generally do not cost the railway a cent for terminals, being loaded and unloaded on the patrons' switch.

(4) On the contrary, traffic such as interests New York requires terminals of the most expensive kind; and the volume is less regular. A close reasoner would expect to find, what actually obtains, the cost of through freights higher than costs for local bulk freights. The Pennsylvania, which handles 10 tons "local" to one "through," conducts transportation 1/4 cheaper than the New York Central, which handles 2 tons "through" to 1 "local." The latter road, which, more nearly than any other trunk line, approximates your ideal physical conditions, charges rates averaging more than five times the mill per ton-mile editorially set forth as its proper compensation.

[* See excerpt therefrom on p. 15.—TR.]

(5) If the people of this country were led to believe that railway freight can be carried for one mill per ton-mile, they would believe that the railways do it; and they would believe the railway officials to be a gang of thieves for whom no treatment could be too severe; and legislation would be so drastic as to smash our business organization and bring our governmental system to anarchy.

Such fallacious reasoning, while out of place in a sober professional journal, and only likely to breed mischief, is less surprising than the misstatements of fact which characterized the editorial in question, as witness the following:

American railway managers have proved to the world that with the steel rail as a roadway, and steam for motive power, freight can be moved far more cheaply than in any artificial waterway or river channel. * * * * *

We have shown above that the railway can carry freight at less cost than any inland waterway, river or canal.

No proof of these assertions was printed in the editorial in question; and no such proof exists. On the contrary, oft-published and well-authenticated statistics prove the very reverse of these statements.

(6) As long ago as 1891 the whole cost of bulk freights on the Ohio and Mississippi rivers was given, in official reports, as half a mill per ton-mile; and recent statistics show much lower costs. Recent reports have given instances of rates on small barge canals in England of 1/4 mill (this figure is from memory, see "Water Commerce Congress," 1893; and others). It is well known that barge transportation on the St. Lawrence approximates lake transportation in cheapness. The cost on the Erie Canal ditch and Hudson River, in 240-ton barges, is even now only 1.5 mills, with more than half the season wasted in port and a considerable part of the remainder lost in unnecessary locks. A sensible revision of terminal arrangements and locks would enable the Erie boatmen to make enough more trips to cut the rate below the mill per ton-mile figure.

The report of Maj. T. W. Symons gives an estimated rate on an enlarged barge navigation of 0.54 mill with the barges losing half their season in ports.

The figures published by me in 1895, based on then-existing conditions, and substantiated by all subsequent valid data, gave the estimated transportation costs between Chicago and New York via the proposed St. Lawrence, Champlain-Hudson ship canal, in a 7,000-ton ship, reduced to ton-mile rates, as follows:

	Mill.
With Full Return Cargoes.	
Conducting transportation	·011
With profits and amortization added	·017
With terminal charges added	·028
With Half Return Cargoes.	
Conducting transportation	·015
With profits and amortization added	·024
With terminal charges added	·035

Further citations are not worth while just now because the forthcoming reports of the N. Y. State Committee on Canals and the U. S. Board of Engineers on deep waterways will shortly bring the data up to date.

The editorial states "Terminals for railways are in general cheaper than those for waterways."

This has no application to canal boat freights, which pay no wharf rent in this port. So far as it applies to the matter, it is an argument in favor of a ship canal and the abolition of breaking bulk at Buffalo. The interest on terminals is a fixed amount; and the longer the haul, the less it amounts to per ton-mile. On shipments from Chicago, taking present pier rentals as a basis, and supposing all the freight to be handled on a pier doing half the business it is capable of, and the entire interest paid by the incoming freight, it would amount to less than 1-50 of a mill per ton-mile. The editorial statement may be true "in general," in

small towns; but in particular, in great ports such as this, it is untrue, as is patent to those who note the great expense and difficulty of enlarging our railway terminals. Every few days I pass a terminal built by one railway which has cost more, probably, than New York has spent on her water front in ten years.

New York water front property taken in condemnation proceedings a few years since, cost about $450 per front foot, or 60 cts. per sq. ft.; and first-class water front property, on the new 40-ft. channel and 1,800 ft. from street to pier line, can be bought for $600 a front ft., or 30 cts. a square foot. The improvements, also, are cheap and efficient. One steamship company here receives and dispatches a 3,000 to 3,500-ton ship every day, using one medium-sized pier.

In view of what precedes it, one cannot wonder that the editorial should "point out that it is to the railways and not to the canal that New York must look for the preservation of its export trade." This sounds like a jest, unless, indeed, the suggestion of a more recent editorial is seriously meant, that New York state should build and maintain a trunk line railroad. It is well known that the railways have for years maintained a heavy differential against the port of New York; and experts know the reason for it, and that it cannot be done away with for at least a generation, except by a modern waterway or a state railway to the Great Lakes, with complete connections and terminals.

(7) The facts are as follows: The only trunk line railroad vitally interested in having rates favor New York is the New York Central. Unfortunately this road has not nursed industry along its line and consequently has a relatively small local tonnage, and depends for its existence on a profitable through traffic.

On the contrary, the Pennsylvania, the Colossus of railways, has coddled its local industries until almost its entire length realizes your ideal of "freight delivered wherever a spur track can be run," and is supported by a local tonnage 10 times its through business. Therefore the Pennsylvania is master of the situation, and will be, so long as controlling rates to the seaboard are made by private corporations. In a rate war its vast preponderance of local business would enable the Pennsylvania to make and maintain through rates which would bankrupt the New York Central in two seasons. The people of this State will not heed your advice because it is contrary to facts and logic.

(8) New York city is a merchant, and lives by the profits of exchange. A merchant cannot long succeed if he permits his rivals to control his trade. The rivals of New York do control New York's traffic in the vital matter of freight rates. If this condition be long continued, the bulk of trade will go where it can go cheapest; and wealth and population will follow it.

These conditions inhere in the existing transportation arrangements, by virtue of which private corporations, not subject to control by New York state, can make the controlling freight rates to the seaboard. New York cannot remedy the situation by legislation affecting railways because the railways which control the situation lie without the state. Attempts at legislative cure-alls would ruin our state corporations without benefiting the situation. But New York can control the situation and her own rivals' business by complying with the laws of trade.

The only practicable water routes from the Great Lakes lead to New York. No other Atlantic port in the United States can be connected with the interior by a good water route at a cost commercially practicable. New York can be connected with the Great Lakes by a 12-ft. barge navigation which can move freight at 1-10 the prevailing rail rates and ½ the dream rate; and by a ship canal which can move freight at 1-20 the prevailing rail rates and ¼ the dream rate.

Wise policy will lead New York to do one or both these things, and forever assure her citizens the controlling freight rates. So to do will not only give her citizens control of

commerce, but also save and profitably invest at least a small part of their surplus earnings. This aspect of the matter should not be overlooked. The only investment savings of the great mass of wage-earners are those invested in wise public works. The average man cannot attain to invested remunerative personal savings. At best he can own a home, educate his children, provide his family the things usual to their condition, and by life insurance, protect his widow and orphans from the worst shocks of fortune.

But the state, by wise public works, not only makes investment savings for the wage-earner, but also for his posterity, saving the present surplus and also that which is to be, and augmenting both, and bringing immediate and constant returns to the wage-earner in higher wages with greater purchasing power. Such works as above contempleted are the best and wisest of their kind; in the present instance they are the only wise procedure: for, whatever our opinion may be as to future state or national ownership or regulation of railways, every intelligent man must admit that the peculiarities of our federal system, and the extreme conservatism inherent in its structure, make it extremely unlikely that such a result can be attained except by long and painful experiments, which will stretch beyond the lives of us and our children. The twelve years' experiment with the Interstate Commerce law certainly confirms this view. To begin experiments in state legislative cures may benefit some future generation, but hardly this—a result desirable but not satisfying.

A state railway would prove a costly disappointment. The Pittsburg & Bessemer road and its 4-mill rate are conclusive evidence to that fact. That road is up to date and cannot be much improved on at present. A state railway, to be of practical value to New York's commerce, must have complete railway connections at Buffalo and terminals at both ends: and the varied character of the commodities it must handle makes it certain that its terminal costs would greatly exceed those of the Pittsburg & Bessemer, which was built to relieve at most two lines of business. It is therefore certain that the suggested state railway could not handle the freights of New York as cheaply as the P. & B. can handle its limited list of commodities, and the resulting freight rates would be little if any lower than the 4 mills obtaining on the P. & B., even allowing that there are no charges for interest and maintenance of way.

On the other hand, rates less than a mill per mile are certain to result from building either the 12-ft. barge canal, or the 20-ft. ship canal—facts which have been theoretically demonstrated and will be fully substantiated by the forthcoming reports above referred to. That of the U. S. Board of Engineers on Deep Waterways in especial, I hear it said, will be an engineering classic, and give the latest and fullest data relating to this most important subject.

Respectfully yours,
Chauncey N. Dutton.

Bowling Green Building, New York city, Nov. 18, 1899.

We have taken the liberty of numbering certain paragraphs, in our correspondent's letter, for greater clearness in replying to some of his statements:

(1) We do not know what our correspondent means by the "percentage of efficiency of the railway as a machine." As nearly as we can gather, however, he means the ratio between the actual cost to a railway company of handling traffic (including in this cost interest on its invested capital), and the rate which it actually charges. Now, as a matter of fact, a railway company does not take into account the cost of moving traffic in making its rates. It figures on what the traffic will bear, and makes its rates to suit that. The Pittsburg, Bessemer & Lake Erie R. R. carries its freight at the lowest cost that its managers can secure by exercise of their best abilities, but it charges for this service all it can get, and so long as railways are run on the competitive basis is within its rights in so doing.

(2) Our correspondent thinks experience with the P., B. & L. E. R. R. is "probably disappointing." Against this we may very fairly set the remarks of Mr. Andrew

Carnegie, who knows a thing or two about railroading in general and about the railroad referred to in particular. In an address before the Pittsburg Chamber of Commerce, Nov. 10, 1898, Mr. Carnegie said:

> I was the first to suggest that the abandoned canal should be replaced by a deeper one which should lead the waters of Lake Erie into the Ohio. Conditions, however, have changed. Such has been the progress of railway development that if we had a canal to-day from Lake Erie through the Ohio Valley to Beaver, and it was opened free of toll like the Erie Canal, we could not afford to put boats on it. * * * It is cheaper to-day to transfer the ore to 50-ton cars and bring it to our works in Pittsburg over our railway than it would be to bring it by canal.

That does not sound to us like disappointment.

(3) If our correspondent would take a job as traffic manager of a railway, he might change his mind concerning the comparative cost per ton-mile of handling local and through freights. As the next thing we suggest that he take a day's ride in the caboose of a through freight and see what an astonishing number of ton-miles are manufactured in the course of ten or twelve hours. If he will then spend another day on a way freight, picking up and setting off cars at every station, he will come to appreciate how much smaller is the output in ton-miles for a given expenditure of labor and a given investment. As for local traffic "not costing the railways a cent for terminals," a very large proportion of the through traffic as well as the local is loaded and unloaded on private sidings. The cutting out and switching of cars to and from these sidings constitute "terminal expenses," however, and is an item of no small proportion in the railway expense account.

(4) We warn our correspondent that it is easy to draw entirely erroneous conclusions from averages of ton-mile rates. His statement that the New York Central carries 2 tons through freight to 1 of local is wide of the mark. The annual report of the New York Central company shows about six times as many tons of local freight handled as of through freight. The difference in the average ton-mile rate of the Pennsylvania and New York Central is not due, therefore, to the former company handling local freight and the latter through freight. We believe it to be due to the great volume of soft coal traffic on the Pennsylvania, which is moved at very low rates, sometimes, we believe, as low as 2 mills per ton-mile.

(5) We have stated plainly under what conditions the railways can move bulk freight traffic at very low rates. One of these conditions, and perhaps the most important one, is that they shall have the traffic to move. So far as the waterway routes are successful in diverting traffic from the railway lines, they increase the costs and necessary scale of charges for rail transportation.

(6) We freely admit that coal from the upper Ohio is carried down the Mississippi, by the peculiar system of transportation which has grown up there, at a rate which is given by Major Symons as 71 cts. per ton for a voyage of 1,970 miles, or 0·36 mill per ton-mile. We know of no other instance anywhere of any such rate being reached for freight transportation by river or canal, and are compelled to doubt the accuracy of Mr. Dutton's memory respecting rates on English barge canals. The important point which we wish to emphasize, however, is that practically all published statements of rates for water transportation, and especially of rates on canals, are seriously misleading in that no allowance whatever is made for the interest on the money spent in creating and improving the waterway or for the annual expenditure for its maintenance.

Now if any fair comparison of the costs of rail and water transportation is to be made, it must evidently be upon an equal basis. If in the case of the Erie Canal we are to omit the interest upon its present value as an investment to the state and the state's annual expenditure upon it, then we should also omit in computing the cost of rail transportation the interest on the cost of the roadway and the annual expenditures for maintenance of way.

The proper way, however, is undeniably to consider all the expenses of the waterway and railway alike. Figure Erie Canal rates on this basis, and they become something quite different from the 1¼ mills per-ton mile which our correspondent claims.

As for the proposed large barge canals and ship canals, the case becomes much worse, as the investment and maintenance account are so greatly increased. Doubtless Mr. Dutton will say the since the Government builds and maintains the canal it is proper to omit its expenditures from the cost of transportation; but this seems to us fallacious and dangerous reasoning. What the public wants to know is what is the cheapest machine for moving freight, the railway or the waterway, and it is entitled to a fair and honest answer.

(7) Since Mr. Dutton's premises as to the proportion of local and through traffic on the New York Central are so faulty, it is reasonable to doubt his conclusions.

The remainder of our correspondent's letter contains some matters already answered above, and others with which we are far from agreement, but to which time and space do not now permit a reply.—Ed.)

Engineering News: *23rd Nov. 1899.*

Railways vs. Canals—*continued.*

From one standpoint the new line may seem a roundabout route from the anthracite fields to New York, but the distance is partially offset by the very favourable grades. It is said that it will be possible to move train loads of 2,000 tons from the collieries to the terminus at Kingston. Here connection can be made with the West Shore Road which, with its ample terminals at Weehawken, should be able to handle a very large coal traffic. The bulk of the coal, however, will doubtless go by water from this point, and as the depth of the Hudson from New York to Kingston is sufficient for large ocean steamers, the cost of this haul should be very small. It is true that the Hudson is closed by ice during part of the winter; but the ice is not so heavy that it would be a difficult or expensive matter to move barges through it, with an ice-breaking propeller to tow them, during any save the most severe winter weather.

A feature of great interest to engineers in connection with the proposed railway is that it follows for the whole distance the line of the old Delaware & Hudson Canal. We know of no more convincing object lesson of the superior economy of the railway to the canal as an agency for handling bulk freight than that here presented. As our readers will remember, the Delaware & Hudson Canal Co. decided a year ago to abandon its famous old canal, as it was found more costly to transport its coal to the New York market over it than no turn it over to the Erie R. R., and haul it over the by no means favourable grades of that road to tidewater at Jersey City. Now comes a new company, purchases the old canal and uses its right of way for a railway line to move exactly the same class of traffic that the canal was built to carry.

The public has often been told by the canal enthusiasts that the reason for decay and disuse of the old time canals was the machinations of the railway companies which competed with them, or the neglect of the public officials in charge, where the canals were under state control.

In the present case, however, the entire transaction has been in the hands of private corporations. The Delaware & Hudson Company had no motive whatever for abandoning its old canal, except the fact the cost of maintenance and operation was too great to make it profitable. The new corporation, if it could see economy in moving its freight by canal instead of by rail, would doubtless repair and use the old canal instead of building a railway line to replace it. Its managers believe it to be cheaper to build a new railway line and pay the interest on its cost than to attempt to maintain and operate a canal already built. We commend this fact to the thoughtful consideration of those who are so persistently urging the expenditure of state and national funds on waterway construction and who have not yet found out that the line of rails on solid ground has proved itself a cheaper highway for freight transporation than any shallow water channel, either natural or artificial.

Engineering News:—" A New Outlet for Anthracite Coal Traffic," 25th January 1900.

What shall New York do with its Canals?—(Extracts from the) Report of the Governor's Advisory Committee.

The commission appointed by Governor Roosevelt in March last to investigate and decide upon the wisest policy for the State of New York to pursue towards its canals has completed its work, and on Jan. 15, presented to the Governor its formal report. This is contained in an octavo volume of 231 pages, of which 43 pages are devoted to the report proper, and the remainder to a number of appendices in which some of the data made use of by the committee in reaching its conclusions are set forth at length. In a separate volume. the correspondence and opinions of various engineers and other experts upon the proper policy for New York to pursue with reference to its canals are given in full. The two volumes constitute a most valuable contribution to engineering literature, and should be obtained and preserved by all engineers interested in modern methods of economic transportation.

Development of Water Transportation in Europe.

In considering this question of the relative advantages and cost of rail and water transportation, we have given much study to what is being done on thé Continent of Europe; and one of our committee, Mr. F. S. Witherbee, has visited Europe for the purpose of gaining information on this point. His report is transmitted herewith, and a large number of documents, plans and photographs which he brought back have been deposited in the office of the State Engineer. It is found that on the Continent of Europe so far from the canals being decadent during the last 30 years, they have been constantly enlarged and improved, enormous sums having been spent for this purpose, and the result has been an extraordinary increase in this class of transportation. It is well known that the railroad rates in Europe are much higher than in America. There are several reasons for this. In Europe there is none of the long-haul traffic, which is so much less expensive to carry, and accounts for so large a part of the lower ton mile rate in America. The management of the railroads is also less efficient. On the other hand, the management of the canals has been more efficient than with us. The result has been a far greater development in water traffic than in rail traffic during recent years in France, Belgium, Germany and Russia.

In France, since the war with Prussia, over 400 miles of new canals, and nearly 500 miles of new river navigation have been constructed, making nearly 7,000 miles of internal water-ways: the water traffic has increased from 1872 to 1897 by 140%, whereas the rail traffic has increased by but 75%. The little State of Belgium has expended since 1860 not less than $50,000,000 for enlarging its canals, and the water traffic increased from 1888 to 1896 by nearly 40%, and it is significant that the increase in the transportation of miscellaneous package commodities during the same period was 54%.

In Germany, the same process of betterment and extension of canals and water routes is continued. During the past year the new canal from Dortmund to Emden has been completed, and opened for traffic; this canal being especially noteworthy for the famous pneumatic lock at Henrichenburg, where vessels are lifted 45 ft. from one canal level to another at one operation. The modern type of canal boat in use on this canal is a barge of 1,000 tons carrying capacity, built of steel, about 230 ft. long, 30 ft. wide, and 7½ ft. draft, and costing only $5,000 each. The propulsion is entirely mechanical—either by steam or electricity. It is well known that the German Government is planning a trunk route between the rivers Rhine and Elbe, and is strongly in favor of a large extension of its canal system; and that its plans would now be in process of being carried out but for the opposition of the agricultural interest, which fears the effects upon its own property of the reduction in rates which would certainly follow the execution of these plans.

In Russia, even greater efforts have been devoted to the development of the water routes on canals and rivers, the sum of $30,000,000 having been expended from 1891 to 1896 for this purpose, and in the same period the internal water traffic has increased by 70%.

This traffic on Russian internal waters accommodates 1,500 steamers, and 60,000 canal boats, with crews numbering 300,000 men. Vessels 200 ft. long can traverse the whole length of the country from the Caspian Sea to St. Petersburg or Archangel (2,500 miles).

We do not think that these facts can be overlooked in the consideration of this problem. They show that in countries where the keenest competition exists not only within each country, but between each and its neighbour, effort is being made to gain an advantage, or, at least, keep on an equality, in the competition, by reducing the rates of transportation, and that to accomplish this large sums of public money are being spent to enlarge and improve the water routes; thus confirming the general proposition that under equal conditions of management the water route, even in a restricted way like a canal, is cheaper than the rail route.

Relative Cost of Rail and Water Transportation.

In our judgment, water transportation is inherently cheaper than rail transportation. It varies slightly with the size of the vessel and the restriction of the waterway. On the ocean, where the waterway is entirely unrestricted and the size of the vessel is the maximum, it averages about half a mill per ton mile; * on the lakes, where the vessels are not so large, and occasional restrictions are encountered on the waterway, it is about six-tenths of a mill per ton mile; † on the canals of New York where the boats are very small, the waterway greatly restricted, and obsolete methods are employed for handling the business, it is about 2 mills per ton mile. By the enlargement of the canal which we recommend, and the introduction of improved methods of management, we believe that the canal rate can be reduced to two-thirds of one mill per ton mile, or very nearly as low as the lake rates. All of these rates have varied in the past and will vary in the future to correspond with prosperity or depression in general business. But there is every reason to believe that they will maintain a corresponding ratio, the ocean, lake, and canal rates being from one-third to one-fourth of those by rail. The reductions which may be made hereafter in the railroad rate can be met by similar reductions in all three classes of the water rates, provided the same methods of skilled management are applied to all.

Moreover, the canals have been largely limited in the past to the lower grades of freight, and this is equally true of the transportation on the lakes. The canal has thus been in competition with the classes of freight which pay only between 2 and 3 mills per ton mile, and which the railroads will carry at a loss rather than lose the business, whereas, the railroads carry other classes of freight, some of which brings as high as 15 to 20 mills per ton mile, and the average freight, including the low grades, as we have seen, being about 6 mills. There is no reason why the canals, if enlarged and properly managed, should not compete for the higher grades of freight, which, at prices far below those charged by the railroad, would bring very profitable returns on the lakes and canal.

Why Railways obtain Traffic at Higher Rates than the Canal.

The statistics which accompany this report show that in 1868 the canals carried 44% of the tonnage across the State, and in 1898 only 5%. In the matter of grain (including flour) in 1868 the canals carried 76%, and in 1898 10%. Yet during all of these 30 years, the rail rate has always been in excess of the canal rate. There must be a reason why shippers and merchants are willing to pay more for transporting grain and other articles by rail than by canal, and the reason is chiefly because the railroad conducts the business according to modern methods and the canals do not. There is, in our judgment, no reason why the same business methods cannot be applied to the canals as to the railroads; and if they are applied they will produce an equally satisfactory bill of lading, equal certainty in the time of delivery and equal responsibility on the part of the carrier.

* It is stated by Mr. E. L. Corthell (Minutes and Correspondence, page 89), that wheat has been carried from California to England for 3-10 mill per ton mile, and coal on the return trip for 1-5 mill.

† On the lakes return cargoes of coal are carried from Lake Erie to Lake Superior ports, about 1,000 miles, or 25 cents or ¼ mill per ton mile.

In order to accomplish this, so much of chapter 934 of the laws of 1896 as limits the amount of capital which shall be employed in the business of canal transportation should be repealed. This law reads as follows: "No corporation organized under this Act, and designed to navigate any of the canals of the State, shall have a capital stock exceeding $50,000." It has been charged on the one hand that this law was passed at the instance of the railroads in order to destroy the usefulness of the canals; and, on the other hand, it is asserted that it was passed for the benefit of the boatmen in order to prevent the formation of large corporations, which, by greater economy, could first drive the small boatmen out of the business, and then by some alliance or understanding with the railroads, increase the rates. Whatever the origin of the law may have been, it has proved in practice to be of no benefit to the boatmen. Their business has continued to diminish and to grow still less profitable year by year since this law was passed. They do not make living wages under existing conditions, and they cannot. They are attempting to maintain an antiquated method of business in competition with the modern methods which have brought about the extraordinary increase of wealth during the last 30 years. They cannot possibly succeed, and the the State is not justified in expending any more public money unless the conditions are so changed as to derive the full benefit from its investment. We therefore recommend in the most positive terms that the above quoted law of 1896 be repealed.

Railway Competition of the Future.

No one disputes these evident facts; but the question which now confronts us is whether the railroads, with their large capital and scientific management, their durable road-beds, powerful locomotives, larger cars, greater train loads, greater speed and more certainty of delivery, will be able now or in the early future to reduce the cost of transportation below what is possible on the canals. If they can do this, then it is obviously unwise and improper to expend any more public money upon a method of transportation which, however important in the past, would no longer be able to compete with other and improved methods. The determination of this question seems to us to lie at the very foundation of the canal problem, and we have therefore given it the utmost attention.

The claim for the railroads has been put forward at great length, and with ability, by the "Engineering News," whose editorial articles on the subject are printed at length in the volume of "Minutes and Correspondence." In brief, they are to the effect that while the average railroad charges in recent years on the railroads of New York State have been about 6 mills per ton mile, yet a lower rate has prevailed on grain, lumber and similar articles, which have hitherto formed the bulk of the goods transported over the canal. The grain rates fixed in April 1899, from Buffalo to New York were as follows per bushel:

Wheat	$3\frac{1}{2}$ cts.
Rye	$3\frac{1}{4}$ "
Corn	$2\frac{3}{4}$ "
Oats	$2\frac{1}{2}$ "

The rate of $3\frac{1}{2}$ cts. per bushel on wheat is about 81·17 per ton, or $2\frac{1}{2}$ mills per ton mile. It is further argued in these articles that the Chesapeake & Ohio R. R. is carrying coal at a profit on a rate of $2\frac{1}{4}$ mills per ton mile; and that on the completion of locomotives now under construction by the New York Central and other railroads, designed to haul trains with from 2,000 to 2,400 tons of paying freight, the rate on such articles as grain, coal, ore, etc., by rail will be reduced to about 1 mill per ton mile. In other words, the argument in favour of the railways is that private enterprise and private capital will at an early date produce on the railroads as low a freight rate as can be produced on the canal by the expenditure of large sums of public money.

If this argument were correct, it is needless to say that no further money should be spent on the enlargement of the canals, but that they should remain in their present condition until plans could be carefully matured for the disposal of them. In our judgment, the argument is not correct. It would carry more weight if it were advanced or approved by practical

railway managers; and we therefore sent the articles to the presidents of the New York Central R. R., of the Illinois Central R. R., and of the Pittsburg, Bessemer & Lake Erie R. R., the last of which was specially built under the most favourable circumstances for the express purpose of carrying ores and low-grade freight at a minimum cost from Conneaut on the lakes to Pittsburg. The reply of Mr. Fish is explicit that there is no probability of a rate of 1 mill per ton mile by rail in the near future. The reply of Mr. Callaway, while not so positive, leaves no doubt in the mind of the reader that the New York Central R. R. has no expectation of quoting any such rate. The reply of Mr. Reed states that during the past summer nearly a million tons of ore were hauled from Conneaut to Pittsburg at an actual cost for transportation alone of 1¾ mills per ton mile: the freight rate being 3·65 mills per ton mile. It is evident, therefore, that the views expressed in the "Engineering News" are not sustained by practical railway managers, responsible to their stockholders for the profitable management of their roads.

The Canal as a Regulator of Rail Rates.

The late Mr. Albert Fink, than whom there was no higher authority on the transportation question, made a statement before the Windom Committee some 20 years ago that the Erie Canal regulated the rates not only on the railroads of New York State, but on every trunk line connecting the lakes with the Atlantic. This statement has never been successfully disputed, and it will continue to be true if the canals continue to keep pace with the railroads in enlargement and in management. If the canals are left stagnant, both in size and management, as they have been for a whole generation, while the railroads are improving year by year, then the time will come, and at a very early day, when this statement will cease to be true. To leave the canals in their present condition is virtually to abandon them. The Constitution of New York distinctly forbids this. For more than 80 years public money has been spent on the waterways connecting the Hudson with the lakes, and during 50 years these waterways were enlarged and improved to keep pace with the increase in the traffic and to decrease the rates. The State has made this expenditure for the purpose of utilizing its natural advantages and keeping within its own limits the route which should produce the minimum freight rate. We believe that the policy which has prevailed in the past, and which has been the chief factor in the commercial prosperity of this State, should be continued in the future. If these views are wrong, then it is in order to stop spending money on the canal and to propose an amendment to the Constitution which will permit of their abandonment and disposal by sale or otherwise. In our judgment, such an amendment to the Constitution would not receive even a respectable minority of votes.—*Engineering News: 1st Feb.* 1900.

Relative Cost of Rail and Water Transportation.

4. Transportation by water is inherently cheaper than by rail, and the less restricted the waterway the less the cost of transportation. The second part of this statement goes without argument. The truth of the first is doubtful.

That the cost of water transportation over long distances on large natural bodies of water, as the ocean and the great lakes, is cheaper than by rail, is probably true, though the margin of difference is much less than is generally supposed. But the cost of water transportation through a restricted artificial waterway is not inherently cheaper than by rail, and depends very much on the degree of restriction, the volume of traffic, and the cost of constructing, maintaining and operating the waterway. It is just as absurd to omit the elements, interest on first cost, depreciation, cost of ordinary maintenance, and operation of the permanent way, in estimating the cost of water transportation by artificial waterways as it is to omit these same elements from an estimate of the cost of rail transportation. For the State of New York to build and maintain a free waterway simply means that the State for some conceived benefit, assumes a part of the legitimate or illegitimate freight rate from the producer to the consumer. It means this, provided the rate after the canal is built is less than the rate before. If this result is not obtained, then the State has simply sunk its wealth in a useless ditch.

In discussing the cost of transportation the Committee uses the word cost in a very loose way; one time it means charge or tariff, and another actual cost to the carrier. The two are very different and bear no relation the one to the other. The tariff is not based on the cost of service, but on what the traffic will bear. The Committee states that on the ocean, where the waterway is entirely unrestricted and maximum vessels may be used, the rate averages one-half mill per ton mile; on the lake, $\frac{6}{10}$ mill per ton-mile, and it estimates that the enlarged Erie canal will develop a rate of $\frac{2}{3}$ mill per ton-mile. The average lake rate in 1898 was, on the freight through St. Mary's Falls canal, 0·79 mill per ton-mile. The wheat rate east was $\frac{2}{3}$ mill per ton-mile, and the iron ore rate was $\frac{3}{4}$ mill per ton mile. The return, or west-bound rate, fixed so low to get the business, was but $\frac{1}{4}$ to $\frac{3}{10}$ mill per ton-mile. With an average rate of 0·79 mill per ton-mile on the lakes, or even $\frac{6}{10}$ mill, it is very doubtful if a rate of $\frac{2}{3}$ mill will ever be quoted on the enlarged Erie canal. It is practically certain that the rate on wheat east-bound will not be so low on the canal as on lake. Let the Committee's estimate of cost of transportation on the canal be examined. It is based on full cargoes east, and one-third cargoes west, carried in fleets of four boats, a steamer and three barges of a combined capacity of 3,900 tons of wheat, making ten trips annually. Each fleet will thus carry yearly 52,000 tons. The value of the fleet is placed at $28,500.

Season's Expenses.

Wages and subsistence	$4,000·00
Fuel, oil waste, etc.	3,300·00
Ordinary repairs	300·00
Insurance on fleet	352·50
Insurance on down cargo	2,925·00
Miscellaneous small expenses	200·00
Interest on investment, 5 per cent.	1,425·00
Deterioration, etc., 5 per cent.	1,625·00
	$13·927·53

Cost per ton, 26 cents.

Cost per ton-mile, 0·52 of a mill.

The statement is further made that improved methods may still further reduce the cost.

It is useless to dispute the accuracy of the several items enumerated in the estimate, and they will be assumed correct. But where is the item of insurance on the return cargo? and where is the item of administration? This being an estimate of bare cost to the carrier, the item of profit is properly omitted; but this being so, the cost obtained must not be compared with quoted freight tariff, and it is manifestly unfair to develop the economic value of the proposed canal by comparing it with the present Erie canal, as the Committee has done, simply because the Constitution of the State prohibits the abandonment of that canal. The Constitution is not greater than the people. Moreover, this bare estimate of cost is further in error because the divisor used to obtain the ton-mile rate is too high; the total distance is not 500 miles. It is doubtful if all the freight will be through freight. The bulk of freight now carried is way, while the estimate is for fully loaded through trips east. There is nothing said of terminal charges, and the Committee is uncertain as to the economy of transhipment at Buffalo.

Is it not singular that this estimate, the work of a theorist, unverified by any transportation manager responsible to his stockholders, should be accepted by the Committee, while the estimates for the cost or rail transportation made by another theorist, of presumably equal ability, should be condemned? The managers consulted by the Committee did not deny the statements of the condemned estimate.

But these errors of omission of small details are not the most important errors in computing the cost of transportation on the enlarged canal. The omission of the items of

interest on first cost, maintenance, and operation of the great waterway are far more serious because such omissions are likely to mislead some people who look only at results and not at the methods of getting at those results. These omitted items are practically constant regardless of the traffic through the canal. The maintenance and operation may vary a little with the traffic, but not much. Therefore, the cost per ton-mile will depend on the tons moved and the distance these tons are carried. The Committee's estimate of the economic value of the canal is based on an annual tonnage of 10,000,000 tons. The interest on the capital to be invested figured at 3 per cent. is $1,800,000; the cost of maintenance and operation will almost certainly be more than as much more. Other estimates add a sinking fund to provide for the ultimate uselessness of the canal. The Committee adds the sinking fund to provide for the extinction of the debt in 18 years. Without any such additions, which without doubt the people must pay, it is plainly unduly favorable to the canal to estimate a perpetual annual outlay by the State of from $3,500,000 to $4,000,000. Assuming the Committee's estimate of 10,000,000 tons, which may not soon be realized, the annual charge to the State is from 35 cents to 40 cents per ton or, on the basis of the Committee's estimate of distance, from 7/10 of a mill to 8/10 of a mill per ton-mile. The total cost per ton-mile will thus be seen to be certainly as much as from one and one-fifth, to one and one-half mills per ton-mile. A particular point to be noted is that the State must pay by far the larger part of it, probably as much as 60 per cent. of the total legitimate cost.

It is not sufficient justification for the omission from the estimates of this important element of cost to say that the State Constitution provides that the canals shall be free. The Committee was asked to outline a policy. Unless it believed that the Constitution is right it should not have accepted it. The inference is, therefore, that the Committee believes the canals should be free; that the people should pay the greater part of the cost of transportation across the State in order to maintain the supremacy of the port of New York and develop manufactures; that the railroads shall pay a considerable portion of this charge for the privilege of having a portion of their business taken away from them. Ethically considered, this last-named policy is certainly open to criticism. Modern views of political economy regard Government as an ethical person whose acts must be based on ethical principles. It may be argued, as has been done by Prof. Haupt, that the canals will benefit the railroad just as it will the State; more business will be developed for the railroad. As has already been indicated, it is right to say that in the past, the canal has built up the business of the State, and has thus created business for the railroad. But this was done when the canal was the only known practical means of transportation. It remains to be shown that with totally changed conditions, with more railroads than are needed, with the actual cost of rail transportation almost if not quite as low as canal transportation, the same benefit to the railroad will accrue from the building of a canal as was secured to it in the past by the same agency. The Manchester Ship Canal may be cited as a case in point to prove that the canal benefits the railroad. The two cases have very different conditions.

It has even been asserted by some people that the New York Central Railroad will be benefited to an extent that would warrant it in paying itself the whole cost of the canal. This may be so, but it does not seem to be apparent that the practical managers of this company, responsible to its stockholders for the management of their property believe this to be true. They may be short-sighted. They use the best sight that has been given them and they are accredited in other matters with considerable discernment. To be sure, they may be lying back waiting for the State to undertake the work and thus save them the expense. If the railroad is to be damaged, can the State assume to destroy one industry of immense proportions to build another which is confessedly of less capacity, and which is established at great cost to the State? Again, suppose the canal benefits the railroad and the State, is it the only, or the cheapest, solution of the transportation problem? Why subsidize a water transportation company yet to be formed, when a less subsidy, if a regulating law would be unjust, will secure from an existing railroad the result sought?

Railroad Gazette: 2 *March*, 1900.

APPENDIX E.

On the Formation of Railway Rates.

(ENG. NEWS.)

It should be clearly understood at the outset—however, that to fix an absolutely equitable rate—one that shall be as fair to the railway company as to the shipper—is a practical impossibility; and it is equally impossible to define exactly what relation the rates on different classes of traffic should bear to each other. All fixing of railway rates, whether done by a railway corporation or a governmental body, whether for roads owned by private corporations or those owned by a State, must be at best an approximation and a rough one at that. It is no more possible to fix and assess an absolutely equitable charge on every class of traffic than it would be to fix the postage on each letter sent in proportion to the actual cost of its carriage and delivery.

What can be done, however, is to lay down the principles which should govern in the making of rates. Their actual determination must be a matter for good business judgment, guided by these principles.

The first of these principles is that each class of traffic ought to pay a rate to the railway company at least high enough to repay the operating expenses incurred in handling it. Upon the correctness of this principle there is uniform agreement. The most case-hardened defender of the old theory of free competition, and of rate-cutting as a means of getting the best of an adversary, will admit that when less money is received for a shipment than is actually spent by the railway company in handling it, the business had better be refused. As nearly as possible, then, each class of traffic ought to bear the cost incurred in handling it.

Now taking the average of all railways in the United States, about two-thirds of their gross earnings are used up in the payment of operating expenses, and the other third goes to pay taxes, interest on bonds and dividends on stock. If, then, we can estimate for each class of traffic and apportion to it the operating expenses due to its carriage, we shall provide for two-thirds of the necessary railway revenue. It only remains then to consider how the additional amount necessary to provide for the payment of taxes and of income to the owners of the railway bonds and stocks shall be apportioned among the different classes of traffic. At first thought it may seem most just to make this apportionment on the basis of the expense incurred and simply increase by one-third the rates found in the manner above stated. Closer study, however, shows this to be really an arbitrary rather than an equitable basis of adjustment. The real principle which should govern is that each class of traffic, after paying for the expense incurred in handling it, ought to pay for the privilege of using the road in proportion to the benefits received. The value of the goods shipped is a better index of this benefit than is the cost of their carriage; and it is, therefore, by no means inequitable that the valuable high-class merchandise should bear a larger proportion of the "fixed charges burden" than the low grade bulk freights.

According to this, the time-honored principle of charging in proportion to what the traffic will bear is founded on an equitable basis, and its observance by the railway managers is in the public interest as well as in the interest of the railway corporations.

To illustrate this by a concrete case: Let us suppose that a railway is to be built in a gold mining district to have for its sole traffic the carriage to a market of the product of two mines, one a mine producing a very rich ore carrying, say, $100 per ton, and the other a low grade ore carrying only $5 per ton. To simplify matters, let us suppose that the railway is built by the owners of the two mines in partnership, and each furnishes his own rolling stock and hauls his own traffic. Each class of traffic, therefore, will pay all its direct

expenses. Before building the line, however, the two partners come together to agree on an equitable basis to proportion the interest on the money borrowed to build the road. The partner with the low grade mine may ship ten times as many tons per annum as the other, but he cannot and ought not to pay ten-elevenths of the cost of the road. To do so, we may suppose, would make his cost of transportation so great as to prevent him from shipping his ore at all. The ore of either mine, we may also suppose, will be valueless without the railway, and neither mine owner can afford to build it alone. It is, therefore, decided that the total value of the ore shipped annually from each mine is a just criterion of the benefit which the railway has bestowed upon each mine owner, and in that proportion it is agreed to divide the cost of the road's construction.

This is, indeed, an extreme case; but it nevertheless fairly illustrates the principle which should control in fixing railway rates. It is not for the interest of shippers of high-class goods that rates on low grade freights should be so raised as to cause the loss of that traffic. The larger the tonnage which a railway carries, the more economically it can handle its business and the less the charge per ton which will have to be made to cover the fixed charges of the company and provide for the dividends.

It will be noted that in the above discussion we have considered only the question of relative rates and have assumed that the net earnings of the companies are to be neither increased or decreased. That is a question by itself and one upon which we will not now enter. Neither have we attempted to defend the practice—happily now far less common than formerly—of rating freight at competitive points far lower than at non-competitive, regardless of its class or the cost of its carriage.

As we have seen above, the actual operating expense cost of moving freight should account for two-thirds of the rate charged upon it. It is chiefly because the cost of moving bulk freights in large shipments over long hauls is generally overestimated, and the cost of handling the small merchandise traffic is generally underestimated that we hear so much complaint of exorbitant railway rates.

We have laid much stress in this journal upon the revolution in rail transportation which has been brought about through the introduction of large locomotives and cars and heavy train loads. We have shown* how, under the most favourable conditions, it is possible to move through bulk freight by rail in great quantities, over long hauls, at a cost as low as one mill per ton-mile. All these conditions must be present, however, to make any such rate even possible, and such a combination is extremely rare. Any departure from it in any particular means a large increase in the cost of transportation.

The great fact which must be comprehended before a correct comprehension of this subject of relative rates is possible, is that in American railway practice the influence of distance has been almost annihilated. Of the total expenditure in the freight department of the railways of the country, taken as a whole, a small proportion is the cost of moving the long through trains over the road on the lines of heavy traffic. The great bulk of the expenses are incurred in loading and making up these trains at one terminal and disposing of them at the other, in collecting from thousands of small stations the many small shipments which are finally united to form the great traffic streams which flow towards the great commercial centres, and finally, distributing the opposite streams in a similar manner. If once this state of affairs is clearly understood, it is easy to see how railway companies can make good profits in the movement of through freight at rates which seem little less than suicidal, and to see, too, how such rates are no criterion whatever as to the cost of movement of freight of other classes.

The greatest item of expense on every railway is its pay roll. We may consider the railway company as a manufacturer, and its trains as so many machines for manufacturing transportation. Watch the movement of the long through trains on any road of heavy traffic

* Eng. News, Oct. 26, 1899.

and see how few men are required to manufacture transporation at the rate of 10,000 or 20,000, or even 40,000 ton-miles per hour. Then watch the process of making up these trains in the terminal yards or at division points. Go out to the local stations and watch the crew of a way freight shifting the cars on the sidings, picking out one here and dropping another there. Remember that on many roads of heavy traffic half as many switching locomotives as freight locomotives are in service. Such studies help one to understand the conditions which we are attempting to make clear.

In the concentration of attention on the through freight traffic, it is apt to be forgotten that the bulk of the freight train mileage of the country is made up of the movement of local or way freights. Even on the large roads this is true. Fully-loaded trains on any of these roads carry from 500 to 2,000 tons of paying load; but the average freight train load on any of these roads is a small percentage of this. Further, of the 190,000 miles of railway in the United States, not more than one-third is comprised in the lines of heavy traffic; less than 12,000 miles are double-tracked. On 100,000 miles or more of the country's railways the only freight train is the way freight.

Very few of these trains are fully loaded, nor can they be from the nature of the case, yet the trains must run regularly to accommodate the public, week in and week out, the year round. The train crew for such lightly loaded, slow moving trains is nearly or quite as large as on the long through freights. The locomotive expenses are only a little less. Probably it would be well within bounds to say that the average cost to the railways of the country of collecting car-load freight and hauling it to division points where it can be made up into full train loads, and of the same process of distribution at the other end of its journey, amounts to 5 to 20 times as much per ton-mile as the cost of hauling the same freight where it can be handled in full train loads, over long hauls on railways equipped and managed for the economical movement of such traffic.

If this is true of freight shipped in car-load lots, it is even more true of the "less than car-load" shipments, which in the aggregate constitute a larger proportion of the total freight business. The average load of the cars engaged in moving this class of freight is and must necessarily be far below their capacity. Careless management in this department can bring about gross waste. A story was told in one of the railway clubs a few years ago of a certain freight car which was hauled half-way across the continent, passing over several different railway lines, and when opened at its destination its entire contents was found to be one 50-lb. tub of butter. Many railways now have systems of inspecting and reloading cars handling this class or traffic; but this again costs money.

A large proportion of the freight department's clerical expenses are due to this class of traffic. Nearly the same routine must be gone through for a shipment of a crate of glassware as for a car-load of lumber. The bulk of the station expenses chargeable to freight traffic, too, are incurred in handling the "less than car-load" shipments. The large shippers do business direct with the heads of the traffic department or their immediate lieutenants.

It is probably true that improvements and economies are possible in handling this class of traffic. Comparatively little attention has been paid to it in American railway development, and the progress in the direction of large cars and large locomotives has tended to increase, on the whole, the expense of its handling. We may yet see ten-ton cars built for local service in some part of the country, when the old light-capacity cars still running have all gone to the scrap heap.

The profits made by the express companies and by the English railway companies, whose traffic is so largely of this character, show that it is possible to make this class of traffic pay well: but it is noticeable that in each of these cases the traffic is subjected to rates considerably higher than those charged by American railways, which sustains our position that this class of traffic is necessarily expensive to handle.

In conclusion, it may be asked what the railways can do convince the public that their adjustments of relative rates are not inequitable. We can think of no better plan than to undertake to compile and open to public inspection some statement of the relative operating expense cost of moving different classes of traffic. We do not mean by this any hard and fast system of record, involving additional clerical labor and expense: but merely such an apportionment of expense as a competent accountant, familiar with operating details should be able to compile from the record already kept by the company. Such a showing would be a valuable guide to the traffic department in the work of adjusting rates, and would do much to justify the railways' contention that its officers are more competent to equitably adjust rates than any State or National Commission unfamiliar with the details of railway operation can possibly be.

Engineering News, March 1, 1900.

APPENDIX F.

Example of the practical application of Commercial Tracing.*

A normal-gauge Subsidiary line is proposed from the Main line station A (pop. 25,000) to the smaller town F (pop. 8,000). Between A and F lie the towns B (pop. 6,000), C (pop. 3,000), D (pop. 1,500) and the hamlet E (pop. 700).

Their relative positions are exhibited in the accompanying Fig.

The question for solution is:—Is such a line buildworthy? and if so, what is its best position?

For the estimate of the traffic the following figures are taken from Table I—p. 88—giving the data regarding Traffic for various German Main and Secondary lines according to the population, viz.

$$p_1 = 19{\cdot}7, \quad p_2 = 14{\cdot}4, \quad p_3 = 10{\cdot}9, \quad p_4 = 7{\cdot}0,$$
$$q_1 = 5{\cdot}4 \quad q_2 = 4{\cdot}3 \quad q_3 = 4{\cdot}0 \quad q_4 = 2{\cdot}8$$

where p_1 and q_1 are the average number of passengers and of goods-tonnes per cap. for localities having 1,000 inhabitants.

p_2 and q_2 ditto 2,000 „
p_3 „ q_3 „ 5,000 „
p_4 „ q_4 „ above 5,000 „

Also let

$$\left.\begin{array}{l} d_1 = 24, d_2 = 115 \\ c_1 = 2{\cdot}85, c_2 = 3{\cdot}63 \end{array}\right\} \text{from Table III—p. 89.}$$

The line will probably be built by the Prussian State.

From the Fig. it is seen that it is simply a question of a main-line $A\ C\ D\ F$ with junctions (nodes) for rail or road diversions, branches, or connexions with B and E. However as regards B it is necessary first to determine whether the building of the sides of the triangle $A\ B$ and $B\ C$ might not be commercially better; but in the case of E it is at once evident that there can be no question of anything but a road from the node.

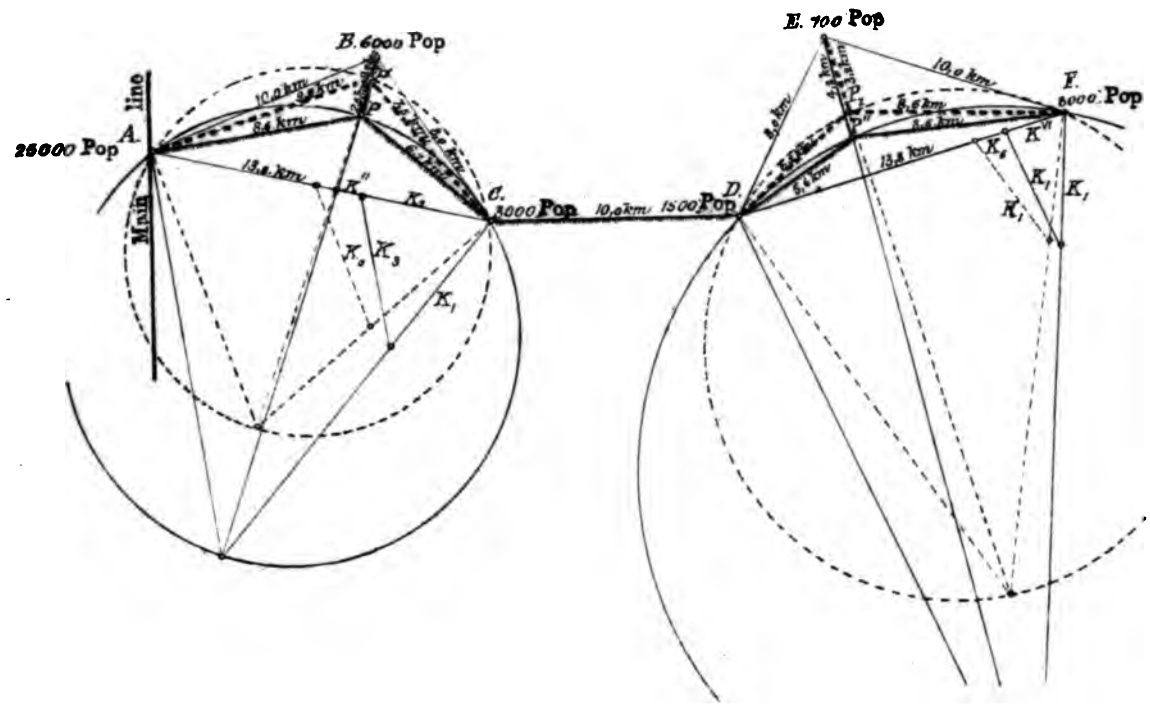
A line is possible from A viâ B and E to F with branch-lines or roads to C and D: however, the present, Example will be limited to the first case. From theoretical considerations only this line would shape itself as shown in Fig. 16, p. 24, that is, only the places A and F would lie immediately on the line, and all other places would be connected by branch-lines or roads from nodes on these lines. But a treatment of the Example in this manner will not be attempted, as it would have but very rare practical application and therefore the usefulness of the present as an Example would be diminished.

To determine the revenue to be expected from the projected railway let the most unfavourable case be supposed, viz., that the line viâ B is the one chosen; then we have from Formula 3,—p. 91—a gross return from the passenger- and goods-traffic of

$$U = \left[19{\cdot}7 \times 2{\cdot}85 \times 24\left(1 + \frac{15}{41{\cdot}8}\right) + 5{\cdot}4 \times 3{\cdot}63 \times 115\left(1 + \frac{23}{41{\cdot}8}\right)\right]\frac{\frac{700}{2}}{10 + 8 + 10 + 13{\cdot}8}$$
$$+ \left[14{\cdot}4 \times 2{\cdot}85 \times 24\left(1 + \frac{15}{41{\cdot}8}\right) + 4{\cdot}3 \times 3{\cdot}63 \times 115\left(1 + \frac{23}{41{\cdot}8}\right)\right]\frac{1500}{10 + 8 + 10 + 13{\cdot}8}$$
$$+ \left[10{\cdot}9 \times 2{\cdot}85 \times 24\left(1 + \frac{15}{41{\cdot}8}\right) + 4 \times 3{\cdot}63 \times 115\left(1 + \frac{23}{41{\cdot}8}\right)\right]\frac{3000}{10 + 8 + 10 + 13{\cdot}8}$$
$$+ \left[7 \times 2{\cdot}85 \times 24\left(1 + \frac{15}{41{\cdot}8}\right) + 2{\cdot}8 \times 3{\cdot}63 \times 115\left(1 + \frac{23}{41{\cdot}8}\right)\right]\frac{6000 + 8000 + \frac{25000}{3}}{10 + 8 + 10 + 13{\cdot}8}$$
$$= 17{,}950 \text{ M.}$$

[* From Meyer and von Willmann's "Handbuch der Ingenieurwissenschaften."—TR.]

Main line
Subsidiary line
Road



Scale:

1 Km = 3,33 m/m
1000 Mark = 3,33 m/m

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and therefore, from Eqn. 4,—p. 96—a gross revenue per **km.** of line of

$$U^1 = 1{\cdot}1 \times 17650 = \mathbf{19,400\ M.}$$

The values of d_1 and d_2 in the expression $\left(1 + \frac{d}{L}\right)$ are the means of the values of d_1 and d_2, respectively, in the 4 railways 31, 32, 33, 34, of Table III, [cols. 44, 45.]
The place A may with certainty be credited in the Traffic estimate with only $\frac{1}{3}$rd of the number of its inhabitants as traffic producers; the town E—regard being had to its distance from the future station—can be credited with at the most only $\frac{1}{2}$ of its population. With these assumptions the results above obtained may claim to be not unduly favourable.

In order to determine the commercially most favourable location it is necessary to know the kilometric traffic-expenses. These consist of the cost of construction plus the cost of the working.

The general preliminary technical studies may be assumed to have indicated a kilometric construction-cost of 1,00,000 M.

The kilometric maintenance- and supervision-expenses including renewal of the line may be put down at 2,500 M. The kilometric cost for both the passenger- and the tonne-km. may be put at 1·8 pf.

Let the construction kilometric-cost of the roads to be made from the nodes to B or to E be 15,000 M. The maintenance-expenses per km. may be put $= 150 + {\cdot}03\ g$, in which g is the number of tonnes carried on the road. And assume ·20 M. as the rate of carriage on paved roads.

From the several places in question, the following traffic is to be expected

From A: $\frac{25000}{3} \times 7 \times 2 = 117{,}000$ coming and going passengers
$\frac{25000}{3} \times 2{\cdot}8 \times 2 = 46{,}000$ „ „ goods-tonnes

of which $\frac{1}{2}$ goes to E, $\frac{1}{6}$ to C and F, and $\frac{1}{12}$ to D and E.

From B: $6000 \times 7 \times 2 = 84{,}000$ coming and going passengers
$6000 \times 2{\cdot}8 \times 2 = 34{,}000$ „ „ tonnes

of which $\frac{1}{2}$ goes to A, $\frac{1}{4}$ to C, and $\frac{1}{12}$ to D, E, F.

From C: $3000 \times 10{\cdot}9 \times 2 = 65{,}000$ coming and going passengers
$3000 \times 4 \times 2 = 24{,}000$ „ „ tonnes

of which $\frac{1}{2}$ goes to A, $\frac{1}{4}$ to B, and $\frac{1}{12}$ to D, E and F.

From D: $1500 \times 14{\cdot}4 \times 2 = 43{,}000$ coming and going passengers
$1500 \times 4{\cdot}3 \times 2 = 13{,}000$ „ „ tonnes

From E: $700 \times 19{\cdot}7 \times 2 = 28{,}000$ coming and going passengers
$700 \times 5{\cdot}4 \times 2 = 8000$ „ „ tonnes

of which $\frac{1}{3}$ goes to A, and F, $\frac{1}{6}$ to D and $\frac{1}{12}$ to B and C.

From F: $8000 \times 7 \times 2 = 112{,}000$ coming and going passengers
$8000 \times 2{\cdot}8 \times 2 = 45{,}000$ „ „ tonnes

of which $\frac{1}{2}$ goes to A, $\frac{1}{4}$ to D, and $\frac{1}{12}$ to B, C and E.

Consequently, assuming two nodes P_1 P_2 there will be the following traffic on the several rays.

On ray $A\ P_1 = 278{,}000$	passengers	$+\ 106{,}700$	tonnes	$=$ **385,000**	passengers	and goods-tonnes.
$B\ P_1 = 174{,}000$	,,	$+\ 69{,}000$	,,	$=$ **243,000**	,,	,,
$C\ P_1 = 252{,}000$	,,	$+\ 95{,}000$	,,	$=$ **347,000**	,,	,,
$C\ D\ = 194{,}000$	,,	$+\ 73{,}000$	,,	$=$ **267,000**	,,	,,
$D\ P_2 = 190{,}000$	,,	$+\ 72{,}000$	,,	$=$ **262,000**	,,	,,
$E\ P_2 = 63{,}000$	,,	$+\ 22{,}000$	,,	$=$ **85,000**	,,	,,
$F\ P_2 = 164{,}000$	,,	$+\ 64{,}000$	,,	$=$ **228,000**	,,	,,

Assuming a rate of interest of $3\frac{1}{2}\%$ for the construction capital, the kilometic traffic-expenses on the different rays are as follows—

$A\ P_1 : K_1 = 100000 \times \cdot035 + 2500 + 385000 \times \cdot018 = 12{,}930$ M.

for a railway : $B\ P_1 : K_2 = 100000 \times \cdot035 + 2500 + 243000 \times \cdot018 = 10{,}370$ M.

for a road : $K_2{}^1 = 15000 \times \cdot035 + 150 + 69000 \times \cdot20 = 14{,}475$ M.

$C\ P_1 : K_3 = 100000 \times \cdot035 + 2500 + 347000 \times \cdot018 = 12{,}250$ M.

$C\ D : K_4 = 100000 \times \cdot035 + 2500 + 267000 \times \cdot018 = 10{,}800$ M.

$D\ P_1 : K_5 = 100000 \times \cdot035 + 2500 + 262000 \times \cdot018 = 10{,}720$ M.

for a railway : $E\ P_2 : K_6 = 100000 \times \cdot035 + 2500 + 85000 \times \cdot018 = 7{,}530$ M.

for a road : $K_6{}^1 = 15000 \times \cdot035 + 150 + 22000 \times \cdot20 = 5{,}075$ M.

$F\ P_2 : K_7 = 100000 \times \cdot035 + 2500 + 228000 \times \cdot018 = 10{,}110$ M.

Now with the aid of these figures, the geometric construction of the nodes is carried out as shown in the present Fig. The nodes are then determined both for branch-lines and roads to B and E. The lengths of the rays in km. are indicated thereon in the Fig.

To enable a conclusion to be come to as to which is the preferable location, *i.e.*, whether branch-lines, roads, or the sides of the triangle $A\ B$ and $B\ C$, or $D\ E$ and $E\ F$, it is necessary to collect the total traffic-expenses, as above defined, for these separate cases.

(1) B.

(*a*) Construction of a a branch-line from the mode P_1 :—

Traffic-expenses $= 8{\cdot}6 \times 12930 + 2{\cdot}5 \times 10370 + 6{\cdot}7 \times 12250 =$ **219,198 M.**

(*b*) A road from node P^1 :—

T. E. $= 9{\cdot}5 \times 12930 + \cdot9 \times 14475 + 7{\cdot}5 \times 12250 =$ **227,738 M.**

(*c*) The sides $A\ B$ and $B\ C$ of the triangle :—

T. E. $= 10 \times 12930 + 8 \times 12250 =$ **227,300 M.**

(2) E.

(*a*) Branch-line from P_2 :—

T. E. $= 6{\cdot}2 \times 10720 + 3{\cdot}2 \times 7530 + 8{\cdot}6 \times 10100 =$ **177,420 M.**

(*b*) Road from P^{11} :—

T. E. $= 5{\cdot}6 \times 10720 + 4{\cdot}2 \times 5075 + 8{\cdot}6 \times 10100 =$ **168,207 M.**

(*c*) Sides of triangle $D\ E$ and $E\ F$:—

T. E. $= 8 \times 10720 + 10 \times 101000 =$ **186,760 M.**

From the above figures it is evident that for B the construction of a branch-line, and for E of a paved road is the best policy financially.

With regard to B, however, it has to be borne in mind that the construction of a branch-line at P_1 and B will require a station at each ; whereas for the construction of the line on the sides of the triangle a station is necessary only at B, and in the present instance it would have to be a large one. If the construction-cost of the station is equal to the capitalised saving in traffic-expenses taken at $3\frac{1}{2}\%$, viz.

$$\frac{227300 - 119198}{\cdot035} = 231{,}400 \text{ M.}$$

then the location on the sides of the triangle is the financially preferable one, since the working-expenses on a short branch-line are always higher than on its main-line besides, the direct connexion of the place itself will always be worth trying for.

To decide as to the buildworthiness of the line it is necessary to determine its working-expenses. This is done in the manner described in § 3, p. 92. In the present example—since the conditions are only imaginary ones—for the sake of simplicity let it be assumed that the entire working-expenses amount—as deduced on p. 95—to 1·812 M. per paying-km.

The number of paying-km. to be done is determined from the traffic figures above obtained. The traffic on $A\ B$ amounts to 278,300 passengers and 106,700 tonnes. To determine the number of trains—since the whole length of a line such as the present will be travelled over equally in both directions—the number is to be halved, thus giving 139,150 passengers and 53,350 tonnes.

There is therefore daily $\frac{139150}{365} = 380$ passengers and $\frac{53350}{365} = 150$ tonnes of goods to be carried.

To handle this traffic it will not be necessary to have mixed trains, but solely goods and passenger trains. Five trains daily will be sufficient to deal with the passenger business, each consisting of a brake-van and 5 passenger-cars. For the goods-traffic, two trains daily will suffice each made up of 15 wagons of 10 tonnes capacity each.

Accordingly, on the main-line $A\ F$ the requisite number of paying-kms. will be :—

$$2 \times 7 \times 365 \times 39{\cdot}5 = 201{,}800 \text{ p. km.}$$

add : —empty-running, and shunting, at 4% = 8100

giving a total of ... 209,900 paying-km.

On the branch-line $B\ P_1$ it is expected that there will be a movement of 174,000 passengers and 69,000 tonnes goods. In determining the number of trains the halves figures are to be taken.

Thus per day we have $\frac{87000}{365} = 240$ passengers, and $\frac{34000}{365} = 95$ tonnes.

Since every train coming from A and F to B will connect at P_1 and assuming two crossings of trains at the station P_1 there will be 12 trains running on the line $B\ P_1$. A special goods train is not under these circumstances necessary. The number of paying-kms. to be done is therefore

$2 \times 12 \times 365 \times 2{\cdot}5$	= 21,900 p. km.
add : empty-running and shunting at 4%	= 900
add as above	= 201,800
giving a total of	**224,600 paying-tonne-kms.**

The toal working-expenses thus amount to

$$224{,}600 \times 1{\cdot}812 = \mathbf{407{,}000\ M.}$$

Consequently, for 1 km. to

$$\frac{407000}{39{\cdot}5 + 2{\cdot}5} = 9{,}700 \text{ M.}$$

The total revenue per km. of the projected line	= 19,400 M.
deduct as above	= – 9,700 M.
giving a total of	**9,700 M.**

as the working gain, or revenue per km. of the projected line.

If the rate of interest on the capital sunk in construction and working is 5%, then its amount should not exceed

$$20 \times 9700 = \mathbf{194{,}000\ M.}$$

The decision as to whether, and in what way (whether as Main, Subsidiary, or Tertiary line) the line should be built and provided with equipment is a matter that is decided when the technical treatment of the project is undertaken later on.

Assuming the ratio existing on the Prussian State Railways for working-expenses to gross returns to hold, then from Eqn. 5_b the whole invested capital of the line should not exceed,

$$19700 \times \cdot 4 \times 20 = \mathbf{158{,}000\ M.}$$

In the above Example in determining the commercially best location the kilometric cost of transportation for the passenger- and tonne-km. has been put at 1·8 pf. A more exact determination of it is only possible when the gradients and curvature of the line are known. The formulæ for the calculation are given in Launhardt's "Technische Trassirung" p. 59* and in any concrete example they would of course have to be employed. These formulæ shew that the rate of 1·8 pf. taken in the above Example would only be reached on a line of severe curvature and grades.

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* See "The Technical Tracing of Railways," p. 31, forming Part II of this version of Launhardt's Work.—Tr.]

APPENDIX G.

Calcul du Trafic Probable

D'UNE LIGNE DE CHEMINS DE FER.

Un des premiers éléments indispensables à connaître et á étudier, pour constituer le plan financier et les études d'un chemin de fer, est naturellement l'estimation de son produit probable et, par suite, de son revenu net, pouvant donner aux capitaux engagés une rente de tant ou tant pour cent.

On comprend aussi que c'est la partie la plus difficile, la plus délicate, et la plus aléatoire de toutes, puisque les renseignements que l'on peut prendre dans le pays sont constamment faussés ou exagérés suivant les intérêts que l'on consulte, et que les seules bases à peu près précises qui existent, sont : les Comptages de colliers faits par les Ponts et Chaussées sur les routes existantes, les statistiques plus ou moins complètes de l'Administration des douanes, les dires plus ou moins variables des entrepreneurs de Messageries, diligences, etc., enfin les *Mercuriales des Halles et marchés* que les maires des localités peuvent avoir dans leurs archives.

En présence de ces nombreuses causes d'incertitude, plusieurs ingénieurs ont essayè de composer des méthodes qui permettent de contrôler tout au moins approximativement les résultats des enquêtes verbales, et de calculer d'avance, avec des éléments certains, comme les *populations* et les *distances*, et les moyennes connues des pays analogues, ce qui peut bien être le résultat probable de la création du chemin de fer.

Parmi ces diverses méthodes celle de M. Michel, ingénieur des Ponts et Chaussées, est une des plus simples, et des plus admissibles comme conformité aux résultats de la statistique *a posteriori*, recueillie sur différentes lignes.

En voici les principales bases d'application :

§ 1.—MÉTHODE DE M. JULES MICHEL,

Ingénieur des Ponts et Chaussées.

Nous consignons ici le passage d'une brochure de M. J. Michel, sur les chemins de fer d'intérêt local, qui donnera une idée complète du mode d'appréciation du trafic :

" Dans toute région agricole, la population est en rapport avec les nécessités de la culture, et cette culture donne des produits à exporter ; le chiffre moyen des exportations annuelles est ainsi dans un certain rapport avec le nombre des habitants. La proportion est d'ailleurs variable avec la nature des produits. Si l'on considère une région de vignobles, par exemple les départements des Pyrénées-Orientales, de l'Aude, de l'Hérault, du Gard, on n'aura pas, entre le tonnage d'exportation et le nombre d'habitants, la même proportion que dans une région exclusivement cultivée en céréales et en fourrages, comme la Beauce et la Brie ou le Haut-Languedoc ; mais dans des régions analogues comme culture, le chiffre moyen du mouvement d'exportation trouvé pour l'une pourra s'appliquer à l'autre sans grande erreur. Ainsi la Brie, région de céréales et de fourrages, donne en moyenne, à chaque station de la ligne de Paris à Lyon, deux tonnes de marchandises à transporter par habitant et par an, et on trouve la même proportion dans les stations du chemin de fer de l'Ouest, entre Versailles et Rennes, et dans celles du Midi, entre Agen et Carcassonne.

" Les stations du Bas-Languedoc présentent un chiffre de trois tonnes à trois tonnes et demie par habitant, et on trouve sensiblement la même proportion dans la Basse-Bourgogne et sur les côtes du Rhône.

" Le nombre des voyageurs n'est pas variable avec la nature de la culture ; on pourra dire d'avance, que, sauf de légères variations dues aux habitudes locales ou au voisinage des grandes villes, le chiffre des voyageurs ne doit pas différer d'un bout á l'autre de la France. Les statistiques donnent en effet une moyenne de six voyageurs par habitant, en laissant de côté les stations des banlieues de Paris, Lyon, etc.

" Tels sont les résultats acquis sur des chemins de fer exploités depuis plus de dix ans, où le trafic s'est développé sous la puissante impulsion imprimée à l'activité commerciale par ces voies de communication perfectionnées.

" Il est facile maintenant de calculer la recette par kilomètre des petites lignes locales. On sait qu'une tonne de marchandises, transportée par chemin de fer, paye 6 cent. $1\frac{1}{3}$ par kilomètre parcouru, et qu'un voyageur paye 6 centimes. S'il y a, par exemple, huit fois autant de voyageurs que d'habitants dans les stations à établir, et trois fois autant de tonnes de marchandises, il suffit de relever le nombre des habitants des centres de populations groupées à des distances de 6 à 8 kilomètres les unes des autres (c'est l'intervalle qui sépare en général les stations), et de compter 0 fr. 70 de recette par habitant pour avoir le produit kilométrique de la ligne projetée. "

L'exemple que nous avons pris admet 6000 habitants dans la région traversée ; la recette kilométrique serait alors de $6000 \times 0{,}70 = 4200$ fr., dont $6000 \times 0{,}48 = 2880$ fr. pour les voyageurs et 1320 fr. pour les marchandises. A ce chiffre on ajoutera ceux qui résulternot des observations fournies par les maires sur les industries spéciales de nature à donner un trafic particulier. On connaîtra ainsi la recette kilométrique probable. On pourra la contrôler par les nombres déterminés d'après les tableaux dressés par les maires.

M. Michel ne s'est pas contenté de ce procédé empirique, lequel, on le conçoit sans peine, pourrait, dans certains cas, conduire à de graves erreurs.

Frappé des inconvénients que présentent toujours, pour certains esprits, les incertitudes qui naissent à la suite des études commerciales, il s'est efforcé de traduire ces eventualités par une formule mathématique.

FORMLUE DE M. MICHEL.

M. Michel admet en principe que :

Le rapport entre les voyageurs expédiés et le nombre des habitants d'une station oscille entre 4 et 9, suivant la richesse de la contrée traversée ; que la moyenne générale est 6, 50, et que, pour les marchandises, la moyenne générale est de 2 tonnes 1/10 par habitant, pour toute la Francc.

Il énonce ensuite la formule du trafic :

Le trafic est la somme des producta des expéditions et des arrivages par le parcours moyen de chaque voyageur et de chaque tonne de marchandises, et il traduit ce principe par la formule :

$$(a) \qquad T = \frac{2\,\Sigma\,(V + t)\,d}{l},$$

dans laquelle :

T représente le trafic ;

V, le nombre de voyageurs ;

t, la demi-somme does tonnes expédiées et reçues par chaque station ;

d, la distance de cette station à l'origine de l'embrachement ;

l, la longueur totale de l'embrachement ;

Σ la somme des produits du trafic par les distances. On multiplie cette somme par 2 pour tenir compte des aller et retour.

On doit faire observer que cette formule suppose que le mouvement général du traffic ira des stations intermédiaires à la gare d'embranchement sur la grande ligne, et de même au

retour, le mouvement des stations entre elles pouvant être considéré comme peu important; cette supposition est vraie pour un chemin de fer d'intérêt local.

(b) $$T = \frac{2\,(m+n)\,\Sigma\,p\,d}{l},$$

où m et n représentent les coefficients des voyageurs et des tonnes de marchandises.

Si l'on suppose la population condensée au centre de gravité de la ligne, dont la distance à l'origine est égale à $\frac{\Sigma\,p\,d}{\Sigma\,p}$, si l'on appelle g cette fraction de la longueur totale on aura : $\Sigma\,p\,d = g\,l\,\Sigma\,p$, et par suite l'expression b devient :

(c) $$T = 2\,g\,(m+n)\,\Sigma\,p.$$

Or, dans cette formule, on connaîtra $\Sigma\,p$, c'est-à-dire le nombre d'habitants à affecter aux diverses stations; on connaîtra m (voyageurs), dont la valeur est comprise entre 4 et 9 suivant la richesse de la contrée, et n, qui a une valeur égale à 2 1/10 ; on calculera g et on obtiendra la valeur probable du trafic T.

Cette valeur ne saurait être acceptée qu'avec de grandes réserves, car les coefficients m et n peuvent varier d'un pays à l'autre dans une proportion considérable.

M. Michel a traduit également en formule la recette brute probable par kilomètre.

Conservant les valeurs m, n et g, on a pour expression générale de la recette brute :

$$K = g\,\Sigma\,p\,(2\,m \times 0{,}05 + 2\,n \times 0{,}061).$$

Dans cette formule, 0,05, et 0,061 sont les moyennes des prix de transport par kilomètre, non compris l'impôt du dixième.

Si l'on fait $m = 6{,}50$, $n = 2{,}16$ et $g = \frac{2}{3}$, on aura $K = 0{,}60\ \Sigma\,p$. C'est-à-dire que, en moyenne, le produit brut par kilomètre, d'un chemin de fer d'intérêt local qui aurait le centre de gravité de son trafic aux 2/3 de sa longueur, serait de 0 fr. 60 par habitant des stations à desservir.

Si, dans les pays riches et industriels, on fait $m = 7{,}50$, $n = 2{,}10$ et $g = \frac{2}{3}$, on aura $K = 0{,}66\ \Sigma\,p$. Si, dans un pays exclusivement vignoble, on fait $m = 6{,}50$, $n = 3$, on aura pour $g = \frac{2}{3}$, $K = 0{,}66\ \Sigma\,p$.

Malheureusement ce genre de recherches se prête difficilement à la rigueur des formules scientifiques. Nous indiquons le procédé de M. Michel, parce qu'il se rapproche du système que nous avons indiqué plus haut et qu'il peut servir à le contrôler utilement dans certains cas.

Opperman : "*Traité complet des chemins de fer économiques.*" 1873.

"Naturally from such calculations based on averages derived from the results of "working of a great network or system of lines only comparatively exact results can be "expected, even when the new line is designed to become a completely uniform member of "the system, and to bring an increase of traffic to the older line. Under exceptional "and abnormal conditions, as on mountain lines, particularly when there is no through "continental traffic, the formulæ are invalid." *F. Kreuter*: "*Linienführung der Eisen-"bahnen*:" 1900. p. 14.

APPENDIX H.

[It is probably no exaggeration to say that in ordinary railway projects the precise estimating of prospective Railway Traffic, Revenue and of Working-Expenses is as yet quite rudimentary, and the results arrived at depend largely on chance and the "sagacity" of the estimator. As the contents of this Work will doubtless have shown, it is quite otherwise on the Continent. Since foreign practice must necessarily be unknown to the ordinary English railway engineer what follows is offered as an illustration of procedure and as a model of method in these problems.

It is extracted from Vol. I, Part I of a monumental work on Railway Engineering, in 5 vols. each of several parts, misnamed a "Handbuch." It is edited by Meyer and von Willmann, and published in 1898.—Tr.]

* * * * * * * * *

The investigation of the "buildworthiness" of a Projected Railway involves the

(1) Determination of the probable traffic.

(2) Calculation of the return from the above traffic.

(3) Estimate of the working-expenses.

(4) Calculation of the necessary capital for construction and for working.

General Principles.

1. Determination of the prospective Traffic.

Local Traffic :—

In calculating the prospective local and direct traffic from the local conditions the country-road traffic is often taken as the standard, modified by the fact demonstrated by experience that the building of almost every railway has been succeeded by an increase in the local traffic of the district served.

The requisite data for this estimate are obtained from Chambers of Commerce, by enquiries and investigations amongst Administrations, executive officials, owners of large industrial or other establishments, and from the statements of experienced and sagacious inhabitants knowing their district.

Now while the collecting the requisite information and data from such statistical sources is a tiresome and laborious business, still not only the information derived from private individuals but also that supplied by the local authorities must be made use of with the greatest caution; since it often happens that the information thus obtained is influenced and vitiated by personal, commercial or other considerations. This method of determining the prospective traffic is thus not only laborious but is very often quite untrustworthy; and, further, the increase of traffic assumed in the calculation, if the line be poorly located, may, under circumstances, entirely vanish, and the total traffic on the line may not even amount to the existing road traffic.*

* * * * * * * * *

Michel—basing his investigations on the Results of the Working of French Railways —found that in 1866 for every head of the traffic-producing population there were 6·5 passengers and 2·1 tonnes of goods moved. These figures fall in purely agricultural districts to 4 passengers and 1·4 tonne goods, and they rise in industrial ones to 9 passengers and 3 tonnes.

* See pp. 108, 109 for an example of how statistical data may be advantageously arranged for use.

In 1894, Michel showed by examples* that his method as applied to branch-lines gave correct results.

Of course in employing these figures (only valid for average conditions) to any particular case, any special conditions of the individual localities which may influence the traffic must be duly taken into consideration. Further, there may possibly be certain junction-stations in which only a part of the population—to be estimated according to the special local conditions—is to be used in the traffic estimate.

In the following Tables such data for a number of Prussian Railways are given based on the Report for the Traffic Year 1893-4 issued by the Imperial Railway Bureau. With the object of making the data as useful as possible to the locating engineer, the lines are chosen systematically from amongst the most various parts of Prussia having the most diverse economical conditions.

As was done by Michel so here also those stations in which the traffic conditions are quite abnormal have been excluded: however, it has been considered expedient to include the smaller station localities having less than 1,000 inhabitants—omitted by Michel in his French investigation—and to classify the stations according to the number of their inhabitants. In this way we are able to emphasise the influence which the neighbouring places and their hinterlands exercise on the small station-sites, and can thus in our estimate more exactly bring in the neighbouring places and the hinterlands than would be possible from the use of the general average.

Finally, it is to be noted that in the case of branch line or junction-stations only a fraction—estimated accordingly to the local conditions—of their population and goods-traffic are to be made use of in estimates of traffic.

In the Tables I and II only Main and Secondary lines are distinguished, on the permissible assumption that what holds for the traffic of Secondary lines holds in general for Local and minor railways: besides which, in many instances, as already mentioned, it is not always possible to draw the line between Secondary, Local, and Minor railways.

* Revue Générale des chemins de fer. 1894.

TABLE I.

Statement of Traffic Statistical Data of Railways arranged according to the population of the Station Sites.

1	2	3	4	5	6	7	8	9
	NAME OF LINE.	Total population of the Station sites forming passenger-traffic.	Movement of passengers outwards.	No. passengers per head of station site population.	Half sum of the received and despatched goods in tonnes.	No. tonnes of goods per head of population of station sites.	Total population of station site producing goods-traffic.	REMARKS.
				p.		g.		
			A.—MAIN LINES.					
1	Insterburg-Memel	5,177	*1. Station Sites of 1,000 inhabitants.*				5,066	
2	Königsberg-Eydtkuhnen	2,946	210,202	40·60	24,253	4·79	2,946	
3	(etc.)	—	88,096	29·90	15,305	5·20	—	
			—	(etc.)	—	—		
1	Insterburg-Memel	—	*2. Station Sites of 2,000 inhabitants.*				—	
			—	—	—	—		
2	Königsberg-Eydtkuhnen	1,019	20,457	20·09	2,129	2·09	1,019	
3	(etc.)	—					—	
			—	— (etc.)	—	—		
			3. Station Sites of 5,000 inhabitants.					
1	Insterburg-Memel	—	—	—	—	—	—	
2	Königsberg-Eydtkuhnen	11,995	156,865	13·08	105,892	8·83	11,995	
3	(etc.)	—	—	— (etc.)		—	—	
			4. Station Sites of 5,000 inhabitants and above.					
1	Interburg-Memel	34,873	223,104	6·4	60,102	1·72	34,873	
2	Königsberg-Eydtkuhnen	12,794	122,649	9·59 (etc.)	34,310	2·68	12,794	
3	(etc.)		B.—SECONDARY LINES.					[And so on similarly for Secondary lines.]

TABLE II.

Statement of the average Traffic Data of the foregoing Lines of Table I (for the year 1893–4).

1	2	3	4	5	6	7	8	9
	NAME OF LINE.	Total station site population producing traffic.	Movement of outward passengers.	No. passengers per head of station site population.	Half sum of the received and despatched goods in tonnes.	No. tonnes goods per head of population of station site.	Total population of station site providing goods-traffic.	REMARKS.
				p.		g.		
	A.—MAIN LINES.							
1	Insterberg-Memel	40,050	438,306	10·82	84,355	2·11	39,939	Rich district, purely agriculture, brisk manufacturing and shopping business in Tilsit.
2	Königsberg-Eydtkuhnen (omitting Königsberg and Gumbinnen)	28,754	388,085	13·52	157,636	5·48	28,754	Moderately rich agriculture, horse breeding, business fairly brisk in towns.
3	(etc.)			(etc.)				(etc.)
	Total and average.							
	B.—SECONDARY LINES.							
1	Insterburg-Lyk	30,048	352,163	8·39	74,525	2·68	27,819	
	Total and average.				(etc.)			(etc.)

[NOTE.—The figures in cols. opposite each Railway are the totals of the figures detailed in Table I for the same Railways under the different Station population groups; thus 40050 = 5177 + 34873—Tr.]

As is seen from Table II the Main lines' averages differ only slightly from those of the individual lines as regards passenger-traffic, but the differences in goods-traffic are very considerable both in excess and defect. The same remark applies also to the Secondary lines. It is in the nature of things, (and is seen in col. 9, Table II,) that the lines with high averages for goods-traffic are those lying in districts of highly developed industry or agriculture, and that the low goods averages are found in the poorer districts. At the first glance it appears surprising that the averages for passenger-traffic should differ so slightly whether the districts traversed by the line be poor or rich. But it must be borne in mind that a district with a highly developed industry or agriculture is usually thickly populated, and that a great percentage of this population consists of workpeople who travel but little; whereas this is less the case in poorer traffic-producing and less thickly inhabited districts; so that the phenomenon is quite intelligible. Further, it appears that in a generally well-to-do district the passenger-traffic figure is above the average.

As already stated, the places with abnormal traffic conditions have been omitted. When it is necessary to apply the given averages in the location of new lines, such places—*e.g.* small localities with large manufacturing industries—must be omitted and separately treated.

2. *Through Traffic.*—In the present investigation local and not through traffic is considered. Through traffic depends on a great number of very various conditions and, consequently, no general rules for it can be given; special investigation is required in each individual case. When a line is to be built to shorten an existing line then the requisite traffic data can be obtained from the latter. If, on the other hand, the new line is to create traffic the data must be obtained from a comparison with other similar lines. In any case attention must be paid to the fact that the new and shorter line may often cause through traffic to disappear, and therefore in most cases it is preferable—if the through traffic is not wholly the main object of the new line—to disregard it entirely, or at least to fix it at a very low figure.

If the new line is a part of an adjoining system then the through traffic almost always brings about a decrease in revenue, since the rates of the older and longer line are fixed at those of the shorter. However, the traffic also of the older line is at the same time increased by the construction of the new one, and thus ultimately the existing traffic will usually be ultimately increased.

In any case, through traffic is an important factor only for the larger Main lines: and the Secondary lines are very few in number at least in Germany on which it is at all a future consideration.

§ 2. Calculation of the probable Returns.

From the foregoing we see that for a projected railway the revenue may be determined in advance, taking into account the particular conditions of the district, from the number of inhabitants of the station sites by the aid of the figures given in Tables I and II.

If p is the annual average number of the outgoing individuals assumed to be traffic producing per head of the total population, g the half of the average annual quantity of goods in tonnes, coming and going, c_1 the traffic-earnings per-passenger km., c_2 the receipts per goods tonne-km., then per head of population and per km. travelled by the traffic there is a return from working of

$$w = 2pc_1 + 2gc_2 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Further, if d_1 is the average length in km. of the passenger journey, d_2 the length of average haul of a goods-tonne, E the total population producing the traffic, L the length of the line in km., then the annual receipts per km. from working are

$$U = 2(p\,c_1\,d_1 + g\,c_2\,d_2)\,\frac{E}{L} \quad \ldots \quad \ldots \quad (2)$$

The coefficient 2 in the above equation only holds for small lines in which the traffic is always carried over the whole length. When this is not the case, viz., when an out-

going passenger or goods-tonne for another station of the same line is an incoming passenger or goods-tonne, the formula gives too large results; consequently, it will be more correct to replace the coefficient 2 by the following expression, viz,

$$\left(1+\frac{d}{L}\right)$$

where d is the number of km. travelled by a passenger or goods-tonne, and L the length of the line.

The value of the above coefficient varies between 1 and 2; it obviously increases in proportion as the length of the line decreases, and as the average haul of passengers and goods increases; and decreases as the length of the line increases, and as the average length of haul of passengers and goods decreases.

In employing this coefficient for goods d is to be replaced by d_1 for passengers, and by d_2; and then the formula becomes

$$U=\left[p\,c_1\,d_1\left(1+\frac{d_1}{L}\right)+g\,c_2\,d_2\left(1+\frac{d_2}{L}\right)\right]\frac{E}{L} \qquad \ldots \qquad (3)$$

Of the factors in the above p and g are to be obtained from Table I or II according as the values are deduced from the size of the station-site, or the average values are adopted ∵ and the factors c_1, c_2, d_1 and d_2 are obtained from the Statistical Returns of the German Railways. Their values are given for individual railways for the Traffic Year 1892-94 in Table III in cols. 44, 45, 46 and 47.

If the projected railway is a part of a uniform network or system of lines under one administration, the values of the coefficients d_1 and d_2 to be inserted in the expression $\left(1+\frac{d}{L}\right)$ are those given in Table III—p. 93—for those lines whose lengths most closely approach to that of the projected line.

Otherwise, the values to be given for d_1 and d_2 are those corresponding to the total length of the system.*

If we employ the above formula to calculate the revenue of the Prussian State Railways, assuming that some 20 million Prussian inhabitants dwell in the station sites—which is very probably not remote from the reality—then for the figures given in Tables I and III for the Traffic Year 1893-94 there is a revenue of

$$U=\left[13{\cdot}47\times2{\cdot}84\times24\left(1+\frac{24}{26000}\right)+5{\cdot}5\times3{\cdot}63\times115\left(1+\frac{115}{26000}\right)\right]20{,}000{,}000$$

= about 650 million Marks. Now the actual revenue was 900 millions: the result is thus 28 % too small.

If, however, allowance be made for the fact that the formula takes no account of the through traffic, and that in deducing the values of p and g localities with abnormal traffic-conditions are excluded, it is evident that the formula when applied to the whole of the Prussian State Railways must naturally give too small results; nevertheless the formula itself may be considered satisfactory. And when applying this formula to a projected railway these facts must be kept in view.**

Table III gives a concise review of the results of working of individual lines.*** In this the important and characteristic feature is the operating gain per km. of the mean working length shewn in col. 43. It is seen that the Prussian State Railways earning 14,723 M. per km., excluding the lesser lines, are only surpassed by the Imperial Railways. Of course this does not mean that they give an equally good return on their capital, for the degree of return depends not only on the amount of the gain from working, but also on the amount of the capital.

This last in the working year 1893-4 for the Prussian State Railways, amounted to 6,749 million M., and the operating gain 381 million M., thus giving a return upon capital of 5·7%.

(*See numerical Example of Commercial Tracing, p. 78.)

[** See following Example, p. 100.]

[*** A few lines only have been given—as an example—Tr.]

Since the development of railways in Germany can be best studied in the history of the Prussian State Railways, the following graphic representation is given showing how the gross returns and expenses per km. have varied yearly from 1854 to 1895. (omitted—TR.)

The diagram also shows that the revenue in general has increased, although from time to time a slight decrease occurred: the former has varied from 13,000 M. in 1854 to 36,600 M. in 1894, or an increase of about 280%. This fact has of course to be duly taken into consideration when employing the above method to determine the revenue to be expected from a projected railway.

The accompanying diagram (omitted. TR.) shows what proportion the returns from the passenger- and goods-traffic are of the total gross-takings of the Prussian State Railways during the period 1854-95.

It is thus seen that during a long period the variations are extraordinarily small, so that we may assume the ratio as practically constant—about 1 : 2·5.

Also the ratio of the other sources of revenue (passengers' luggage, rents from restaurants, leasing slopes of bank, etc.) to the gross revenue varies only slightly.

* * * * * * * * *

§ 3. Determination of the Working-Expenses.

The total working-expenses may be subdivided into those of

1. General Administration.
2. Way and Works.
3. Motive-power, or Traction.

The expenses of these departments are subdivisible into those of Staff and Materials.

(a) STAFF.

Under this head comes

(a) The wages and salaries of the State-appointed officials inclusive of their fixed share of the profits (Tantièmen).

(b) The dietary allowances, day wages, and wages of subordinate staff.

(c) House allowances, local allowances, scarcity allowance.

(d) Other personal expenses.

To determine the amount of the above for a new line it is necessary to fix the number of the officials and subordinate staff required, and to consider each one with reference to his annual amount of allowances. The permanent-way gangs are not to be included—their pay is properly included in the outlay under materials for maintenance of way.

* * * * * * * * * *

When the line under consideration is not an important one the calculation of the total number of officials and subordinate staff required is easily and rapidly done in the manner shown hereafter in the calculation of Revenue.

According to the statistics of the German Railways for 1893/94, the expenses under staff on 55 private Railways under their own management was as follows.—

GENERAL ADMINISTRATION

$$1 \text{ paying-km.*} = \frac{3415000}{21516000} \quad \dots \quad \cdot 159 \text{ M.}$$

WAY AND WORKS.

$$1 \text{ paying-km.} = \frac{2861000}{2156000} \quad \dots \quad \cdot 133 \text{ M.}$$

TRANSPORT DEPARTMENT.

$$1 \text{ paying-km.} = \frac{13695000}{2156000} \quad \dots \quad \cdot 638 \text{ M.}$$

Total... ·930 M.

* The paying or earning-km. comprises the work done by the locomotive in express, passenger, or mixed trains, in construction and material trains, as also in extra engine service on grades where two engines are employed either hauling or pushing.

TABLE III.

Actual Construction- and Working-Expenses, Capital Cost, and Results of Working in 1893-94 of the "Verein" Railways.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
								I Land.		II Earthwork.		III Fencing.		IV Level Crossings.		V Culverts & Bridges.		VI Tunnels.		VII Track.		VIII Signals.		IX Stations.	
Current No.	Railway.	Date of opening.	Length of line. L.	Length of double track.	Maximum grade.	Length of curves of rad. < 300″ in % total length.	Smallest radius outside stations.	per km. M.	% of total.	per km. M.	% of total.	per km. M.	% of total.	per km. M.	% of total.	per km. M.	% of total.	per km. M.	% of total.	per km. M.	% of total.	per km. M.	% of total.	per km. M.	% of total.
1	Prussian State Railways (including Private lines worked by the State) ...	1838/94	25,907	10,399	$\frac{1}{30}$	1·03	100	26,004	10·37	31,657	12·63	670	·27	6,131	2·44	19,804	7·9	4,313	1·72	57,563	22·96	3,087	1·23	31,922	12·73
2	Bavarian State Railways (including as above) ...	1839/93	5,073	1,109	$\frac{1}{25}$	3·60	150	20,168	9·01	37,996	16·91	514	·23	2,951	1·32	27,179	12·10	3,210	1·43	53,084	23·64	2,876	1·28	31,613	14·08
3	Saxon State Railways (including as above) ...	1837/93	2,388	766	$\frac{1}{34}$	7·84	50	28,654	9·15	57,034	18·20	737	·24	10,526	3·36	39,541	12·63	3,962	1·25	59,239	18·91	5,204	1·66	39,629	12·65
	etc.						etc.									etc.								etc.	

—	—	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
		X Workshops.		XI Extra-ordinary Works of Art.		XII Rolling-Stock.					XIII Administration.		XIV General.		Capital Cost in 1,000 M.		Operating Gain.				Revenue from Working.		
	Railway.	per km. M.	% of total.	per km. M.	% of total.	per km. M.	% of total.	Driv. axles per Loco. per km.	No. of seats per km.	No. of goods-axles per km.	per km. M.	% of total.	per km. M.	% of total.	Total.	per km. M.	On the whole in 1,000 M.	per km. of average worklength.	Length of passenger haul or journey. km. d_1	Length of goods haul. km. d_2	per passenger km pf. c_1	per goods km. pf, c_2	Current No.
1	Prussian State Railways (including Private lines worked by the State) ...	4,879	1·95	4,610	1·84	45,819	18·28	1·05	29·86	17·05	10,018	4·0	4,207	1·68	6,776,021	261,549	381,418	14,723	d_1 24	d_2 115	c_1 2·85	c_2 3·63	1
2	Bavarian State Railways (including as above) ...	115	·08	1,846	·82	32,746	14·56	·62	25·03	7·97	8,805	3·92	1,818	·81	1,200,186	236,581	38,464	7,582	34	143	3·32	3·93	2
3	Saxon State Railways (including as above) ...	3,512	1·11	1,573	·50	49,153	15·56	·95	40·70	19·95	7,987	2·55	7,134	2·28	685,675	287,133	31,861	13,342	23	71	3·14	4·44	3
	etc.							etc.								etc.							etc.

Since the railways in question are quite small and unimportant ones and the number of officials is usually limited to the barest necessity, the above figures may very well serve as a basis for the determination of working-expenses.

The above values are those for a Secondary normal-gauge line of a higher class; and consequently for smaller lines, especially for narrow-gauge lines, they must be reduced correspondingly.

(*b*) Material Expenses.

The determination of average figures for these expenses which shall be generally applicable is much more difficult, because they involve many factors which are very different in each case according to the method of working and the constructional characteristics of individual lines.

In what follows the Statistics of the German Railways for 1893/94 are made use of being applicable to most of the normal-gauge lines yet to be built in Germany. In the case of narrow-gauge railways, a suitable reduction of the figures must be made according to the individual circumstances in each case.

The figures extracted from the statistics relate exclusively to the 55 private railways under their own Administrations. Figures derived from the whole of the German railways would give too large mean values.

General Administration.

The 55 self-managed railways expended in the year 1893/94 in General Administration a gross sum of 569 million Marks. Of this 341 millions went in staff expenses and 228 millions in material or non-staff expenses. Accordingly, the staff expenses approximately amounted to 60°/$_0$ and the other expenses to 40°/$_0$ or, in other words, the ratio between them was as 3 to 2. Now since the staff-expenses are known, the non-staff expenses are at once ascertainable. Employing the already determined value of the staff expenses, *viz.* ·159 M. per paying-km., the value of the non-staff expenses for the same unit would amount to ·106 M.

Way and Works.

1. The maintenance-expenses of the line outside stations inclusive of the through line in stations (but excluding the cost of renewals of rails, switches, and sleepers) amounted to 2,105 million Marks. Of this amount 1,171 million Marks went for the maintenance of the track. Now since part of the line is double-track, there being 4,513 km. of through track on 3,660 km. of line, the expenses per km. for single-track amount to

$$\frac{2105000-1171000}{3660}+\frac{1171000}{4515}= 515 \text{ M.}$$

The maintenance-expenses may accordingly be put at 500 M. per km. of normal-gauge single-track. For narrow-gauge lines this figure may be reduced to 400, 300 and 200 M. according as the gauge is 1_m, $·75_m$, or $·60_m$.

2. The maintenance-expenses of inside stations amounted to 1,161 million Marks; consequently per km. of line worked they amounted to

$$\frac{1161000}{3660} = 320 \text{ M.}$$

Accordingly, for a normal-guage Secondary line an average of 300 M. will be amply sufficient. For narrow-guage lines this amount may be reduced, say, to 250, 200, and 150 M.

3. Maintenance-expenses of telegraph, signals and accessories amounted to 153 million Marks, or per km. of line worked, to

$$\frac{153000}{3660} = 42 \text{ M.}$$

Usually the signal and telegraph installations on new lines are comparatively simple, and the expense per km. may be put at 40 M. for a normal-gauge line, and for narrow-gauge lines at 35, 30, 25 M., respectively.

4. The track renewal expenses (rails, switches, sleepers) amounted to 2,893 million Marks: consequently per paying-km. of truck they came to

$$\frac{2893000}{5831} = 500 \text{ M.}$$

This figure may be retained for normal-gauge lines; and for narrow-gauges it may be reduced to 400, 300 and 250 M., respectively.

The determination of the expense of track renewals can be also made on the basis of the work it has done. This is given by the number of paying-km. that are made thereon. In the traffic year under notice this amounted to 21,516 million. Consequently the cost of renewals per km. of the track was

$$\frac{2893000}{21516000} = \cdot 135 \text{ M.}$$

Since this method is based on the work done it is undoubtedly preferable to the preceding.

5. The general expenses of Way and Works amounted to 217 million Marks, the expenses under heads 1—4 to 6, 312 milions. Accordingly, the general expenses are to be taken as 3½ per cent. of the expenses under 1—4.

TRANSPORTATION.

According to the statistical Returns for the traffic year 1893/1894 the expenses were:

1. Train-expenses:—Engine fuel, cleaning and lubrication of locomotives and vehicles, disinfectants for vehicles, fuel for beating trains 'and water cranes, shunting with horses, &c., per earning-km	·223 M.
2. Maintenance of rolling-stock:—Locomotives, passenger-vehicles, luggage and goods-waggons, also tarpaulins, hand-lanterns, whistles, guards' satchels, &c., per km.	·159 „
3. Renewal of rolling-gtock:—Locomotives, cars, waggons, luggage-vans, &c.	·088 „
Total ...	·470 M.

The above non-staff expenses are of course only mean values and are only valid for a normal-gauge single-track with average grades and curves. For other lines they are to be increased or decreased according as the grades and curves lines are more severe or less.

Collecting the above items of the cost of a paying-km. we obtain the following results for the working-expenses—

STAFF EXPENSES.

General Administratien		·159 M.
Way and Works		·133 „
Transportation		·638 „

NON-STAFF EXPENSES.

General Administration		·106 „
Way and Works.		
1. Maintenance, outside stations:	$= \frac{2105000}{21516000} =$	·100 „
2. „ inside stations:	$= \frac{1161000}{21516000} =$	·054 „
3. „ Telegraphs, signals, &c.	$= \frac{153000}{21516000} =$	007 „
4. Renewal of track (rails, switches, sleepers):	$= \frac{2893000}{21516000} =$	·135 „
5. General expenses:	$= \frac{217000}{21516000} =$	·010 „

TRANSPORTATION.

1. Train-expenses	·223 „
2. Maintenance of rolling-stock	·159 „
3. Renewal of rolling-stock	·088 „
Total per paying-km. =	1·812 M.

The total number of goods and passenger tonne-kms. was (990,556 + 670,667) millions = 1,661,233 millions and paying-kms. 21,516 millions. Consequently, per passenger or goods-km. the cost was $\frac{21516000 \times 181 \cdot 2}{1661233000} = 2 \cdot 35$ pf.

The expenses of the staff and non-staff items of the transportation per km. amounted to

$$\cdot 638 + \cdot 223 + \cdot 159 + \cdot 088 = 1 \cdot 108 \text{ M.}$$

Consequently, the actual transportation expenses per passenger or goods trains-km. amounted to

$$2 \cdot 35 \times \frac{1 \cdot 108}{1 \cdot 012} = 1 \cdot 44 \text{ pf.}$$

It is here assumed that the expenses per passenger-km. are the same as those per goods tonne-km., which is approximately the case.

From the Graphical Table (omitted—Tr.) it is seen that on the German State Railways the working-expenses for years have varied between 55 per cent. and 65 per cent. of the gross returns, and may for the purposes of estimate be assumed at 60 per cent.

§ 4. Determination of the amount of the Capital required for the Construction and Working of a Railway.

As already stated, Eqn. 3, p. 91, gives only the probable returns from local passenger- and goods-traffic. There is still to be estimated the revenue derived from passengers' luggage, the sale of old materials, receipts from the Telegraph, Refreshment-room leases, rents of Lands and Buildings, subsidies from the Postal Department, &c., and lastly the revenue derived from through traffic. From the Graphic Table 2 (omitted—Tr.) it is seen that on the Prussian Railways the former at the date in question amounted to some 4 per cent. of the gross returns. It has already been stated with reference to through traffic that it is judicious to take it at a quite low figure. In order to be on the safe side, both the above sources of revenue may be taken at 10 per cent. of the gross returns.

Thus the gross revenue will be :—

$$U' = 1{\cdot}1\ U \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad 4$$

I. If the gross working-expenses are B and the capital K sunk in construction and working is to give a return of at least 5 per cent. then :—

$$K \leq (1{\cdot}1\ U - B)\ 20. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad 5a$$

Assuming the ratio of the working-expenses to the total returns to be the same as that obtaining on the Prussian Railways, then for the same rate of interest,

$$K \leq 1{\cdot}1 U \cdot 4 \times 20 = 8{\cdot}8\,U. \ \ldots \quad \ldots \quad \ldots \quad \ldots \qquad 5b$$

In Table III (a few lines of which are given as a sample—Tr.) the expenses per km. for the lines under the jurisdiction of the Imperial Railway Department are given. Comparing the capital cost of similar lines in this Table with that given by Eqn. 5 an approximate idea of the commercial value of any projected line can be formed. However, it is the duty of the general preliminary studies to definitely determine whether and in what form—Main line, Secondary or Tertiary line, normal-gauge or narrow gauge—the line should have for the amount of the available capital.

The procedure outlined above for the estimation of the probable revenue of a projected railway has been widely employed, particularly in Italy and Austria. Of course it cannot give absolutely exact results since the factors

$$c_1\ c_2\ d_1\ d_2\ p \text{ and } q$$

vary in each particular case. But with careful and intelligent application it will give results which will certainly enable a good idea to be formed beforehand of the commercial build-worthiness of any projected line.

With a view to uniform procedure on the Prussian State Railways in the preparation and presentation of estimates of prospective revenue attention to the following points is regarded as essential.

A.—Revenue.

(1) From Passenger-and Goods-Traffic.

(a) The probable passenger-traffic is to be determined on the basis of the number of probable journeys under the prevailing local conditions of the inhabitants of the traffic-areas of the localities through which the projected line is to run. On this determination adequate consideration is to be given to the distance of the individual localities from the nearest station, to the commercial occupations of the inhabitants, and their standards of comfort, proximity to manufactories, large towns, markets, &c. A suitable basis for this estimate will be afforded by the data which similar lines already existing in the same or in neighbouring districts and having similar traffic and industrial characters to the district under comparison, have given.

(*b*) A statement is to be furnished for the purpose of estimate of each locality with its population and the number of their journeys in the form of a Table in which is especially to be entered the probable length of the journeys in kiloms., both in the case of the projected line and for the already existing lines when an increased traffic flows to the latter,

(*c*) A special estimate is required for any prospective special traffic in addition to the general traffic, such as artisan trains, tourist trains, excursion trains to watering-places, pilgrimages, &c.

(*d*) Regarding the effect of the projected line on the passenger-traffic of existing lines, it has to be examined whether and in what degree there will be an increase of traffic caused to the latter, or a deviation of the traffic therefrom to the advantage of the new line and to the injury of the old ones. If the new line will shorten older routes, it is to be particularly and carefully estimated what the expected through traffic on the new lines as a consequence of this will be.

(*e*) The kilometric unit-rate, which is taken as the basis in the estimate of the probable returns, is to be worked out and reasons briefly given for its amount.

(*f*) As to the revenue from luggage as a rule, a simple estimate and statement in per cents. of the revenue derived from passenger-traffic will suffice.

II. From Goods- and Cattle-Traffic.

(*a*) When feeder lines are in question, the following points as a rule will require notice in the forecast.

(1) What goods—kind and volume—hitherto exclusively carried on the high roads or on waterways are despatched to and from the localities in the traffic-area of the new line?

(2) What goods—kind and volume—are at the present time carried to and from the neighbouring stations of State and private railways?

(3) Within what area, and up to what distance, will the volume of goods under (1) and (2) fall to the share of the new railway?

(4) What are the principal goods (classes), and what revenue will they yield to the new line in terminals and mileage rates?

(5) What amount is lost to the older lines in terminal and, under given circumstances, in mileage charges on the volumes of goods under (2)?

(6) what amount of revenue can the older lines reckon on as derivable from the volume of goods determined under (1)?

(7) What new traffic is reasonably to be expected from the opening-up of the districts concerned (arising from the enlargement of existing businesses, or the founding of new industrial enterprises, from the export or import of agricultural and forest productions, or from the increased demand for agricultural implements and machinery, &c.), and up to what average distance will it probably be moved on the old and new lines respectively?

(8) What revenue from the terminal charges and mileage rates may be expected from the traffic under (7) for both the old and the new lines?

The results of the above investigation are to be tested and examined as to their completeness and trustworthiness and, if necessary, to be corrected, and finally exhibited in one or more tabular statements.

(*b*) When the projected line will unite already existing lines (State or private) the participation of the new line in the through traffic to and from the existing lines is particularly to be examined (in addition to the information required under (*a*). For this purpose the shortenings of distance effected by the new line are to be determined for the most important stations—or nodes—and are to be shown either in the calculation of the revenue itself, or in a tabular statement.

The decrease in revenue on the older lines due to their traffic having been in part deviated from them and to the lowering of rates arising from the shortening of route are to be determined from the available data, and after deduction of the saving in working-expenses is to be compared with the revenue accruing from the traffic deviated to the new line. Now the building of a new line which connects older existing lines is financially only justifiable if its injurious influence on the revenue of the older lines can be restricted to the smallest extent possible. Therefore particularly in the case of Secondary lines it is necessary to examine whether and to what degree, under the particular working conditions obtaining, a deviation of the traffic is probable and advantageous, and whether in fixing the rates the full amount of the shortened distance is to be allowed for, or whether the existing rates should be lowered only so far as is necessary to avoid re-booking.

(*c*) In determining the increased working-expenses on the older existing lines resulting from an increase of traffic thereon it is to be borne in mind that the expense of working the new additional traffic is usually less than the average outlay for the existing traffic. In most cases on the older lines the increase of business can be handled adequately by the existing staff and arrangements and with the ordinary trains without any noteworthy increase of cost.

On the other hand, a saving in working-expenses arising from a decrease in traffic is only to be reckoned on when it can be foreseen with clearness and certainty.

(*d*) The income expected to be derived from cattle-traffic, duly taking into account the local conditions, is to be shown separately both for the new line and for the existing lines.

(*e*) As to the subsidiary sources of revenue from goods- and cattle-traffic, what has been said under (1) holds.

III. Other Sources of Revenue.

In the majority of cases, the consideration of other sources of income may be neglected; in other cases when this is not so an estimated percentage of the gross revenue will suffice.

B.—Outlay.

(*a*) The anticipated administration, maintenance, and working-expenses are to be estimated under the heads of "staff" and "materials"; and the probable outlay in additional staff is to be roughly determined.

(*b*) For Secondary lines it may be assumed that when opened to traffic there will not be any considerable extra expense in staff in the Administration Department nor of Inspectors' staff; and for the posts already existing on the older lines there is only the extra expenses to be considered arising from the new line.

(*c*) The determination of the "material" expenses is made with the help of the Plan of Working * the traffic and on the basis of the probable work to be done by the rolling-stock. Their estimation according to the unit averages for the whole of the Prussian State Railways published in the Statement of the Results of Working—per 1,000 locomotive paying-km. or vehicle-axle-km.—will be for Secondary railways necessary only in exceptional circumstances, because their working and traffic conditions are much simpler than those of Main lines.

(*d*) With the object of making the estimate as trustworthy as possible it is desirable that the expenses determined per km. be contrasted as far as possible with those which experience shows to obtain on other lines similarly situated as traffic and working.

C.—Return on the Capital.

(*a*) As capital is to be reckoned, the contributory payments by the State for the

(1) The construction of the line:

(2) Acquisition of land.

(3) Purchase of the requisite rolling-stock—on Secondary lines 10,000 M. per km.

(*b*) The return on the capital to be expected on the basis of the net revenue is to be calculated both with and without taking account of the reflex effects on the State lines already existing or under construction, and expressed to one decimal place as a percentage of the capital.

TRAFFIC REPORT.**

(*a*) Name and Object of the Proposed Railway.

The line is to be named as indicated in the Order authorising the undertaking of the preliminary studies.

The object of the proposed line is to be briefly stated in consonance with the facts exhibited in the Technical Report.

(*b*) **Length: Districts and Circles** passed through by the line. The approximate length of the whole line as well as the several parts of it lying in each district and circle are to be taken from the plan and expressed approximately to one decimal place, and care is to be taken that the sum of the individual lengths so shown is exactly equal to the total length of the line; in addition the area in q. kms. and the population in round thousands are to be added in brackets.

(*c*) **Location of line**: when there is more than one project or possible line for the opening-up of a district under consideration, a short but exhaustive statement is to be given of the grounds for the choice of the line selected and the rejection of the others.

(*d*) **Economic and Traffic condition** of the districts run through. The economic and commercial conditions are to be thoroughly and exhaustively treated on the basis of the information supplied by trustworthy and competent authorities and the Engineer's own observations. This is to include not only a detailed investigation of the necessity for the railway connexion but also a statement of the prospective advantages and increased traffic facilities.

[* The Programme of the proposed method of working the prospective traffic is the subject of a special study.—Tr.]

[** The Traffic Report discusses the projected line in its commercial aspect and is the work of the Traffic experts. It is accompanied by a Technical Report the work of the Engineer expert—here omitted—giving a complete description of the proposed line from a technical (*i.e.* engineering) point of view.—Tr.]

The traffic- or feeder-area of the line is in general to be assumed as being 5 km. wide on both sides of the line. Its size is, accordingly, in respect of the local conditions (for example, already existing routes, mountain railways, etc.), to be determined and expressed in sq. kms. in units of 10 accompanied by the number of the population in round thousands. A short description of the existing economic and traffic conditions is to be added. This will include data regarding the different forms of agriculture and the productivity of the soil, minerals, &c., existing railways, roads, rivers, streams, &c., and proposed works of art in reference to their influence on the traffic of the new line. Further, are to be stated the most important towns with their population in round hundreds, the principal branches of trade, and any other noticeable factors influencing the future traffic of the new line resulting from the particular direction taken by the new line.

The omission here of these details with a mere reference to them in the part of the project dealing with calculation of the revenue will not suffice.

Only industrial and manufacturing installations which are important in regard to the future of the line such as mines, quarries, smelting works, machinery factories, other large factories, mills, tileries, breweries, distilleries, &c., are to be noticed in their respective districts. It is also desirable that the number of workmen engaged in large works be mentioned.

Finally, the influence of the new line on the future condition of the economic and traffic conditions of the districts opened-up is to be stated provisionally.

As regards the objects of transportation, only the most important and, in particular, the bulkier need be mentioned, but they should be distinguished as export and import.

(*e*) **Fiscal Property.** The extent to which the new line will affect Government (fiscal) landed property is to be indicated, particularly as regards domains and forests; and where necessary, after conferring with the local authorities, the individual domains, farms, head-quarters of Forest Rangers in the district are to be indicated with their areas in hectares.

(*f*) **Construction-cost,** shares of the co-partners: State contribution to purchase of land.

In exact coincidence with the scope of the memos. subjoined to projects of law as hitherto usual, are to be stated :—

(1) The land-acquisition expenses which fall to the share of the parties concerned; also the estimated capital for the construction of the line, exclusive of those expenses, expressed in round thousands of marks.

(2) The amount per km. of line of the construction-expense in round hundreds of marks.

(3) Any extra cash contributions to the construction-expenses by the parties concerned, or the State contribution to the cost of the acquisition of land, with a short statement of the grounds for being sanctioned.

(4) When a case under (3) occurs the total amount of the expenses to be incurred on the part of the State for the construction.

As the Memoranda will form the basis of the project of law to be brought before the Government particular care should be given to the editing, especially as to the accuracy and mode of expression.

Foreign expressions are to be strictly avoided when equivalent German terms exist. All data must be completely up-to-date, and if possible should be official, and the sources thereof exactly indicated.

APPENDIX I.

In order to a better understanding of the preceding and to facilitate its application to any given case in what follows is shown by a concrete example *the form in which certain of the preliminary studies for a railway are to be submitted to the Director of German Railways for sanction.*

TRAFFIC REPORT

REGARDING

A projected Normal-gauge Railway

FROM BRAUNDORF TO GRÜNWALDE.

A. Designation and Object of the Proposed Railway.

The object of the line is to connect the districts lying between the Friederichsfeld-Raupach, Raupach-Braundorf, Braundorf-Lindheim, and Lindheim-Friederichsfeld lines, thus opening up an area of some 600 q.km.

The districts passed through are generally fertile and carefully cultivated, and contain valuable minerals (stone, turf): there are also numerous industrial concerns.

The proposed line will confer railway connection so indispensible for their progress on the district run through and on its inhabitants, both with the individual districts and also with the existing lines of railways.

More particularly a connection will be made with Friederichsfeld the adjacent capital of the principality of R. which it is calculated will facilitate to a still greater extent the present brisk traffic, and at the same time increase it.

The new line will also provide a shorter rail connexion between Braundorf and Friederichsfeld and will thus join-up a considerable part of the traffic-area lying on the further side of these two stations.

The shortening in question will be somewhat as follows:—

Friederichsfeld and Berlin	17 km.
,, ,, Braundorf	17 ,,
Grünwalde	31 ,,
Berlin and Annaberg	13 ,,

This shortening of distance will render possible a much more rapid movement of goods in an important traffic district, especially between Berlin and Friederichsfeld.

From a military point of view also the line will be an important one, and will therefore be so located that complete military trains can be despatched at intervals of 2 hours by means of powerful locomotives.

B. Length: Governmental districts and circles traversed.

The length of the line between the railway-stations of Braundorf and Grünwalde will be about 29·2 km. Of the existing station track in Braundorf and Grünwalde however ·7 km. will be utilized, so that the length of the line to be built will be only 28·5 km. Of this distance 7·4 km. lies in Prussian territory, namely, 3·2 km. in the royal district A, circle Musterfeld (13,379 q. km. 52,000 inhab.), and 4·2 in the royal district B, circle Raupach, (802 q. km. 31,000 inhab.). The remaining length of 21·1 km. lies in the principality X; of this, 17·8 km. lies in the circle Lindheim (812 q. km. 62,000 inhab.), and 3·2 km. in the circle Friederchsfeld (543 q. km. 128,000 inhab.).

C. The Location described.

The proposed line runs from the Braundorf station some 2 km. southwards along the Braundorf–Raupach line, then goes in a south-westerly direction to Bruchhausen—an important place, on account of its stone quarries, and from there to Bohndorf, in the neighbourhood of which there is a sugar factory, and continuing in the same direction *viâ* Nehrheim and Grünwalde until the Lindheim–Friederichsfeld line—some 2 km. east of the Grünwalde railway station—is reached. From this point the new line will run along the track of the Main line and join it at Grünwalde. From Grünwalde to Friederichsfeld, the Main line will be run over.

Beside this projected line there are two other lines from Nehrheim under consideration. One of these would run in the Steinbach valley along the right bank, and possibly join the Friederichsfeld-Raupach line at Dreuheim. The other proposed line would follow the right bank of the Steinbach as far as Dristadt, cross the Steinbach at this point and then run along the left bank *viâ* Nesselrode and Hehren and join the Friederichsfeld-Lindheim line at the flag-station at Delben.

By the two lines these distance is about 7·5 km. and 8·3 km. longer. The construction would involve an additional expense of 1,000,000, or 600,000 M. Besides this, the ground in the Steinbach is mainly swamp.

Consequently, the first and shortest line to Grünwalde which is also the cheapest to construct is the one taken for further study.

D. Economic and Traffic conditions. (1) The traffic area described.

The district opened-up by the line Braundorf–Grünwalde has an area of about 200 q.km. with 11,000 inhabitants. The line itself will directly touch some 40 q.km. and will serve a population of 8,000. The soil of the strip of country in question is mostly fertile. There is a well-developed system of agriculture, grazing, and a considerable business in cattle-rearing. Amongst the principal agricultural objects produced may be named corn, potatoes, and beet-root. Further, there are extensive forests and valuable stone quarries, also, clay, sand, turf and loam pits.

In the district under notice there is a well-built carriage road. The use of made roads to the nearest railway station is, however, for distances up to 16 km. associated with so much expense that any competition with other districts having more favourable means of communiciation is very difficult and almost impossible. But in the case of a new railway. the existing roads would form appropriate and convenient ways of approach and departure for the railway traffic.

The most important towns touched by the proposed railway and their populations are as follows:—

Bruchhausen	1000
Bohndorf	300
Nehrheim	500
Grünaue	200
Herzdorf, Barzheim, Florbach, each	400

Of these Bruchhausen, Bohndorf, Nehrheim and Grünaue are proposed as railway stations.

At Bruchhausen, the principal branch of industry is the quarrying and working-up of stone—in the remaining places agriculture is the principal employment.

As regards more distant commercial districts there are the following important places:—Dristadt 800, Benzheim, Derdorf, Wehrheim, each with 600 inhabitants, Gr. Klosbach 500, and Gr. Kirchheim, Friesbach, Florbach and Wehrburg each with 400 inhabitants.

Here also agriculture is the main occupation of the inhabitants.

In the district to be opened-up there are the following industrial concerns—important in view of future traffic.

(*a*) In the Lindheim Circle.

There are extensive stone quarries in the neighbourhood of Benzheim and Bruchhausen which employ regularly some 400 workmen; a beetroot-sugar factory and chicory manufactory in Bruchhausen in which 50 workmen are employed, a sugar factory in Kirchheim employing 150 hands, three tileries in Bruchhausen, Herzdorf and Zarndorf employing 30 workmen, a creamery, a lime-burning business, and several mills.

(*b*) In the Münsterfeld Circle.

There are sandpits at Wehrburg and Lehmsburg employing 14 men; several tileries at Wehrheim and Kl. Friesbach employing 15 hands; also a distillery.

A packing-case and cask manufactory, turf bogs at Kl. Friesbach and Florbach, and several mills some of which are worked by steam.

In the neighbourhood of Dristadt and Nixdorf there are extensive loam deposits. Up to the present owing to the high cost of transportation little or no use has been made of these deposits, although they would provide an excellent manure for less fertile districts, such as, for instance, the adjacent part of the province of A. The business would be an advantageous one not only for the owners, but also for a distant circle of buyers.

For the same reasons the sand of Wehrburg and Lehmsdorf, which is so valuable for glass-making, and the great turf deposits at Kl. Friesbach have been but imperfectly utilized.

Finally, there are in this district run through by the railway extensive forests, especially the (fiscal) Gleinbach wood in the Raupach circle: and these owing to the imperfect railway communications are productive in only a minor degree. The proposed railway will confer on agriculture, forestry, and the manufacturing industries, and on the mining undertakings in the district to be opened-up the possibility of a larger and more rapid sale of their productions resulting in a more convenient and at the same time cheaper satisfaction of their many requirements.

2. Commodities to be dealt with.

The principal objects of transportation will consist of agricultural products and cattle together with an export business in sandstone, bricks, manure (loam), gravel, turf, sand, beetroot, beetroot-parings, sugar, and timber. The imports will consist of manufactured goods small coal, coke for domestic use, steam coal, artificial manures, seed, cattle food and poor underfed cattle.

3. Effect on the State Lines.

The new line will withdraw traffic for considerable distances from the existing State railways and mainly from that between Friederichsfeld on one side, and Braundorf-Wilhelmsthal and Grafenstein and Berlin and their hinterlands on the other. The existing lines will suffer a further reduction of traffic since the rates (per unit) over a considerable traffic-area will fall owing to the shortening of distance by the new line. On the other hand, the new line will bring new traffic to the older State lines from the newly opened-up districts.

It is probable that the annual loss of revenue to the State Railway lines may amount to 63,000 M. per annum.

The greater part of this falling-off of revenue will be profit to the new line and is derived mainly from the through-traffic, and the balance will go to the freighters, owing to the reduction of rates consequent on the shortening of the distance.

4. Government Lands.

The new line will be important as opening-up the (fiscal) Government lands. The domain Nonneuberg, situate in Prussian territory, and the Gleinbach forests amount to about 190 and 580 hectares, respectively.

5. Cost of construction—Share therein of the parties concerned.

The amount due for land-contribution from the communes and landowners affected by the proposed line is estimated at 298,000 M. and the construction expenses (exclusive of these payments on account of land, and of the special funds to be subsequently agreed upon for the rolling-stock) at 2,450,000 M. or for a length of 28·5 km., say, 86,000 M. per km.

The Government of the principality of Z has offered to pay a cash contribution of 8,000 M. per km. Accordingly, this contribution for the 21·1 km. lying in this principality, will amount to a total of 168,000 M.

The share of the State contributions to the construction-cost will therefore be (245—168) 000=2,281,200 M.

Calculation of Revenue of a Subsidiary Railway,

FROM BRAUNDORF TO GRÜNWALDE.

I. Revenue from the new Line.

A.—General.

The working-length of the line is 29·2 km. or, say, 30 km. Beside the terminals, stations are proposed at Grünaue, Nehrheim, Bohndorf and Bruchhausen.

The distances apart of the several stations are as follows:—

From Grünwalde	to	Grünaue	...6·7 km.	0· km.
Grünaue	„	Nehrheim	...6·3 „	13 „
Nehrheim	„	Bohndorf	...7·7 „	20·7 „
Bohndorf	„	Bruchhausen	...4·0 „	24·7 „
Bruchhausen	„	Braundorf	...4·5 „	29·2 „
			29·2 km.	

B.—Passenger-traffic.

From Statement I—p. 108—in which are given the traffic data based on the results of working of existing lines under similar traffic and revenue conditions, it is seen that the probable *local* passenger-traffic on the new line will amount to 1,340,000 passenger-kms., and the additional traffic on the old existing lines will be 452,000 passenger-kms.

Thus there is a total of 1,792,000 passenger-kms., and assuming a return per passenger-km. corresponding to the present conditions of the Prussian State Railways, viz. 3 pf., there will be a total revenue of 53,760 M.

Assuming that the revenue from luggage to amount to 4% of the gross returns from the passenger-traffic, then the revenue of the new line from the local passenger and goods business per annum is

$$53760 + 53760 \times \cdot 04 = 56{,}000 \text{ M.}$$

In this calculation all special and extraordinary descriptions of traffic such as workmen's trains, tourist-tickets, excursions, watering-place trips, pilgrimages, etc., are under the assumed conditions excluded, and consequently are not to be considered in the estimate of the new line's probable traffic, since a district of only average traffic character is in question.

Since the projected line will considerably shorten existing lines, it will enjoy a considerable **through traffic**. The revenue from this, according to the investigations of competent statistical authorities, for passenger and goods will amount to 9,000 M. Consequently, the probable annual gross revenue from passengers and parcels of the projected line will be

$$56000 + 9000 = 65{,}000 \text{ M.}$$

C.—Goods-traffic.

According to data funished by the local authorities, the goods-traffic will comprise

Imports: Stone, sugar, beetroot, beet-refuse, grain, potatoes, butter, milk, sand, turf, timber, beet-extract, cattle.

Exports: Coal, beet, limestone, manures, fodder, grain, building-materials, manufactured goods, cattle.

In Statement II—p. 109—is shown the volume of and distances for which the above goods will probably be offered for transportation based on information supplied by the local authorities, together with the revenue that will accrue therefrom to the Administration. Accordingly, the projected line will draw an annual revenue

from the goods-traffic of	...	... 161,800 M.
„ „ cattle „	...	... 15,700 M.
		Total = 177,500 M.

The annual revenue from the through goods- and cattle-traffic according to the calculations of competent statisticial authorities is put at 64,000 M, so that there will be an annual gross revenue of

$$177500 + 64000 = 241{,}500 \text{ M.}$$

D.—Gross revenue.

The total revenue of the projected line accordingly is

$$65000 + 241{,}500 = 306{,}500 \text{ M.}$$

This gives for the tariff-km. a revenue of

$$\frac{306500}{30} = 10{,}200 \text{ M.}$$

Employing the method of determining the revenue given in § 2—p. 91—we obtain the following results.

The formula for the total revenue derived from local passenger and goods business for the whole line is

$$U = p\,c_1\,d_1\left(1 + \frac{d_1}{L}\right) + G\,c_2\,\,d_2\left(1 + \frac{d_2}{L}\right)E.$$

The population of the station areas is taken as under:—

Braundorf and Warmshausen: 2600 + 1000 = 3,600:

owing to their proximity to the existing lines only $\frac{1}{6}$th of this figure is to be taken

Bruchhausen	...	... 950
Bohndorf	...	... 300
Nehrheim	...	... 500
Grünaue	...	... 200
Grünwalde	...	... 450

As in the case of Braundorf, and for the same reason, only $\frac{1}{3}$rd of this last figure is to be taken, viz. 150.

Since the projected line will be a State Railway, the terms in the above formula will have the following values.

(*a*) The average annual number of passengers and goods-tonnes per head of population: $p = 19{\cdot}7$, $g = 5{\cdot}4$ (see Table II—p. 89—Secondary lines, for station sites of less than 1,000 inhabitants.

(*b*) Revenue per passenger-km.; c_1 = about ·03 M. and for the goods-tonne-km., c_2 = about ·04 M. (see Table III.—p. 93— cols. 46, 47, Prussian State Railways).

(*c*) The average length of journey or passenger-haul on the Prussian State Railways is $d_1 = 24$ **km.**, and of a goods-tonne $d_2 = 115$ **km.**; on a line of 29·2 km. in length, d_1 will $= 12$ $d_2 = 18$. (see Table III., cols. 44, 45. These two values of d_1, d_2, are means).

(*d*) The total number of inhabitants in the station sites is

$$E = 600 + 950 + 300 + 500 + 200 + 150 = \mathbf{2{,}700}.$$

(*e*) The length of the line,

$$L = 29{\cdot}2 \text{ km}.$$

Inserting these values in the formula we have

$$U = \left[19{\cdot}7 \times {\cdot}03 \times 24\left(1 + \frac{12}{29{\cdot}2}\right) + 5{\cdot}4 \times {\cdot}04 \times 115\left(1 + \frac{18}{29{\cdot}2}\right)\right] 2{,}700 = \mathbf{162{,}000\ M.}$$

To this sum has still to be added extra revenue derived from other sources—passengers' luggage, rents, etc., which at the present time on the Prussian Railways amount to 4% of the whole; so that the total is thus raised to **169,000 M.**

Now since in deriving the Formula (3) extra-ordinary traffic conditions have been expressly excluded, and since also, the values, for p and g in the Tables I and II, have been deduced under the same limitation, the traffic from the great stone quarries at Bruchhausen must be allowed for separately. According to the Techenical Report accompanying this Estimate (omitted—Tr.), this traffic at the present time amounts to 8,500 double loads = 85,000 tonnes. The stone goes wholly to the large towns of Friedrichsfeld, Grafenstein, and Eldenau, and in the proportions of $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$, respectively.

Consequently, the projected line will carry—

$\frac{85000}{3} = 28{,}000$ tonnes a distance of 24·7 **km.** (Bruchhausen-Grünwalde.)

$\frac{85000 \times 2}{3} = 57{,}000$ t. „ „ „ 4·5 **km.** (Bruchhausen-Braundorf)

The revenue from this amounts to

$$(28000 \times 24{\cdot}7 + 57000 \times 4{\cdot}5)\ {\cdot}04 = \mathbf{38{,}000\ M.}$$

The revenue to be derived from its own traffic by the new line will therefore be

$$169000 + 38000 = \mathbf{207{,}000\ M.}$$

To this is to be added the revenue derived from the through traffic, viz.

$$9000 + 64000 = \mathbf{73{,}000}\ \text{M}.$$

So that the gross revenue derived from the projected line is

$$207000 + 73000 = \mathbf{280{,}000\ M.}$$

II. Effect of the Projected Line on the Traffic of existing Lines.

The shortening of existing routes by the building of the projected line will bring about the reduction of the existing passenger-and goods-rates. Owing to this, beside the falling-off in revenue on the existing lines, put at 73,000 M, there will be a second falling-off in returns due to the deviation of the through traffic on to the new line; and according to the calculations of the official traffic statisticians this will amount to

For the passenger and luggage business	... ...	5,000 M.
„ goods and cattle „	... ...	48,000 „
	or a total of	53,000 M.

The total loss to the older lines will therefore be 126,000 M.

There are important stone-quarries, turf bogs, clay, marl, and sand-pits, sugar manufactories, brick manufactories, distilleries, creameries, mills, and woods on the new line. The increased facilities of sale of commodities and the opening-up of districts will increase the general business done, and thus in a short time, the transportation on the old lines may be expected to increase, so that eventually there will be a considerable reduction in their original or initial losses.

III. Working Expenses.

A. Staff.

Increased number of officials in the General Administration	3,000 M.
1 P.-Way Inspector	2,800 „
8 Line Inspectors and line-walkers	8,000 „
1 Station Master (Bruchhausen)	2,800 „
4 Pointsmen, I Class (Bruchhausen, Bohndorf, Nehrheim and Grünwalde) ...	7,200 „
7 Pointsmen, and Assistant Pointsman (Bruchhausen 2, Bohndorf 2, Nehrheim 1, Grünaue 2)	9,100 „
7 Gangmen (Braundorf 1, Bruchhausen 1, Bohndorf 1, Nehrheim 1, Grünaue 1, Grünwalde 2)	5,600 „
3 Engine-drivers	7,500 „
3 Stokers	4,500 „
3 Guards	5,400 „
7 Brakesmen, Assistant Brakesmen, Porters	9,100 „
Total ..	65,000 M.

B. Material Expenses.

According to the proposed programme of working (omitted—Tr.) the projected line there will be daily three mixed passenger trains and one goods train; consequently the paying-load km. per annum will amount to

$2 \times 4 \times 29{\cdot}2 \times 365 = 85{,}264$ pay.-load-km.

add : for returned empties and shunting, at 4% 3,410

Total ... **88,700 pay.-load-km.**

1. General Management:—
   According to § 3,* the non-personal items of General Administration amount to ·106 M per pay-load-km. *i.e.*, 88,700 at ·106... = 9,400 M.

2. Way and Works:—
   (*a*) Maintenance of track between stations, inclusive of the through track in stations—excluding rail, sleeper, and switch renewals—per pay-load-km. @ ·1 M. ∴ 88,700 at ... ... ·1 M = 8,900 M.
   (*b*) Maintenance of side track in stations per pay.-load-km. @ ·054 M. ∴ 88700 at ... . ·054 M = 4,800 „
   (*c*) Maintenance of telegraph, signals, per pay.-load-km. @ ·007 M. ∴ 88,700 at ... ... ·007 M = 600 „
   (*d*) Renewal of the track—@ ·135 M. ∴ 88,700 at ... ·135 M = 12,000 „
   (*e*) General expenses of road management-@ ·01 M. ∴ 88,700 at ... ... ·01 M = 900 „

3. Transportation Expenses:—
   (*a*) Train-expenses @ ·223 M. ∴ 88,700 at ... ·223 M = 19,800 „
   (*b*) Maintenance of vehicles @ ·159 M. ∴ 88,700 at ... ·159 M = 14,100 „
   (*c*) Renewals of Rolling-Stock @ ·088 M. ∴ 88,700 at ... ·088 M = 7,800 „

Total ... = 78,300 M.

[* See p. 94.]

The total expenses are thus

$$65000 + 78300 = \mathbf{143{,}300\ M.}$$

and therefore per km. operated they are

$$\frac{143300}{29{\cdot}2} = \mathbf{5{,}000\ M.}$$

[If we take the total cost of a paying-km. as deduced in § 3, viz. 1·812 M. then we have as the outlay a sum of

$$88700 \times 1{\cdot}812 = \mathbf{160{,}700\ M.}$$

or per km. of length operated

$$\frac{160700}{29{\cdot}2} = \mathbf{5{,}500\ M.}]$$

IV. Calculation of the Profits.

Omitting the making good the loss to the existing lines, the profit-producing capacity of the projected line is

gross revenue:		280,000 M.
gross expenses:		−143,300 „
	Profit	**136,700 M.**

or per km. operated is **4,680 M.**

1. The cost of construction exclusive of land and rolling-stock amounts to 2,450,000 M. This gives a return of 5·6% on the invested capital.

2. The equipment-expenses inclusive of rolling-stock amount to 2,450,000 + 528,000 = 2,978,600 M.: under these circumstances the returns fall to 4·6%.

3. If the loss in revenue of the old lines be taken into account and it is assumed that half of it is recovered by the increased traffic arising from the new line then there remains a profit from the working of

$$136700 - \frac{126000}{2} = \mathbf{73{,}700\ M.}$$

which gives a return on the invested capital—

exclusive of rolling stock—of 3%.
inclusive „ „ —of 2·6%.

Estimate of probable profits from the Traffic of the Districts Opened-up.

PASSENGER-TRAFFIC.

Districts Connected-up.	Population.	Railway station.	Distance apart.	Probable Traffic of New Line.		Total passenger-kms.	Probable increased Traffic on the Lines Grünwalde-Freidrichsfeld Braundorf-Grafenstein.						Remarks.
				Journeys per inhabitant.	Length of single journeys.		Journeys per inhabitant.	Length of single journeys.	Total passenger kms.	Journeys per inhabitant.	Length of journeys.	Total passenger kms.	
Grünaue	135	Grünaue ...	0	2·5	7	2,680	2	14	4,280	...	...	...	Agriculture: land of average quality.
Herzheim	274	Grünaue ...	3	2·5	7	4,800	2	14	7,670	...	...	...	
Garberg	257	Grünaue ...	2	2·5	7	4,390	2	14	7,030	...	...	...	
Zarndorf	249	Grünaue ...	2	2·5	7	4,360	2	14	6,970	...	...	...	
Karbach	247	Grünaue ...	2	2·5	7	4,330	2	14	6,900	...	...	...	
Nehrheim	600	Nehrheim ...	0	2	13	13,000 etc.	1·5	14	10,500	...	...	...	
				Total	=	1,340,000	passenger	-kms. =	350,000			102,000	

452,000 passenger-kms.

SECONDARY LINE BRAUNDROF-GRÜNWALDE.

STATEMENT II.

GOODS AND CATTLE-TRAFFIC.

				Probable Goods Traffic.															Probable Cattle Traffic.								
District.	Population.	Railway station.	Distance apart.	Parcels.			General Car-loads class.			Special rate, Class V.			Special rate, Class III.			Special rate, Class II.			Horned cattle.			Other cattle.			Revenue from		Total.
				Import: tonnes.	Export: tonnes.	Revenue.	Import: tonnes.	Export: tonnes.	Revenue.	Import: tonnes.	Export: tonnes.	Revenue.	Import: tonnes.	Export: tonnes.	Revenue.	Import: tonnes.	Export: tonnes.	Revenue.	Import: tonnes.	Export: tonnes.	Revenue.	Import: tonnes.	Export: tonnes.	Revenue.	Goods-traffic.	Cattle-traffic.	
Grünaue ...	153	Grünaue	0	10	...		...	...		20	200		70	1,200		65	...		40	60		...	300				
Herzheim ...	274		3	2	12		...	...		...	10		...	290		...	...		90	120		350	350				
Garberg ...	251		2	9	...		...	...		...	260		150	1,900		100	40		30	150		...	950				
Zarndorf ...	249		2	2	25		...	...		...	415		575	2,470		90	...		20	160		500	1,000				
Karbach ...	247		2	8	15		...	...		...	200		125	1,750		90	...		60	170		...	500				
Nehrheim ...	500	Nehrheim	0	...	...		...	...		...	...		...	...		...	...		...	...		...	...				
						100			0			913			5118			231			780			301	6,362	1,081	7,443
																								Grand Total Revenue...	161,800	15,700	177,500

APPENDIX J.

The trial of Dr. Arons, the Socialist lecturer on mathematics at the Berlin University, on a disciplinary charge, has now, after more than twelve months' delay, reached its anticipated conclusion. The Ministry of Education, acting as the final Court of Disciplinary Appeal, has decided in accordance with the terms of the so-called "Lex Arons," which it framed and passed through the Prussian Chamber, with a view to the exigencies of this special case, that Dr. Arons must be deprived of his position as a privat-docent, and consequently of his right to lecture at the University. In July of last year, the Philosophical Faculty of the Berlin University, acting as a Court of Lower Instance, decided that there was no ground for proceeding against Dr. Arons, who though known to hold Socialist opinions, had not taken part publicly in the agitation of the party. The case was then referred to the Education Department, which has now condemned Dr. Arons in accordance with the law framed for the purpose. The Socialists, as the *Vossische Zeitung* observes, are attended with excellent good fortune. Dr. Arons has hitherto taught mathematics. In the future he will teach Socialism—

The Pioneer, 26th March, 1900.

DEDICATED

TO THE

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OF

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BY THE SAME TRANSLATOR.

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The Theory of the Trace:

BEING A DISCUSSION OF

The Principles of Location.

From the German
of
Wilhelm Launhardt.
Privy Councillor: Professor at the Polytechnic,
Hannover.

Premiated
by
The Union of German Railway Administrations.

Part II.

The Technical Tracing of Railways.

BY
A. BEWLEY,
FORMERLY
Ex. Eng.,
P. W. D. INDIA.

Madras:
PRINTED AT THE LAWRENCE ASYLUM PRESS, MOUNT ROAD, BY H. PLUMBE, SUPERINTENDENT.

1902.

"But to such engineering as is needed for laying out railways, ... the definition "given is literally applicable, for **the economic problem is all there is in it** . . . . A "little practice and a little study of field geometry will enable any one of ordinary intelligence, "without any engineering knowledge whatever in the larger sense, to lay out a railway from "almost anywhere to anywhere, which will carry the locomotive with perfect safety, and "perhaps shew no obstrusive defects under what is too often the only test—inspection after "construction from the rear end of a palace car.

"Therefore . . . . . one may fairly say that the locating engineer has but the one "end before him to justify his existence as such—to get the most value for a dollar which "nature permits; and but one failure to fear—that he will not do so . . . . His true "function and excuse for being an engineer, as distinguished from a skilled workman, "begins and ends in comprehending and striking a just balance between topographical "possibilities, first cost, future revenue, and operating expenses . . . . .

"But whereas the 'operating expenses' of bad bridge engineering come in a series "of startling catastrophies which shock the community . . . . . . the operating expenses from "bad railway location come by a gentle but unceasing ooze from every pore which attracts "no attention, albeit resulting in a loss vastly greater than any possible loss from bad con- "struction: **for it requires some training and experience even to appreciate the loss a "existing, and still more of both to appreciate it as remediable . . . . .**

"Hence it happens that Railway Location tends more and more to be entrusted to "those to whom it is a mere temporary incident in their professional career, and who consider "the work mainly from the constructive standpoint, without much attention to those larger "economic questions . . . . . to which, in well conducted work, the mere constructive details "should be wholly subordinate."*——**Wellington: The Economic Theory of the Location of Railways: pp. 2-3.**

"It is conceded to-day that a thorough acquaintance with all the branches of science "connected with his profession is an essential requirement for the engineer. A man who is "merely an expert in material matters is a good workman, mechanic, or artisan, but he is not "an engineer. By courtesy he may be called so; but his conscience must tell him that he has "no more right to the title than the medical man with only the parchment of a licensing body "has to be called doctor."

Engineering: 2 March, 1900.

"As regards the survey operations, it should be noted that there is hardly any branch "of engineering science which has made such strides in advance, in recent years, as that "of railway location. It is now more thoroughly understood that the extra expenditure "in employing competent men, and in spending sufficient time in working out the several "problems that present themselves in the prosecution of such work, is trifling in comparison "with the benefits derived from such a course in lessening the cost of construction and "of working.

C. O. Burge: Railway Construction in N. S. Wales.
[*Mins. Procs. Inst. C. E., Vol. CXXXII.*]

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* *Ex. gr.* "The report of Major G. F. Wilson, R.E., on the Ceylon railway system, makes serious charges "against the engineer responsible for the location of the coast-line extension from Alutgama to Galle. It is stated that "heavy grades have been put in which could have been avoided, whilst sharp reverse curves are frequent.

"The official explanation given is that the chief resident engineer consulted the general manager as to the "sharpest curve and steepest grade on the existing line, and believed that if he got nothing worse on the extension "he would be doing all that was desired. This somewhat haphazard method of locating the line has resulted in "extremely heavy working expenses.—*Engineering.*

AUTHOR'S PREFACE.

IN Part I of my "Theory of the Trace" the problem of location was considered solely from the *ecomonic* standpoint as 'Commercial Location.' We have now to consider in detail the *technical* conditions which depend on the form and character of the ground and on the type of construction and mode of working. The method of treatment differs for Railways and for Roads; and in this Part II the discussion is restricted to Railways.

The constructional details of the track, the substructure, and of all works of art are assumed to be known, as is also the existence of a geodetic survey, maps of the country and of its configuration together, and a knowledge of the physical and geological character of the ground based on an actual examination of it.

The technical laying-out of railways in its entirety comprehends all the considerations, numerical investigations, rules derived from experience, and the prescribed Government Regulations, which come up for consideration and disposal in the choice of a line of railway. All technical conditions are to be constantly regarded and disposed of from the broader point of view of more or less remote *financial contingencies.*

It is the object of the present Work on the Theory of Location to withdraw the problem of the determination of the trace and of its shape in plan and elevation from the region of vague guesswork and to place it as far as practicable on a mathematical and numerical basis. The work of location is not thereby, however, reduced to a simple problem of figures, it demands in a degree scarcely found in any other technical work a comprehensive study and a clear critical judgment of the many-sided economical and technical matters in which calculation serves as a tool.

Particular circumspection is required in fixing many of the numerical values employed in the calculations, since some of these, for example, prices, are subject to temporary and local fluctuations; others again are based on contingent future conditions which can only be dealt with by a guess.

It is inherent in the nature of the problem that there is uncertainty in the determination of several of the quantities entering into the discussion; this is unavoidable but that by no means lessens the value of these theoretical investigations from which we obtain a clear and rational conclusion in place of an opinion based frequently only on a merely indistinct intuition.

HANNOVER.
April, 1888.

Wilh. Launhardt.

Les questions de dépenses en relation avec les difficultés rencontrées dans l'exploitation des voies ferrées par suite de leur tracé deviennent de jour en jour plus importantes avec l'accroissement du réseau secondaire. En Allemagne, en Autriche-Hongrie et en Suisse où les fortes rampes sont fréquentes, cette question de la variation des dépenses d'exploitation avec le profil des lignes a été suivie de très-près pendant ces dernières années. Parmi ces travaux, une étude intéressante récemment publiée en Allemagne par M. Launhardt, directeur de l'École polytechnique de Hanovre, nous a paru digne d'être exposée dans ses traits généraux à cause de l'originalité de la méthode et des résultats remarquables auxquels elle conduit.

*   *   *   *   *   *   *   *   *

En résumé, les formules de M. Launhardt permettent d'aborder analytiquement une foule de problèmes relatifs à l'affectation des dépenses d'exploitation sur lignes accidentées, ou à l'étude approfondie du tracé des lignes projetées. Les données suffisamment précises manquent toutefois, mais il est juste de dire que celles obtenues par l'auteur sont d'une approximation bien supérieure aux évaluations que l'on fait aujourd'hui lorsque l'on projette une ligne ferrée, et en cela les formules qu'il donne peuvent être d'une incontestable utilité. Nous ajouterons *qu'avec les documents statistiques d'une Compagnie on peut arriver à établir avec suffisamment d'exactitude les coefficients numériques, pour procéder à la révision de certaines parties de la voie qui offrent des difficultés spéciales en service courant*, et, par example, rechercher s'il y a avantage à augmenter les frais de premier établissement pour diminuer les dépenses journalières d'exploitation par un abaissement des *rampes nuisibles*, le redressement de courbes trop raides.

(**Ch. Gerhardt**: *Revue Générale des chemins de fer*: *Oct.* 1878, *p.* 222).

REVIEW.

THE TECHNICAL TRACING OF RAILWAYS.

Technische Trassirung der Eisenbahnen. Zweites Heft der Thorie des Trassiens von Wilhelm Launhardt, Geh. Reg-Rath, Prof. an der Techn. Hochschule zu Hannover. Hannover: Schmorl und von Seefeld. 1888—8°, 259s. mit 23 Holzschn.

In the literature of the subject it is undoubtedly the economic aspect of Railway work that for a variety of reasons has hitherto been the least cultivated. Whereas Mechanics and the Strength of Materials have a well-defined field of their own, *scientific* Civil Engineering presupposes a knowledge of this economic aspect of its subject; and a thorough treatment of the subject from an economic point of view is only possible when based on a knowledge of the science of Engineering. But as the scope of the economic side of engineering widens, and in proportion as the principles of Mechanics are applied to economic concerns, the individual problems become more complex and many-sided, and therefore less amenable to systematic classification and treatment.

The supreme merit of the present Work of the distinguished Author **is that it treats from the economic point of view the considerations, numerical investigations, and rules and precepts derived from experience, which determine the choice of the string of lines forming a railway centre line.** Consequently, it would be unreasonable to look for either completeness and still less, unlimited applicability in its conclusions. The work is based on the Statistics of the Prussian State Railways for 1885—86: *its figures thus have reference solely to the average character of the whole Prussian State Railway system and must be correspondingly modified if its conclusions are to be applied to other railways.* Further, the Author in order to obtain results of general applicability has had to make in several instances certain assumptions which limit the applicability of his formulæ. Thus (see § 2) in his determination of the Cost of Construction he has omitted all Stations and Major works; and the remaining line-expenses he has treated as being in direct proportion to the length of the road: also the question (see § 33) whether a line in the bottom of a valley or one on the valley-side is the better is investigated without consideration of the fact that a valley bottom-line generally costs less in maintenance than one located on the side of the valley. Nevertheless in most cases the results obtained in the work under notice are of immediate practical application.

The first Section begins with an analysis and subdivision of the Construction-cost and Working-expenses into those items which are independent of the traffic and of location of the line, those which increase with the traffic; viz. Station-expenses, and the Road-expenses; those directly proportional to the tonne-kms. and passenger-kms. performed; and the cost of motive-power dependent on the traffic, length of line, and character of the line. Formulæ and numerical data are given connecting these expenses with the length, curvature, and gradients of the line and the direction of the goods- and passenger-traffic.

Section II contains investigations of the best value of the details, viz. the width of gauge width of ballast and accessories, curvature and transition-curves, the gradients, the optimum ruling gradient, momentum-grades, lost height, distance apart of stations, and finally, virtual lengths and grades.

Section III treats of Location under the heads of general and detailed tracing, location in flat or undulating ground, in hilly ground, and in mountains; of the relative financial merits of two locations in hilly regions, of continuous and broken grades, the development of the trace, and of its location by means of contours.

From the above description it is evident that the contents of this work are most varied; and every Railway Engineer ought to be grateful to the esteemed Author for his attempt to remove a difficult subject more and more thoroughly from its present region of blind guesswork into the domain of scientific certainty.

FORCHHEIMER.

[In the "**Organ für die Fortschritte des Eisenbahnwesens.**"]

CONTENTS.

SECTION I.

The Working-Expenses of Railways as influenced by the Length, Curvature, and Gradient of the Trace.

SECTION II.

Discussion of the Details of the Trace.

SECTION III.

The Location of the Trace.

APPENDIXES.

EXPLANATION

Of the more Important Symbols occurring in this Work.

A = Kilometric capital cost.
B_0 = The cost of the locomotive-km. independent of the tractive-force exerted.
$B.$ = Cost of an engine-km. when the locomotive exerts its maximum tractive-force.
$B = \frac{B_0 + B_1}{2}$ = The mean cost of a locomotive-km.
L = Weight of locomotive and tender in tonnes.
Q = Weight of the train, *excluding* engine and tender, in tonnes.
P = The number of passengers } carried per annum.
T = The number of tonnes of paying-load } carried per annum.
U = The kilometric cost of line-maintenance and supervision.
Z = The tractive-force, in tonnes, of the locomotive.
a = The expense per engine-km. per tonne of tractive-force exerted—or
a = The part of the coefficient of resistance, $w = a + b\,v^2$, independent of the velocity.
b = The load-coefficient—that is, the ratio of the gross load (exclusive of engine and tender) to the paying-load—or
b = The coefficient of v^2 in the expresison for the coefficient of resistance, as above.
c = The coefficient of curve-resistance.
e = The coefficient of the maximum gradient in the expression for the cost of braking per gross tonne-km.
f = Way-expenses per gross tonne-km.
g = Acceleration due to gravity. ($= 9{\cdot}8^{m}$.)
h = The height surmounted on a gradient.
i = Rate of interest.
l = Length or distance in km.
m = Width of gauge.
r = Curve-radius—or
r = Ratio of the volume of traffic in the direction in which it is less to that in which it is greater.
s = Ruling gradient
s_0 = Non-injurious gradient. *viz.* $< w$.
s_1 = Injurious gradient. *viz.* $> w$.
s_2 = Equivalent gradient.
v = Speed of train in m/sec.
w = Coefficient of resistance of train.
z = Coefficient of engine's tractive-force.
α = Central-angle in degrees of a curve.
α_0 = " " " " " " on a non-injurious gradient.
α_1 = " " " " " " on an injurious gradient.

Erratum.

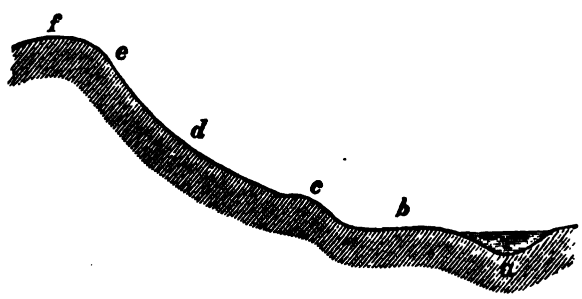
The Commercial Trace.

P. 6. Equ. (2). For $\frac{Ai}{c}$ read $\frac{i}{c}$

SECTION I.

THE WORKING-EXPENSES OF RAILWAYS AS INFLUENCED BY THE LENGTH, CURVATURE, AND GRADIENTS OF THE TRACE.

Fig. 1.



§ 1.

INTRODUCTORY.

The determination of the **Commercial Trace** is based on a consideration of the traffic-movement conditions, and assumes a uniform horizontal surface of the ground. When such a trace is next considered with reference to the working of the traffic thereon and to the configuration of the ground it undergoes changes more or less considerable through which it becomes the **Technical Trace.**

Amongst the conditions affecting the **working**, exclusive of the volume and kind of loads to be carried, the **type of vehicle** and the **mode of employment of the motor-force** are of capital influence on the form of the trace.

The **conditions affecting the ground** which have to be taken into consideration are :—

1. The **Shape** of the ground.

This may be represented by a plan on which horizontal points at fixed equal vertical intervals are connected by continuous lines termed Contour Lines, or horizontal curves.

If on such a plan lines are drawn cutting each contour perpendicularly we obtain the lines of maximum fall or steepest ascent, which if continued indefinitely across country form a series of alternately rising and falling lines. By joining the upper points of curvature of adjacent lines of steepest grade we obtain the watershed line or "divide'; and, similarly, by joining the lower points of curvature, the valley-line or Thalweg.

The longitudinal section of a line of steepest descent exhibits in the immediate neighbourhood of the valley—**Fig. 1**—the river-bed, *a*, with its banks; and immediately adjacent to these, and slightly ascending the valley bottom, *b*, which is bounded by the upper shore, *c*, from which latter the sides or declivities, *d*, of the valley rise, usually becoming steeper the higher they rise. The valley-sides as they ascend are frequently cliffs, and form the border or edge of the valley, *e*. Still higher up we have the crest, *f*, usually more flatly rounded, rising up to the watershed. The general form of valley as above described, of course, rarely occurs with all its parts clearly marked.

Valleys from their upper ends where they strike a watershed down to their lower ends where they debouch into a larger valley or on to the shores of a lake always have a descent decreasing in degree from the higher to the lower end. Within this general regularity there are flatter sections or steeper falls which form steppes, rapids, or waterfalls. Sometimes valleys rise for small distances in their general fall and so form lakes.

Watersheds, also, fall longitudinally from above downwards but much less uniformly than valleys, often forming a series of rising and falling lines.

The lowest depression in a watershed is variously named, according to the local usage, a Pass, Saddle, Col, Furka, Scheideck, etc.: and the highest elevations are fancifully named according to their shapes, as Summits, Crowns, Peaks, Crests, Eggs, Horns, Cones, Points, Needles, Pics, etc.

The area bounded by watersheds and which is drained by a definite water-course is called the basin or *catchment area;* and according to the importance and size of the water-course it is either a river or stream basin. The largest catchment area in Europe is that of the Volga which has a length of about 3,400 km. and an average width of 500 km. or, say, an area of 1,700,000 q. km.

The risings of the ground classified according to their height, are termed, Elevations (up to 150 m); Hills (150 to 300 m); Mountains (above 300 m); and the valleys, according to the heights of their surrounding watersheds, are lowland valleys, hill-valleys or mountain valleys; or according to their shapes, Ravines, Rifts, Canyons, Clefts, Troughs, Becks, etc.

Adhering approximately to the above gradations of rise (*i.e.*, from under 150 m, between 150 m to 300 m, and over) we distinguish low-lying plains, middle-plains, and high-lying plains; and further plains, hilly districts, and mountain regions. If within the same limits of height, the formation of the ground continues uniform over a large area, say above 5,000 q. km. it is termed a Plateau or Table-land; and we distinguish low, middle and high table-lands.

2. The **geognostic character** of the ground.

The geognostic investigation of the ground should not extend merely to the determination of its geological character yielding information as to sources of building material and to the determination of the various kinds of rock and earth; it should also be directed to ascertaining the bearing power or support that the soil can afford, its resistance to climatic disintegrating influences, its capabilities as regards drainage and excavatability; and the examination should further be directed to ascertaining the strike and dip of the strata. By the strike is meant the direction of the imaginary horizontal contours of the outside or externnal surface of the strata; and by dip, the inclination with the horizon of the imaginary lines of the steepest descents.

With regard to the excavatability or cohesiveness of soils, we distinguish

(*a*) Loose earth, capable of rolling, and which can be excavated with the shovel or after breaking up with the pickaxe—such as sand, gravel, rubble, clay, light vegetable and field soil.

(*b*) Light or soft earth, which can be loosened with the spade and removed in pieces of considerable size such as fat vegetable earths, loams, etc.

(*c*) Stuff requiring crowbars, pickaxes or wedges to detach it, for example, marls, indurated clays, conglomerates, consolidated gravels.

(*d*) Material requiring the use of explosives—including, beside rock, dense and firmly-sticking stuff.

When determining the resistance to detachment, that is, the cohesiveness or bearing power of different kinds of ground, it is to be particularly examined whether landslips are to be expected. Landslips may arise from one of two causes: either the stability of the ground has been injured by the entrance of water and has become semi-fluid; or, an incumbent mass, which in itself may have great stability, may slide on the inclined surface of a stratum permeable by water and which by the infiltration of water has become slippery.

Finally, the examination of the ground is to be directed to the discovery of building material, such as quarry-stone, sand, gravel, etc.: although it is only exceptionally that this has any influence on the position of the trace.

3. **Climate** and **Weather**.

As a rule it is only necessary to examine how the trace lies with respect to the sun, and whether it is exposed to frequent cross-winds or snow-drifts. In mountainous regions these points acquire importance because there it has to be ascertained in advance what difficulties and obstacles threaten the working in winter of the open-cut line; where and to what extent avalanches occur; and whether showers of rocks and stones due to frost and weathering are likely to occur.

4. **Hydraulic** conditions.

The height of the highest and lowest level of the ground-water: the low, mean, and high-water level of rivers and lakes, and their highest navigable levels; also the maximum volume of flow, flood-velocity, and flood-area are most important points as regards the position of the trace. In mountainous regions torrents and their cones of deposit and moraines demand especial attention.

The volume of flood-water depends on the size of the catchment area. For North-west Germany, for example, it may be assumed per q. km. per second, somewhat as follows:

In the neighbourhood of the source: high water, ·25 to ·35 c.m.: low water, ·0025 c.m.

In mountains: high water, ·20 to ·35 c.m.: low water, ·002 c.m.

In hilly country: high water, ·16 to ·18 c.m.: low water, ·002 c.m.

In plains: high water, ·10 to ·15 c.m.: low water, ·0015 c.m.

The character and position of the water-ways often compel an alteration in the position of the trace, both in plan and elevation, with a view to obtain the best position for bridging; also to avoid repeated crossings, or the crossing of a flood-area.

5. The **Economic exploitation,** *i.e.*, the culture of the surface of the ground.

The position of the trace may have to be altered in order to avoid traversing valuable plots of ground, especially where built upon, or to avoid meeting churches and graveyards, or to avoid dividing-up and separating property. In addition, there is the location of the protective cuts—against fire—in moorland and forest to be attended to, although this would rarely be a reason for altering or modifying the trace.

6. **Relation to lines of communication.**

The volume of the traffic on the lines of communication crossed—*i.e.*, Cart roads, water-ways—and the paying-load in both directions thereon as dependent on their grades has to be ascertained. This is required in order to decide the question whether crossings are to be made at rail-level, by an over-bridge, or by an under-ground passage or tunnel; and what gradients and approach-ramps are necessary. The lines of communication met with will frequently cause the trace to be raised or moved sideways; and will determine the position of stations.

Further, the necessity for making parallel roads—which bunch unimportant Cart roads, etc., crossed by the trace—is to be examined.

§ 2.

Analysis of Cost of Construction and Working Expenses of Railways for the problems of location.

In order to gain an idea of the influence which the vehicle and the features of the ground, just discussed, have on the position of the trace we must determine how the traffic working-expenses depend on the length, gradients, and curvature of the line.

In the official Statement of the Working-Expenses of Prussian Railways the disbursements are distributed under 3 heads, viz.:

General Administration:
Way and Works:
Transport.

This grouping of the **working-expenses** is useless **for location purposes**; they must be distributed under the following heads, of which the 3 last groups only concern our subject, viz.:

1. **General**—which are independent of the volume of traffic and of the length, gradients and curvature of the trace.
2. **Station**—which comprise all expenses in connexion with the collection and distribution of the traffic. These expenses therefore increase with the traffic, but are independent of the character of the trace.
3. **Way and Works or Road**—which include the *interest on the capital cost* of the line—exclusive of stations and all extra-ordinary works of art—and the ordinary *maintenance* and supervision expenses of the line between stations. Only those expenses are to be included under this head which can be regarded as increasing directly with the length of the line. Thus, all the outlay on the Major bridges and road crossings, which are requisite whatever the location of the line, also protection-works and such like extra-ordinary outlays which are independent of the length and position of the line or of the volume of the traffic, are to be excluded.

   Finally, the wear of rails, which is dependent on the traffic, is to be omitted.
4. **Train or running.**—These expenses are to be kept distinct for goods- and passenger-traffic and are to be evaluated on the basis of the tonne-km. and the passenger km., respectively. They are independent of the gradients and curvature of the line, but increase directly with its length and with the volume of the traffic.
5. **Traction or Motive-Power**—dependent on the volume of the traffic, on the length of the line, the gradients, and the curvature. These expenses are to be evaluated per locomotive-km., and in them the wear of the rails is to be included.

The sum of the last 3 items gives the expense of the service, *i.e.*, the net cost of working the traffic, which is to be taken as a guide in locating the line, because they are the items of cost affected by changes in the length, gradient, and curvature of the trace. Thus the cost of general administration, major bridges, over-bridges, tunnels, protective works, etc., being indispensable whatever the shape of the trace may be, are omitted, also the station expenses, cost of loading and unloading of vehicles, train-staff expenses, ticket distribution, luggage and goods management; and finally, the expenses due to halts of trains at stations, and the starting of trains.

§ 3.
Way and Works or Road Expenses.

The road-expenses, excluding stations and all *major* works of art, comprise the cost of land, cost of earthwork and culverts, minor bridges, road-crossings and retaining walls, permanent-way, fencing, telegraph, signals, and huts of sorts; and in any particular case will be easily determined by means of an estimate. For the Prussian Railway System in 1874 these expenses amounted to 134,000 M. per km. From amongst the items in the official Statement of the Results of the Working of the Prussian Railways—just published*—only the average gross sum per km. of the construction-cost is obtainable, and this according to the "Report of the Working of the Prussian State Railways for the year 1885-86," amounted in that year to 277,752 M. After deducting the expenses of stations and extra-ordinary disbursements and the cost of the rolling-stock there remains, approximately, 140,000 M. as the expense per km. of road, the interest on which at, say, 4% amounts to 5,600 M. per annum.

Similarly, the cost of track-maintenance and track-inspection per km. of the line between stations, including the through track in stations, can only approximately be distinguished in the data supplied by the Railway Administrations.

We may assume that of the total disbursements of the Administration on staff, for office-contingencies, heating, lighting, for substitute-service, &c., for large outlays due to *force majeure*, and finally for telegraphs and signals and accessories, which in all amounted to 30,442,200 M.—75% or 22,831,650 M., is to be charged to the account of the line between stations.

Of the expenses incurred for major supplementary works, completions, extensions and improvements, one-half, or $4,365,723 \times \frac{1}{2} = 2,182,861$ M., is to be debited to the line outside stations.

The cost of rail-renewals only comes into account in determining the cost of traction or motive-power, and consequently does not appear here.

Sleeper-renewals cost 13,447,813 M.; and the total length of the track between stations plus the portion of it lying within station limits was to the length of sidings as 3 to 1. If we assume that the cost of maintenance of the sleepers in sidings was only half as much as that on the main line, we obtain as the outlay on sleepers in the line between stations as $\frac{6}{7} \times 13,447,813 = 11,526,697$ M.

To the items thus obtained, viz., $22,831,650 + 2,182,861 + 1,152,697 = 36,541,208$ M. there is to be added the disbursements for the maintenance of the line between stations specified in § 13*a* of the above cited Statistical Return, viz. 23,538,424 M., whence we obtain a total of 60,089,632 M.

Since in the year under notice (1885-86) there were 21,089 km., of track operated, the expenses for track-maintenance and supervision amount to an average of 2,850 M. per km.

The cost of General Administration was 11·6% of the other disbursements on the working of the traffic; so that adding this percentage, the cost of line maintenance and supervision is increased to 3,180 M. per km.†

In the following investigations the road expenses will be represented by U, the kilometric construction-cost of the section or length of line under consideration by A, and the rate of interest by i; so that the kilometric track-expenses per km. is

$$A\,i + U.$$

[† This was written in 1888. Tr.]

* Baumeister: "*Organ*": 1880, "Summarische Veranschlagung der Betriebskosten von Adhäsionseisenbahnen," puts these expenses for the Prussian Railway System, 1874, at 3,016 M.

TABLE I.

Analysis of the Cost of the Transport Department.

According to the Report of the Working of the Prussian State Railways the outlay in the traffic year 1885-86 was			Of which are here omitted from consideration the following items	The remaining expenses are distributed in the columns A. B. and C. in the proportion of			Despatching and Station service. A.	Train-Expenses. B.	Traction-Expenses. C.
Head.	Sub-head.	For							
			M.		M.		M.	M.	M.
1 and 2	—	Salaries of superior officials, of the Accounts and Office staffs ...	...	34	24	42	690,368	487,319	852.807
3	---	Station Staff	...	100	0	0	24,974,124	...	...
4	1, 2 and 3	Despatching staff	...	100	0	0	4,402,370	...	...
4	4 and 5	Drivers and Stokers	..	0	0	100	...	...	13,087,697
4	6 and 7	Guards and Baggage Clerks ...	...	0	0	100	...	...	3,635,439
4	8	Conductors	...	0	100	0	...	2,375,421	...
4	9	Brakemen	...	0	0	100	...	...	2,820,167
4	10 — 14	Workshop Superintendents and Foremen, etc.	...	0	0	60	...	285;497	428,245
4	15 — 21	Ships' crews	72,336		...		...	...	...
5	—	House allowances	...	46	16	33	3,318,972	1,154,425	2,741,760
6	1	Food allowances	...	88	5	7	7,922,972	450,169	630,236
6	2	Substitute expenses	...	76	4	20	790,537	41,607	208,036
6	3	Local and Famine contribution ...	..	75	0	25	68,496	...	22,802
6	4	Daily wages and contract-work ...	...	63	13	24	24,711,730	5,099,246	9,413,993
6	5	Pensions of former officials... ...	180,241		...		...	...	...
7	1	Travelling and similar allowances...	...	71	12	17	763,645	129,067	182,844
7	2	Extra mileage, overtime, and night-allowances	...	0	74	26	...	8,869,519	3,116,318
7	3	Premiums for coal and other economies...	...	0	0	100	...	...	2,969,611
7	4	Uniforms	...	12	57	31	133,881	635,934	345,858
7	5	Cashiers' Deposits	...	100	0	0	70,078	...	...
	6	Private telegraph business	46,607		...		..	...	...
8	—	Rewards	...	66	16	18	734,393	178,035	2[illegible]2,202
9	—	Charitable institutions	...	67	23	10	69,200	23,783	10,340
10	1	Office expenses	...	90	7	3	1,981,908	154,143	66,063
10	2	Heating offices	...	89	1	10	5,377,590	60,422	604,223
10	3	Maintenance of Records and Plant.	...	70	5	25	944,021	67,430	337,150
10	4 to 8	Rent of land...	...		...		62,707	...	...
12	—	Compensations	...		...		338,139	...	...
14	1 and 2	Firing and watering engines ...	...		...		...	...	19,003,768
14	3 and 4	Lubricating and cleaning engines ..	...		...		...	...	2,411,184
14	5	Lubricating vehicles	...		...		...	476,030	...
14	6	Lighting of trains	...		...		...	994,102	...
14	7	Heating of trains	...		...		...	855,707	...
14	8	Heating, illumination, etc., of steamboats and ferries	42,951		...		...	...	...
14	9	Shunting with horses	...		...		691,997	...	...
15	1	Maintenance of locomotives ...	...		...		...	...	22,408,247
	2	" passenger cars ...	...		...		...	6,403,049	...
	3	" luggage and goods waggons ...	..		...		...	16,402,066	...
	4	" tarpaulins ...	...		...		...	375,089	...
15	5	" auxiliary establishments ...	91,147		...		...	..	...
	6	Maintenance and renewals of instruments	...		...		...	..	233,518
17	1	Renewals of locomotives	...		...		...	..	6,821,337
17	2	" " passenger vehicles ...	...		...		...	2,737,019	...
	3	" " luggage and goods waggons	...		...		...	4,464,397	...
18	—	For the use of foreign lines and services of officials	1,620,354		...		...	...	...
19	—	For the use of foreign rolling-stock..	5,570,100		...		...	...	...
		Totals...	7,623,736		...		78,047,208	52,699,081	92,643,955

§ 4.

Running- or Train-Expenses.

In the Report of the Results of the Working of the Prussian State Railways the outlay for the reception and distribution of goods (station and terminal expenses), the transport- or train-expenses, and the cost of the motive-power are not shown separately but are all lumped together in the expenditure of the Transport Department. In the summaries of the several departments these transport-expenses are separated into 4 groups, viz. (*a*) for outside station service (*b*) station and terminal service; (*c*) train service; (*d*) locomotive or traction expenses.

In Table I the cost of the Transport Department of the Prussian Railway System for 1885–86 is distributed in 3 groups: (*a*) Station and terminal service, (*b*) Train or running service, and (*c*) Locomotive or Traction service, in the proportion they had during the same period in the Hannover District. In so doing no notice has been taken of the outlay on the staff employed on ferries, nor for the pensions of former officials, outlay for private telegrams, the expenses for illuminating steam-boats and ferries, nor for the maintenance of the latter. Also, the payments for use of foreign lines, stock, and staff, have been excluded because these outlays are approximately covered by the revenue obtained from the payments by foreign lines for the use of the Prussian lines, rolling-stock, and service of their officials.

Of the items of outlay incident to the **train- or running-service** amounting in all to 52,690,081 M. shown in Table I the following items are to be charged to the **goods-traffic** and for the accompanying reasons:—

(1) Lubricating vehicles—In passenger service 1,510 million axle-kms., and in goods-service 5,087 million axle-kms. were run; and of this in the case of the axles of mixed-trains $\frac{2}{3}$rds are reckoned goods-train axles, and $\frac{1}{3}$rd as passenger-train axles. Consequently three-fourths of the cost of lubrication of vehicles or 3/4 × 476,030 M. = 357,023 M. is to be allocated to goods-traffic.

(2) Lighting of trains—$\frac{1}{5}$th of the total amount (198,820 M.), is, as an approximate estimate, to be put down to goods-trains; and after deducting 15 % for halts at stations there remains for the line between stations 169,000 M.

(3) Maintenance and renewals of goods-vehicles—$\frac{1}{28}$th of the whole amount for goods and luggage-vans is to be deducted, for the luggage-vans running in passenger-trains, thus leaving for the goods-traffic 20,121,810 M.

(4) For maintenance of the truck tarpaulins, 375,089 M.

(5) Of the remaining disbursements on the train or running service the wages of guards, cost of heating of vehicles, maintenance and renewals of vehicles incident to the passenger-traffic, are to be deducted; leaving 17,636,301 M. This sum is to be divided between goods and passenger service in proportion to the number of the trains, viz. one-half each. Of the 8,818,157 M. under notice 15 % is to be deducted for halts of trains in stations, and accordingly there remains 7,495,428 M.

(6) To the above items amounting to 28,518,350 M. there is to be further added 11·6 % for General Expenses, *i.e.*, 3,308,129 M.

(7) The capital cost of the goods-waggons amounted to 481,322,767 M. the annual interest charge on which at 4 %, is 19,252,911 M.

(8) The capital cost of luggage-vans amounted to 21,803,957 M.; and charging one-half of this to the goods-traffic there is an item for interest thereon of 436,079 M. to be added.

From the interest on the capital-cost of goods- and luggage-vans determined under 7 and 8 is to be deducted not only the part incident to the schedule-halts of the trains at stations but also a considerable sum for the loss of time in loading and unloading, in the making-up of trains, and in the nightly stabling of the same; so that for the running service of the section only $\frac{1}{3}$rd, say, of that interest is to be taken into account, viz. 6,562,997 M.

The train-service in goods-traffic accordingly cost annually 38,389,476 M. distributed over 10,866 million of paying tonne-kms., and consequently amounts to ·35 pfg. per paying tonne-km.

Since the vehicle-weight per goods-train axle is taken at 2·85 tonnes, 5,087 million goods-traffic axle-km. give 14,498 tonne-kms. done by the weight of vehicle. Thus for each tonne of paying-load there was 1·35 tonnes, or roughly 1½ tonnes, of vehicle carried, and 2½ tonnes gross-load. The train-service expenses per tonne-km. gross-load consequently amount to ·15 pfg., while per axle-km. of goods-train they are ·75 pfg.

For the **passenger-traffic** we have the following items for **train-running expenses** from Table I:

(1) The salaries of guards, viz. 2,375,421 M. of which 10 % is to be deducted for the halts of the trains at stations, thus leaving a balance of 2,137,879 M.

(2) For lubricating vehicles, $\frac{1}{4}$th of the whole amount (compare No. 1 of the outlay on train-expenses of goods-traffic), viz. 119,008 M.

(3) For train lighting $\frac{4}{5}$ths of the whole amount (compare No. 2 of the goods train-expenses), viz. 795,282 M. from which 10 % is to be deducted to allow for the halts at stations, leaving 715,754 M.

(4) The cost of heating vehicles, 855,707 M, which when reduced by 10 % for station-halts becomes 770,137 M.

(5) Outlay for maintenance and renewals of vehicles—9,110,068 M.

(6). The $\frac{1}{28}$th part (compare No. 3 of goods-traffic) of the expenses for maintenance and renewals of goods and baggage-waggons, allowing for the luggage-vans atttached to the passengers-trains—745,253 M.

(7). The $\frac{1}{2}$ of the other items of outlay (compare item No. 5 goods-traffic)—8,818,150 M., of which after deducting 10 % for station-halts there remains 7,936,335 M.

(8). To the total amount of these items—21,534,434 M.—there is to be added 11·6 % for general expenses, viz. 2,497,994 M.

(9). The first cost of the passenger-vehicles amounting to 107,323,532 M, which at 4 % gives an annual charge for interest of 4,292,941 M.

(10). The first cost of luggage-vans amounted to 21,803,957 M., and half the amount of the interest thereon is to be allocated to the passenger-traffic—namely, 436,079 M.

Of the interest on first cost detailed under (9) and (10), only $\frac{2}{3}$rds say, viz. 3,152,680 M., as the allowance for station-halts is to be allocated to the train-service of the section.

The train-service in passenger-traffic required consequently an annual outlay of 27,185,108 M., which distributed over 5,033 million passenger-km. gives ·54 pfg. as the cost per passenger-km.

There was 1,570 million axle-kms. performed in passenger-traffic: and since we may assume an average load of 4·4 tonnes per axle of passenger-trains there was done 6,908 million tonne-kms. by passenger-trains, so that per passenger there was hauled a load of 1⅓ tonnes of passenger-train.

The cost of the tonne-km. of passenger-train service works out therefore to ·405 pfg. and cost per axle-km. of passenger-train to 1·75 pfg.

For the train-service of mixed-trains the cost per tonne-km. of train-load may be assumed at ·25 pfg.; and per axle-km. about 1·1 pfg.

In what follows the expenses of train-service per tonne-km. of the gross load of goods-trains will be indicated by f, that per tonne-km. of passenger-trains by f_1, and the ratio of gross-load to paying-load by b.

§ 5.

Cost of Traction.

In the Traction-expenses, depending on the grades of the line, are to be included naturally the *Brake-Staff wages* likewise so dependent.

From Table I the outlay for brakesmen amounted to 2,820,167 M. which by the addition of 11·6% for General expenses is raised to 3,147,306 M. Of this, after deducting 15% for train-halts at stations, there remains 2,675,210 M. to be added as the road-expenses of the section. But this is the outlay on brake-staff only; the actual outlay in interest on the prime cost of the brake installation and its maintenance, for brakes, and for wear of tires is included in the train-expenses already determined, and the cost of the rail-wear due to brakes is included in the outlay for rail-renewals. These material expenses of braking as distinct from the staff expenses ought, strictly speaking, to be separately determined and their dependence on the grades of the line ascertained.

According to the "Technische Vereinbarungen" for goods-trains, on gradients

of $\frac{1}{500}$	there is should	$\frac{1}{12}$th	of the axles	braked.
" $\frac{1}{300}$	"	$\frac{1}{10}$th	"	"
" $\frac{1}{200}$	"	$\frac{1}{8}$th	"	"
" $\frac{1}{100}$	"	$\frac{1}{7}$th	"	"
" $\frac{1}{60}$	"	$\frac{1}{5}$th	"	"
" $\frac{1}{40}$	"	$\frac{1}{4}$th	"	"

The total number of axles being N and the stiffest gradient s the number of axles to be braked is approximately

$$n = (s + \cdot 01)\ 7\ N$$

and the cost of the braking may be assumed as increasing in proportion to the number of axles braked. As the stiffest gradient, s, on any section of the Prussian State Railway System is about ·0055, $\left(=\frac{1}{182}\right)$ only a little over ⅓rd of the number of brakes employed are due to the gradients of the line; consequently some ⅔rds of the cost of brakes would, as being independent of the gradients of the line, have to be included in the train running-expenses. Now as the actual amount of the material, *i.e.* non-personal, items of the cost of braking cannot be determined from the statistical data available, and as these have already been included in the train-expenses, it may be assumed for the purpose of allocation that the whole amount of brake-staff wages still remaining to be charged is proportional to the stiffest gradient on the section. There will be no serious error made in the final result of the calculation if the brake-expenses are charged entirely to the goods-traffic. The brake-staff cost is accordingly to be put equal to $e\, s\, T$, where e is a coefficient, s the average value of the severest gradient on any section = ·0055, and T the number of tonne-kms. gross-load of the goods-trains, *i.e.* 26,354 million tonne-kms. The coeff. e is accordingly about 2 pfg.

After deducting the brake-staff expenses there remains over from Table 89,823,788 M. for the traction-service, the individual items of which in determining the cost of a Loco.-km are to be distributed as shewn in the following Table.

TABLE II.

Analysis of the Cost of the Locomotive-km.

Item of expense.	Amount per Locomotive-km. Pass. Train.	Amount per Locomotive-km. Goods Train.	Station Expenses.
	pfg.	pfg.	M.
1. For Guards and Baggage clerks the outlay was 3,635,439 M. which is to be distributed proportionately to the number of train-km. on the assumption that goods-trains cost 1½ times as much as passenger-trains. The number of passenger and express-train-km.			
was ... ... 71,853,306 M.			
add 1½ times for goods- and mixed-trains ... ... 121,326,856 „			
Total ... 193,180,162 „			
∴ expense per passenger-km. = 1·88 pfg.			
and per goods train-km, 1½ × 1·88 = 2·92 pfg.			
From this is to be deducted 10%, for halts at stations of passenger-trains, and of goods-trains 15%. and the amounts added to the Station expenses—			
Thus ·1 × ·0188 × 71853306 =	1·C9	..	135,084
and ·15 × ·0282 × 80884571 =	...	2·40	342,142
2. For Engine-drivers and Stokers, there was, including mileage and overtime allowances, expended 16,204,015 M. which is to be divided by the number of locomotive-km. performed. But, it has to be borne in mind that the number of kiloms. done by passenger-trains, owing to the smaller expenditure of time, costs only ⅔rds as much as the other trains. Consequently, we have for the distribution,			
⅔ × passenger-and express-trains ... ... 47,902,204 M.			
Goods-trains, including yard-work ... ... 154,986,007 „			
Total ... 202,888,211 „			
Consequently, we obtain for the cost of the locomotive-km. in goods-trains or in yard-work, 7·99 pfg. and for the engine-km. for passenger-trains, 5·33 pfg. From this is to be deducted for halts at stations 10% and 15% as before, and the amounts added to the Station expenses. Accordingly,			
for passenger-trains ·1 × 0533 × 71853306	4·80	...	382,971
and for goods „ ·15 × ·0799 × 80884371	...	6·79	969,399
Yard-expenses (shunting, sorting, etc.), ·0799 × 41680255	...	...	3,330,252
3. Maintenance and renewals of locomotives amounted to a total of 29,319,584 M.			
of which ⅔rds, viz., ... 15,546,389 „			
may be taken an independent of the performance of the engines. Since according to statistical data the maintenance of engines amounted to 257632533 loco-kms.			
therefore per loco-km. we have...	7·C0	7·60	...
and of this for yard-work, ·076 × 83089700	...	...	6,314,826
4. Other outlays for Staff and materials independent of the work done by the locomotive amounted to:			
Salaries of the superior officials ... ... 852,807 M.			
Workshop-managers, etc. ... ... 498,245 „			
House-rent allowances ... ... 2,741,760 „			
Food allowances ... ... 630,236 „			
Substitute salaries ... ... 208,0'6 „			
Local and other extra-allowances ... ... 22,832 „			
Extra daily allowances and travelling expenses ... ... 182,844 „			
Uniforms ... ... 315,858 „			
Premiums ... ... 202,282 „			
Assistance, charitable ... ... 10,340 „			
Office contingencies ... ... 66.063 „			
Heating offices ... ... 604,223 „			
Maintenance of records ... ... 331,150 „			
Lubrication and cleaning of locomotives ... ... 2,411,184 „			
General, for tools, etc. ... ... 233,518 „			
Total ... 9,271,378 „			
This divided over 226839313 Engine-kms. gives per km.	4·09	4·09	...
and for yard-service, 0409 × 41680235	...	...	1,704,361
5. To this is to be added the share of the General Administration charges @ 11·6%, viz.	2·11	2·42	1,528,768
Carried over ...			27,451,955

TABLE II.

Analysis of the Cost of the Locomotive-km.—continued.

Item of expense		Amount per Locomotive-km. Pass. Trains	Amount per Locomotive-km. Goods Trains	Station Expenses
	Brought over ...	pfg.	pfg.	M. 2,[illegible]
The capital Outlay in locomotive-stock amounted to ...	[illegible] M.			
add [illegible] ...	[illegible]			
...	[illegible] „			
The interest on the above at [illegible] is ...	[illegible] „			
Since goods-train locomotives run annually on an average only [illegible] of the distance annually done by passenger-trains and yard-locomotives, therefore the amount of interest on the former is [illegible] times greater per km. than on the latter.				
Since the number of goods-train km. form some [illegible] of the whole number of loco-kms., then before making the division the number of Engine-km. is to be increased by [illegible], i.e. raised to				
1 [illegible] × 229[illegible]13 = 245[illegible]				
The distribution is then for passenger loco-km. — — — — —		7·[illegible]	—	—
... ...for goods do. do. — — — — —		—	9·75	—
and for yard-service, [illegible] × 41[illegible] — — — — — — —		—	—	2,[illegible]
7. Expenses to be assumed as proportional to the work done by the locomotive are—				
Renewal of rails	14,[illegible]1 M.			
Track-labour	9,[illegible] „			
Premiums for Savings	2,9[illegible]1 „			
Locomotive-firing and water	15,[illegible]75[illegible] „			
Add ⅓rd of the outlay, for loco. maintenance and renewals ...	9,773,[illegible] „			
	50,7[illegible] „			
Add the share of the expenses of the General Administration @ 11·6%	6,[illegible],175 „			
Total ...	6[illegible],701,[illegible]5 „			
This gives for 229[illegible]12 loco-km. 29·4 pfg. per loco-km.				
From the above is to be deducted for the stopping and starting of trains, say, 3%, so that we obtain		28·52	28·52	1,[illegible],008
and for yard-service ·294 × 41[illegible]23[illegible]				12,754,152
Total ...		—	—	[illegible]2,002,573

If the expenses which vary solely with the distance travelled and which are wholly independent of the power of the locomotive are B_0, per km. and the cost of the performance of a tonne-km. be a, then the cost of a locomotive-km. B for the exertion of a tractive-force of z tonnes, is

$$B = B_0 + a Z.$$

The average tractive-power of locomotives on the Prussian State Railways in 1885-86 may be taken as 1·15 tonnes.* This figure agrees very well with the coal-consumption which, excluding the wood required to light the locomotive, amounted to 2,191,266 tonnes. After deducting 3% for halts at stations and for the getting-up of steam there is a balance of 2,125,528 tonnes. According to **Wöhler**** a kg. of coal produces 7·06 kg.

* The average tractive-power of locomotives for the Prussian State Railways in 1874 was estimated by me in my work—"Die Betriebskosten der Eisenbahnen in ihrer Abhängigkeit von den Neigungs—und Krümmungsverhältnissen der Bahn": Leipzig: Engelmann, 1877—somewhat too liberally at 1·37 tonnes.

Schübler—"Ueber Selbstkosten und Tarifbildung der deutschen Eisenbahnen": Stuttgart: Paul Neff. 1879—fixes this value at 1·25 tonnes.

Baumeister—"Summarische Veranschlagung der Betriebskosten der Adhäsionsbahnen": *Organ für die Fortschritte des Eisenbahnwesens*—determines the average tractive-power of locomotives on the Prussian State Railways, 1877, as 1·18 tonnes.

** See *Centralblatt der Bauverwaltung*: 1882. No. 40.

of steam; and according to Prof. Frank* 1 kg. of water produces in a locomotive 17,254 m-kg. of work: so that 1 kg. of coal produced

$$7 \cdot 06 \times 17{,}254 = 121{,}813 \text{ m-kgs}$$

or 1 tonne of coal was consumed in the performance of 121,813 tonne-kms.

The 2,125,528 tonnes of coal consumed in locomotives gave consequently a performance of 259,116,940 tonne-km.; and thus for 226,839,813 loco-kiloms., a mean tractive-force of 1·14 tonnes. The tractive-force of course is that requisite to haul the locomotive itself and to overcome the frictional resistances of its machinery.

From the mean tractive-force of 1·15 tonnes and from the cost 28·52 pfg. of the work done per locomotive-km. given in Table II, it follows that the production of a tractive-force of one tonne cost 25 pfg. per locomotive-km.

Consequently, the cost of a locomotive-km. for a tractive-force of Z tonnes is, for passenger-trains,

$$B = 27 + 25\ Z$$

and for goods-trains,

$$B = 32 + 25\ Z.$$

[* ("Widerstände der Locomotiven und Bahnzüge": 1886, p. 64.) For an English version of this work, see a pamphlet by the present translator: On the Resistance of Locomotives and Trains......determined experimentally. Higginbotham: Madras.]

§ 6.

Station and Terminal Expenses.

The Station and Terminal, *i.e.*, collection and delivery, expenses have no connection with the problems of location and therefore do not call for any detailed investigation.

They amounted according to Table I to:	78,047,208 M.
plus.—the share of the expenses of General Administration at 11·6%	9,048,476 „
plus.—the amount deducted from the Train-expenses—in § 4—for the train-halts at stations	17,645,228 „
plus.—the amount deducted in § 5 for the Traction-expenses ...	32,002,573 „
Total ...	136,738,485 M.

From the statistical data available it is scarcely possible to distribute these expenses accurately between the goods- and passenger-traffic. **Baumeister** estimated the Station and Terminal expenses at 15·28 pf. per person, and at 69·98 pf. per goods-tonne; but in this are not included the shunting and marshalling expenses which he treated and charged as road-expenses. In the year 1885-86 on the Prussian State Railways, there were moved 161,812,362 persons and 80,406,992 goods-tonnes and on this basis the Station and Terminal expenses would amount to 85 million M.—almost exactly the total of the Terminal expenses given in Table I, with the addition of 11·6% for the General Administration expenses.

Accepting the unit-figures determined by Baumeister, $\frac{2}{7}$ths of the total amount of the Station and Terminal expenses are incident on the passenger-traffic, and $\frac{5}{7}$ths on the goods-traffic.

If the Station and Terminal expenses given in Table I are distributed in this proportion, adding the corresponding share of the expenses of General Administration, and also the amount deducted from the train-expenses in § 4 for the halts at stations, and if the Station expenses given in Table II (arising mainly from the shunting-service and the halts at stations, which is particularly the case with the goods-traffic), are divided in the proportion of 1 to 10, then of the total Station and Terminal expenses, 29,802,171 M. are incident to the passenger-traffic, and 105,936,314 M. to the goods-traffic. For the given volume of passengers and goods moved we obtain the Station and Terminal expenses per head as 18·4 pf. per person and 122·6 pf. per tonne-goods. In these figures the interest on capital cost and the maintenance-expenses of stations are not included.

But attention must be directed to the fact that a distribution of Station and Terminal expenses made in this way per person and per goods-tonne cannot lay claim to any high degree of exactitude.

§ 7.

Train-Resistance.*

The well-known experiments of **Veuillemin, Dieudonné, and Guebhard** on the resistance of railway trains carried out in 1868 on the Eastern Railway of France are well known and have frequently been made use of in theoretical discussions. The formulæ representing the results of these experiments involve in an unsatisfactory and misleading manner the first power of the train-velocity multiplied by a coefficient which increases with the velocity. It is now generally admitted that the influence of the velocity V in train-resistance can only enter in as the second power; and accordingly the results of experiments on the resistance of trains have almost universally—especially by German engineers—been represented by an expression of the form,

$$w = a + b\,v^2$$

In certain earlier investigations I deduced the well-known expression,

$$w = \cdot 0015 + \cdot 00003\,v^2$$

v being in m/sec.

Baumeister arrived at almost exactly the same expression: he obtained

$$w = \cdot 0018 + \cdot 0000023\,v^2$$

v being in km/sec— or

$$w = \cdot 0018 + \cdot 00003\,v^2$$

v being in m/sec.

Subsequently, I became convinced from the results of the experiments of the above French engineers— which give only the resistance of the vehicles exclusive of the locomotive —that the influence of velocity had been too highly estimated, and I concluded that it was nearer the truth to put for the whole train, including the locomotive,

$$w = \cdot 002 + \cdot 00002\,v^2$$

The experiments made by **Frank** ** support the opinion that in these French investigations too much weight had been given to the influence of velocity.

Frank found for the resistance of a passenger-train on a level-straight—putting

L = the weight of the locomotive and tender:
Q = the weight of the train:
$w_1\ w_2$ = the coefficients of resistance of locomotive and vehicles, respectively:
F_1 = the frontage of the locomotive exposed to the atmosphere in q. m($\square^m$):
F_2 = the frontage of the remaining part of the train in $\square^m$:
v = the velocity of train in metres/second:

that

$$W = 1\cdot 033\left[w_1\,L + w_2\,Q + \cdot 1225\,(F_1 + F_2)v^2 + 31\right]$$

in which L, Q, and W, are in kgs.

The frontage or air-resisting area of the locomotive F_1 may be taken as $= 7\square^m$, and F_2 for luggage-vans $= 1\cdot 7,\square^m$ and for every succeeding one, $= 0\cdot 5\square^m$.

If n is the number of vehicles, including the luggage-van, then

$$F_1 + F_2 = 8\cdot 2 + \cdot 5\,n.$$

Since the weight of a vehicle in a passenger-train is on an average 8800 kgs., then $n = \dfrac{Q}{8800}$; and therefore

$$F_1 + F_2 = 8\cdot 2 + \cdot 0000568\,Q.$$

[* Vide Appendix B.—Tr.]

[** See: "On the Resistance of Trains......determined experimentally." Higginbotham, Madras.]

The coefficients of resistance were found to be

$$w_1 = \cdot 0032, \text{ and } w_2 = \cdot 0025.$$

Consequently, inserting these values

$$W = \cdot 003306\, L + \cdot 00258\, Q + (1 \cdot 038 + \cdot 0000072\, Q)\, v^2 + 31.$$

For the locomotive alone the resistance-coefficient is consequently

$$w = \cdot 003306 + \frac{31}{L} + \frac{1 \cdot 038}{L} v^2 :$$

or, for the average weight of a passenger-locomotive of $L = 54000$ kg.

$$w_1 = \cdot 00388 + \cdot 0000192\, v^2,$$

whereas the resistance-coefficient for the train of vehicles alone is

$$w_2 = \cdot 00258 + 0000072\, v^2.$$

If the vehicles are m times the weight of the locomotive, then the average resistance-coefficient for a passenger-train is

$$w = \cdot 00258 + \frac{\cdot 0013}{1+m} + \left(\cdot 0000072 + \frac{\cdot 000012}{1+m}\right) v^2$$

For passenger-vehicles m may usually be taken as $= 2$, so that the resistance-coefficient of passenger-vehicles becomes

$$w = \cdot 00301 + \cdot 0000112\, v^2$$

For express-trains, on the other hand, $m = \frac{3}{2}$ thus

$$w = \cdot 00310 + \cdot 0000120\, v^2$$

For goods-trains Frank gives the expression

$$W = 1 \cdot 04 \left[w_1 L + w_2 Q + \cdot 1225\, (F_1 + F_2)\, v^2 + 50 \right]$$

in which $w_1 = \cdot 0038$, $w_2 = \cdot 0025$, and F_1, the locomotive's fore-end area, $= 8$, and of the succeeding goods-vehicles, $1 \cdot 7$, and for the rest of the cars on an average, $= \cdot 6$: so that for n vehicles

$$F_1 + F_2 = 9 \cdot 1 + \cdot 6\, n$$

and since the average weight of a goods-vehicle may be taken at 9800 kg., then

$$F_1 + F_2 = 9 \cdot 1 + \cdot 0000612\, Q.$$

Consequently,

$$W = \cdot 003952\, L + \cdot 0026\, Q + (1 \cdot 149 + \cdot 00000078\, Q)\, v^2 + 52$$

and accordingly the resistance-coefficient for the locomotive alone, if the average weight of a goods-locomotive be taken as 60000 kg., is,

$$w_1 = \cdot 00482 + \cdot 0000192\, v^2$$

and for goods-vehicles,

$$w_2 = \cdot 0026 + \cdot 0000078\, v^2.$$

If the weight of the train be m times that of the locomotive, then the mean resistance-coefficient for the whole trains is

$$w = \cdot 0026 + \frac{\cdot 00222}{1+m} + \left(\cdot 0000078 + \frac{\cdot 0000114}{1+m}\right) v^2.$$

On an average for the Prussian Railway System $m = 6$ say, then for goods-trains

$$w = \cdot 00292 + \cdot 0000094\, v^2$$

Thus for $v = 7$, $w =$ **·00338.**

On lines having stiff gradients both m and v become smaller, for example—m on mountain railways for a grade of ·025 falls to $2\frac{1}{4}$, and v to $3\frac{1}{2}$; so that the resistance-coefficient $w =$ **·00337.** Thus **for goods-trains the resistance-coefficient for flat or undulating regions is the same as for mountainous districts,** which is a fact of importance for the simplification of our calculations.

Although the train-weight of Passenger-trains on mountain lines diminishes, it does not do so to such a degree that with the necessarily occurring reduction of velocity the resistance-coefficient has the same value as in level or undulating districts. But since, as will be subsequently shown, the passenger-traffic is of much less importance for the location of the trace than the goods-traffic, the resistance-coefficient for Passenger-trains may be taken without error as indentical in hilly ground and in flat or undulating country.

The resistance-coefficient is thus on the level-straight for the average train-weights on the Prussian Railway System, as follows:

For Goods-trains,

$$w = \cdot 00292 + \cdot 0000094\, v^2$$

For Passenger-trains,

$$w = 00301 + 0000112\, v^2$$

For Express-trains,

$$w = \cdot 00310 + \cdot 0000120\, v^2$$

These formulæ—which for velocities of 7^m, 13^m and 18^m give resistance-coefficients of ·00338, ·00490, and ·00699 respectively—can be comprehended with sufficient accuracy in a uniform formula valid for all kinds of trains within the limits of the ordinary velocity of each; viz.,

$$w = \cdot 00273 + \cdot 0000131\, v^2.$$

This expression gives for the above velocities of 7^m, 13^m and 18^m the values of $w = \cdot 00337$, ·00494, ·00697 respectively.

But since the velocity of trains is not uniform the average resistance is greater than that corresponding to the arithmetic mean velocity. For example, if on a journey of length l the velocity on $\frac{l}{16}$ is $\frac{v}{2}$—as actually is the case more or less at the starting and stopping of the trains,—and $\frac{3v}{4}$ on $\frac{l}{16}$, and finally $\frac{9v}{7}$ on $\frac{3}{8}\,l$, then the value of the mean velocity v is given by the Eqn.

$$\frac{\frac{l}{16}}{\frac{v}{2}} + \frac{\frac{l}{16}}{\frac{3v}{4}} + \frac{\frac{l}{2}}{v} + \frac{\frac{3l}{8}}{\frac{9v}{7}} = \frac{l}{v}$$

and the train-resistance in the several sections depending on the velocity would be

$$\cdot 0000131 \left[\frac{1}{16}\left(\frac{v}{2}\right)^2 + \frac{1}{16}\left(\frac{3v}{4}\right)^2 + \frac{v^2}{2} + \frac{3}{8}\left(\frac{9\,v}{7}\right)^2\right]$$
$$= \cdot 0000131 \times 1{\cdot}218\, v^2$$
$$= \cdot 000016\, v^2$$

The discrepancies from the mean velocity, which have been assumed by way of example in the above calculation, arise in a greater or lesser degree from the starting and stopping of the train and from the influence of the gradients; and everything considered, they warrant our putting the coefficient with which the square of the mean velocity v is to be multiplied in most cases at ·000016.

In addition the wind produces an increase of resistance. For a wind velocity v_1, that part of the resistance-coefficient dependent on the velocity is increased in one direction to $\cdot 0000131\ (v + v_1)^2$, and diminished in the other by $0000131\ (v - v_1)^2$; or on an average by $0000131\ (v^2 + v_1{}^2)$. The resistance-coefficient thus becomes increased by $\cdot 0000131\ v_1{}^2$. If also the wind blows against the train perpendicularly with a velocity v_1 then it produces a pressure on the side surfaces per q.m. of $\cdot 1225\ v_1{}^2$ which, for a coefficient friction of $\frac{1}{6}$, gives a resistance of $\cdot 0204\ v_1{}^2$.

Assuming that 2 q.m. of train-surface are equivalent to a tonne of train-weight, the increase in the resistance-coefficient due to a side wind is thus $\cdot 0000408\ v_1{}^2$.

If it be assumed that ordinarily a wind of 1 m/sec. velocity blows both in the direction of the train and also right across it then the constant part of the resistance-coefficient increases by ·0000131 + ·0000408 = ·0000539, or roughly, to ·0028.

Consequently, from the above, the resistance-coefficient of trains is

$$w = \cdot 0028 + \cdot 000016\, v^2 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

And therefore the resistance-coefficient for Goods-trains of which the mean velocity is 7^m., is to be assumed as

$$w = \cdot 0036.$$

and for Passenger-trains at a mean velocity of 13^m,

$$w = \cdot 0055;$$

and for Express-trains at a mean velocity of 18^m,

$$w = \cdot 008.$$

The mean velocity is to be determined excluding all halts at stations, *i.e.*, for the journey from station to station.

The increase of train-resistance in curves. As is well known very many experiments and theoretical investigations are extant without up to the present providing any generally recognised valid results. So many variables enter into curve-resistance that in general it is impossible to take account of them all.

The most suitable and satisfactory expression for the curve-resistance coefficient is given by the very simple formula of **von Kaven** and **Baumeister**, viz.,

$$c = \frac{1}{R}$$

where R is the curve-radius in metres*

If the central-angle of a curve be α° and R the radius in metres; then λ the length of the curve in metres is

$$\lambda = \cdot 0000175\, \alpha\, R.$$

Consequently, the work to be done in overcoming curve-resistance in the haul of a tonne is—in tonne-km.

$$c\lambda = \frac{1}{R}\cdot 0000175\, \alpha\, R$$

$$= \cdot 0000175\, \alpha$$

or approximately, $$= \cdot 000018\, \alpha$$

Thus for location purposes to determine the power required to overcome curve-resistance it is only necessary to know the sum of the central-angles of all the curves.

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(1) In previous investigations, instead of the above, I gave $c = \frac{1\cdot 7}{R} - \cdot 002$, which is fairly satisfactory for values of R between 250^m and 600^m, but for smaller and larger values this formula is less in agreement with experiment than the above simpler one.

§ 8.

The Tractive-Power of the Locomotive.

The tractive-force that the locomotive is capable of exerting is limited by the amount of the sliding friction or "adhesion" of the driving-wheels on the rails, or by the steam-producing power of the boiler, or by the dimensions of its machinery.

Let L = the weight of the locomotive and tender ; *
αL = the weight on the driving-wheels ;
β = the coefficient of (sliding) friction between wheel and rail ;
then the locomotive's maximum tractive-force is

$$Z = \alpha \beta L.$$

If the steam-producing power of the boiler be equivalent to N horse-power, when

$$N = n L$$

and if the velocity to be attained is v m/sec., then the maximum-tractive force exertable is

$$Z = \frac{75}{v} n L.$$

If the piston-area be $\frac{\pi d^2}{4}$, the steam-pressure p, the degree of cut-off be a, and the length of the piston-stroke h, then for one revolution the work done in both cylinders is

$$a p h d^2 \pi$$

during which the locomotive, for a driving-wheel diam. D, has travelled a distance πD: whence the tractive-force of the locomotive is

$$Z = \frac{a p h d^2}{D}$$

For example: if d = 55 cm., h = 60 cm., D = 120 cm., p = 10kg/q. cm. then $Z = a$ 15125 kg.

As a matter of fact, as these data chosen simply as examples clearly shew, it is very rarely that the tractive-force is ever practically limited by the dimensions of the locomotive.

The cofficient α may vary between 1 and ·2 according as all the wheels of the locomotive are coupled, or whether there is only one driving-axle. And since the coefficient β in flat ground may be taken as $\frac{1}{6}$ to $\frac{1}{7}$, in hilly land and mountains as $\frac{1}{7}$, in tunnels as $\frac{1}{8}$, or in unfavourable circumstances only at $\frac{1}{10}$, the frictional resistance will limit the tractive-force to between $1 \times \frac{1}{6}$ and $\frac{1}{5} \times \frac{1}{10}$, namely to something between ·167 L and ·02 L.

The number of Horse-Power—the weight of the locomotive being given in kgs.—may be assumed as ·006 L, *i.e.* $n L$. The production of steam will therefore cause the tractive-force to rise to ·15 L for a velocity of 3 m/sec., but will cause it to fall to ·02 L for a velocity of 22·5 m/sec.

If z being the **tractive-force coefficient** we put,

$$Z = z L,$$

then under the influence of the production of steam and the frictional-resistance of the driving-wheels on the rails, z may vary from ·02 to ·16, and in any given case the smaller of the values obtained under the above two conditions is the one to be used for the tractive-coefficients in calculations.

[* *i.e.*, a Tank engine—TR.]

§ 9.

Ruling-Gradient: "Injurious" and "Non-injurious" Gradients.

Let Q be the weight of all the vehicles forming a train,
L „ that of the locomotive and tender,
w „ the resistance-coefficient of the train,
s „ the gradient of the line;

then for the ascent the tractive-force required is

$$Z = (Q + L)\ (w + s)$$

To determine the weight of train which a locomotive having a tractive-coefficient z can haul up the incline, we have

$$z\,L = (Q + L)\ (w + s)$$

whence
$$Q = \frac{z - w - s}{w + s}\ L$$

This gradient, s, which serves to determine the weight of train haulable is termed **the ruling gradient.**

Usually, the steepest gradient in the line is the ruling gradient. But steep ascents when short may be disregarded in determining the weight of trains haulable up an incline when they can be 'rushed,' viz., mounted by making a call on the momentum of the train.

When curves—of which the resistance-coefficient is c—occur on the maximum grade the curve-resistance coefficient is to be added to the maximum gradient in order to obtain the ruling gradient.

Thus, for example, suppose there were straights on gradients of ·012 and curves of 300^m radius of which the resistance-coefficient is c = ·0033 on a grade of ·010. In that case, the ruling gradient would be the virtual gradient of (·010 + ·0033) = ·0133, and not that of ·012.

The **ruling gradient** is thus, leaving aside all ascents of exceptional occurrence which may be rushed (*i.e.*, Momentum Grades), fixed from those parts of the line in which **any grade plus the resistance due to any curve thereon is a maximum.**

On grades which are so flat that in descending them the engine's tractive-force must be employed, and which are therefore less than the train-resistance co-efficient w, the tractive-force required is

$$Z_2 = (Q + L)\ (w - s)$$

and that required to make the ascent, is

$$Z_1 = (Q + L)\ (w + s)$$

and the mean of this
$$Z = (Q + L)\ w,$$

viz. identical with that required on the level.

When the loads hauled in both directions are equal, ascents which are equal to or less than the resistance-coefficient w cost no more to work than the level.

Those ascents which are **less** than w are consequently termed "**non-injurious**": while those ascents which are **greater** than w are termed "**injurious**".

For goods-traffic, gradients up to $\cdot 0036\left(= \frac{1}{277}\right)$ are "non-injurious;" but for passenger-traffic they may rise to $\cdot 0055\left(= \frac{1}{181}\right)$· without becoming "injurious."

The assumption that the traffic is equal in both directions is approximately true for most railways: since the weight of the locomotive and vehicles which must, under any circumstances, be despatched in both directions form the larger half, or even ⅔rds, of the total load.

But if the traffic be greater in one direction than the other, then all ascending grades in the direction of the main traffic are "injurious"; while all grades falling in the same direction are not only "non-injurious," but are cheaper in working even than the level, so long as the descent does not exceed a certain quantity.

How to treat this case will be disscussed later on (§ 14): for the present we shall assume an equal traffic in both directions, which is most commonly the actual case. The limit value of "non-injurious" ascents is in general equal to the resistance-coefficient: it may, however, under circumstances, be greater or less than this.

In flat land lines* the ruling gradient will sometimes be less than the resistance-coefficient w; and thus this smaller value becomes the limit of the "non-injurious" ascents. In curves, on the other hand, the limit of the "non-injurious" ascents is increased.

Thus if c = the curve-resistance, then the tractive-power required for the ascent is

$$Z_1 = (Q + L)\ (w + s + c):$$

and for the descent, so long as brakes are not required,

$$Z_2 = (Q + L)\ (w - s + c).$$

The mean for both directions is thus

$$Z = (Q + L)\ (w + c),$$

precisely as on the level.

Consequently, the limits of "non-injurious" ascents when on curves exceed the the resistance-coefficient by the amount of the curve-resistance coefficient.

[* For precise definition, see § 33, p. 109—Tr.]

§ 10.

The Working-Expenses.

The Working-expenses are to be calculated excluding all outlay for collection and delivery of the traffic and, therefore, exclusive of the station and terminal expenses so that only **the cost of the actual moving** of merchandise is to be considered and this only in so far as it depends on the volume of traffic; consequently, the maintenance of way and supervision-expenses, being independent of this, are likewise excluded; and thus the only items that come into consideration are the train-or **running**- and **the traction-expenses.**

The cost of a locomotive-km. has been determined in § 5 as

$$B = B_0 + a Z.$$

Assuming that the locomotive exerts its full power so that the tractive-force is, according to § 8,

$$Z = z L$$

and indicating the cost of the engine-km. in this case by B_1 so that

$$B_1 = B_0 + a z L$$

then for a train-weight—excluding the engine and tender—of Q tonnes on a straight and level line the working-expenses per tonne-km. is given by

$$k_0 = f + \frac{B_1}{Q}$$

where f is the cost of transport per tonne-km. determined as per § 4. Substituting the value of B_1 we obtain

$$k_0 = f + \frac{B_0}{Q} + \frac{a z L}{Q}.$$

Equating train-resistance and locomotive tractive-force,

$$(Q + L)\, w = z L.$$

we obtain the weight of train hauled as

$$Q = \frac{z - w}{w} L$$

And substituting it in the above

$$k_0 = f + \frac{B_0}{L} \frac{w}{z - w} + \frac{a z w}{z - w} \qquad \dots \quad (2)$$

Of the terms on the right-hand side, the first is the train-running cost; the second, the locomotive-running cost; and the third, the cost of the work done by the locomotive.

If for goods-traffic we insert the values (§§ 4, 5, and 7), $f = \cdot 15$, $B_0 = 32$, $a = 25$, and $w = \cdot 0036$; and assume the weight of a goods-train as 60 tonnes and the tractive-coefficient z as $= \cdot 05$, then we have

$$k_0 = \cdot 15 + \cdot 041 + \cdot 097 = \cdot 288 \text{ pfg.}$$

Taking the load-coefficient b—*i.e.* the ratio of the gross-load, exclusive of locomotive and tender, to the paying load,—at $2\frac{1}{3}$—which corresponds to the actual conditions of working obtaining on the Prussian Railways in the year 1885–6—then the working-expenses on the level straight and line per paying tonne-km. amount to ·672 pfg.

For passenger-traffic, $B_0 = 27$, $a = 25$, $w = \cdot 0055$, and f per tonne passenger-trains $= \cdot 405$: so that for a passenger-locomotive weighing 54 tonnes and for a traction-coefficient of ·02 the working-expenses per tonne-km. for a passenger-train is

$$k_0 = \cdot 405 + \cdot 190 + \cdot 190 = \cdot 785 \text{ pfg.}$$

And assuming that $1\frac{1}{3}$ tonnes of train-load correspond to one passenger, the cost of the passenger-km. on a straight and level line is 1·047 pfg.

For a flat country line in which the ruling gradient is s and on which only "non-injurious" descents and curves of resistance c occur, the mean tractive-power z requisite is

$$(Q + L)(w + c).$$

The cost of an engine-km. is therefore

$$B_0 + a(Q + L)(w + c)$$

and the **working-expenses per tonne km.** on **non-injurious** grades are

$$k_1 = f + es + \frac{B_0 + a(Q + L)(w + c)}{Q}$$

or inserting the value

$$Q = \frac{z - w - s}{w + s} L$$

we obtain

$$k_1 = f + es + \frac{B_0}{L} \frac{w + s}{z - w - s} + \frac{a z w}{z - w - s} + \frac{a z c}{z - w - s} \quad \dots \quad (3)$$

In this equation the first term is the train-expenses; the second, the cost of the use of brakes; the third, the running-expenses of the locomotive; the fourth, the cost of the performance of the locomotive on the straight; and the fifth, the cost of the locomotive's extra exertion in curves.

If "injurious" grades s_1 occur in the line of which the ruling gradient is s and in which the curve-resistance is c, then the locomotive-cost for the ascent is

$$B_0 + a(Q + L)(w + s_1 + c)$$

whereas for the descent where the train is braked, and the engine runs down without exerting traction, it is B_0. Consequently, the average value for both directions is

$$B_0 + \frac{a}{2}(Q + L)(w + s_1 + c).$$

The **working-expenses per tonne-km.** on **injurious** grades are consequently

$$k_2 = f + es + \frac{B_0 + \frac{a}{2}(Q + L)(w + s_1 + c)}{Q}$$

And inserting the value $\qquad Q = \dfrac{w + s}{z - w - s} L$

we obtain

$$k_2 = f + es + \frac{B_0}{L} \cdot \frac{w + s}{z - w - s} + \frac{a z}{z - w - s} \cdot \frac{w + s_1}{2} + \frac{a z}{z - w - s} \cdot \frac{c}{2} \quad \dots \quad (4)$$

A comparison of this expression with that for the cost on "non-injurious" grades shows that **the increase of the working-expenses on "injurious" gradients due to curve-resistance is only half as great as on "non-injurious" grades.** This is evident also without calculation, seeing that "non-injurious" gradients require extra exertion on the part of the locomotive, whereas on "injurious" grades this is so only in the ascent.

§ 11.

Equivalent Gradient,

The working-expenses for the *whole line* may be easily determined from the formula just deduced for the working-expenses per tonne-km. on *various grades.*

The train-expenses, the expense due to the use of brakes, and the locomotive, running-expenses are the same for sections with "injurious" or with "non-injurious" grades both in straights and curves: they increase simply with the length of the line, and for a line l km. in length per tonne-km. are

$$K_0 = \left(f + e\,s + \frac{B_0\,(w+s)}{L\,(z-w-s)}\right) l$$

The cost of the work done by the locomotive on straights on "non-injurious" gradients is the same as on the level: and if l_0 is the length of all the parts of the line having "non-injurious" grades the cost is

$$K_1 = \frac{a\,z}{z-w-s}\,w\,l_0$$

The cost of the work of the engine on an "injurious" grade s_1 of length l_2 per tonne gross-load moved is

$$k_2 = \frac{a\,z}{z-w-s}\,\left(\frac{w\,l_2}{2} + \frac{s_1\,l_2}{2}\right)$$

Since $s_1\,l_2$ is the height surmounted on the grade s_1 in a length l_2—which may be represented by h_1—we may write

$$k_2 = \frac{a\,z}{z-w-s}\,\left(\frac{w\,l_2}{2} + \frac{h_1}{2}\right)$$

Now if l_1 is the sum of all lengths on "injurious" grades, and h the total height surmounted by them expressed in km., then the cost of the engine's performance per tonne- km. on all "injurious" grades is

$$K_2 = \frac{a\,z}{z-w-s}\,\left(\frac{w\,l_1}{2} + \frac{h}{2}\right)$$

The curve-resistance $c\,\lambda$ per tonne for a curve of length λ and a central-angle α is, according to § 7, $\cdot 000018\,\alpha$. On "non-injurious" grades the cost of the performance of the engine in running through a curve of resistance c and length λ is therefore

$$k_3 = \frac{a\,z}{z-w-s}\,c\,\lambda$$

or

$$k_3 = \frac{a\,z}{z-w-s}\,\cdot 000018\,\alpha$$

whereas the cost would be only half as great if the curve were on an "injurious" gradient. If then the sum of the central-angles of all the curves on "non-injurious" gradients $= \alpha_0$ and the sum of the central-angles of all curves on "injurious" grades $= \alpha_1$ then the cost of the work done by the locomotive due to all the curves in the line is

$$K_3 = \frac{a\,z}{z-w-s}\,\cdot 000018\left(\alpha_0 + \frac{\alpha_1}{2}\right)$$

We thus obtain the working cost per tonne gross-load moved on the line of length $l = l_0 + l_1$ on a ruling gradient s as

$$K = K_0 + K_1 + K_2 + K_3$$

or

$$K = \left[f + e\,s + \frac{B_0\,(w+s)}{(z-w-s)\,L}\right] l + \frac{a\,z}{z-w-s}\left[w\,l_0 + \frac{w\,l_1}{2} + \frac{h}{2} + \cdot 000018\,(\alpha_0 + \frac{\alpha_1}{2})\right]$$

In order to simplify this expression, the terms in brackets in the last member has been named by me the **Trace-Modulus** and indicated by p, so that we now have

$$K = \left[f + es + \frac{B_0 (w + s)}{L (z - w - s)}\right] l + \frac{a z p}{z - w - s} \qquad \dots \quad (5)$$

Baumeister and **Schübler**, who have employed the above formula in their investigations, have proposed simplifications of this expression. The former obtains the mean train-resistance m from the term in brackets of the last term of the equation: viz. by putting

$$m l = w l_0 + \frac{w l_1}{2} + \frac{h}{2} + \cdot 000018 \left(\alpha_0 + \frac{\alpha_1}{2}\right)$$

consequently

$$K = \left(f + es + \frac{B_0 (w + s) + a z m L}{L (z - w - s)}\right) l$$

whereas Schübler determines for the whole line a uniform straight ascent s_2 giving the same working-expenses as the variably formed trace.

This uniform straight ascent, termed by Schübler the 'Representative' gradient, but which I have named the '**Equivalent**' gradient, is obtained from the Eqn.

$$\frac{w l}{2} + \frac{s_2 l}{2} = w l_0 + \frac{w l_1}{2} + \frac{h}{2} + \cdot 000018 \left(\alpha_0 + \frac{\alpha_1}{2}\right)$$

viz.

$$s_2 = \left[w l_0 + h + \cdot 000018 \, (2 \alpha_0 + \alpha_1\right] \frac{1}{l} \qquad \dots \qquad \dots \quad (6)$$

or

$$s_2 = \left[w l_0 + h + 2 c \lambda_0 + c \lambda_1\right] \frac{1}{l}$$

where λ_0 is the total length of curves on "non-injurious" grades, and λ_1 that on "injurious" grades.

Thus the equivalent gradient is obtained by assuming that the whole length of the line l surmounts a height which is equal to the height h surmounted on all the "injurious" ascents plus a height $w\, l_0$ representing the sum of all the heights attained on "non-injurious" ascents supposing they were on a grade w, plus twice the resistance of all curves on "non-injurious" grades, and the simple resistance of all curves on "injurious" grades.

By inserting the equivalent grade, s_2 we obtain the cost of the transport of a tonne of gross-load over the whole line, viz.

$$K = \left(f + es + \frac{B_0 (w + s) + \frac{a z L}{2} (w + s_2)}{L (z - w - s)}\right) l \qquad \dots \quad (7)$$

Now although the ruling gradient has thus disappeared from the line assumed for the purpose of calculation as having equivalent gradient, it is nevertheless to be made use of in fixing the loading of trains. It is further to be noted that the equivalent grade for goods-traffic is not the same as that for passenger-traffic, as the following Example will show:—

On a railway of 100 km. in length there is 50 km. with grades under ·0036; 30 km. with grades between ·0036 and ·0055 surmounting a height of ·14 km.; and 20 km. with grades above ·0055 rising ·26 km. In the first group of ascents there are curves of a total of 1000° central-angle; in the second group occur curves of 900° central-angle; and in the third group, 600° of central-angle.

For a goods-traffic, $w = \cdot 0036$, $l_0 = 50$, $l_1 = 50$, $h = \cdot 4$, $\alpha_0 = 1{,}000°$, $\alpha_1 = 1500°$: whence

$$s_2 = \frac{1}{100}\left[\cdot 0036 \times 50 + \cdot 4 + \cdot 000018 \; (2 \times 1000 + 1500)\right] = \cdot 00643.$$

For passenger traffic, $w = \cdot 0055$, $l_0 = 80$, $l_1 = 20$, $h = \cdot 26$, $\alpha_0 = 1900°$, $\alpha_1 = 600°$: whence

$$s_2 = \frac{1}{100}\left[\cdot 0055 \times 80 + \cdot 26 + \cdot 000018 \; (2 \times 1900 + 600)\right] = \cdot 0077.$$

§ 12.

Determination of the Traction-coefficient.

The increase of locomotive tractive-force obtained by coupling the axles enables heavier trains to be hauled; and in this way those expenses of the locomotive which are independent of its power exerted are distributed over a greater number of units, and the working-expenses per tonne-km. are correspondingly diminished.

For the average weight of **Goods-locomotives**, *i.e.*, 60 tonnes, inclusive of the tender, the weight of train haulable up an incline of ·0036 by such an engine is in tonnes

$$Q = \frac{z - \cdot 0036 - \cdot 0036}{\cdot 0036 + \cdot 0036} 60$$

or

$$Q = 8333\, z - 60$$

If the H.P. of the locomotive is ·006 L or 75 × ·006 L = ·45 L metre-kgs. then the possible velocity in metres per second with a tractive-force $z\, L$ kgs. is

$$v = \frac{\cdot 45}{z}$$

The working-expenses on a flat land line which has no "injurious" gradients are, by Eqn. 3, per tonne-km. gross-load.

$$k = 15 + 2 \times \cdot 0036 + \frac{32\ (0036 + \cdot 0036)}{60\,(z - \cdot 0036 - \cdot 0036)} + \frac{25.\, z.\ \cdot 0036}{z - \cdot 0036 - \cdot 0036}$$

or

$$k = \cdot 157 + \frac{\cdot 00384 + \cdot 09\, z}{z - \cdot 0072} \text{ pf.}$$

For various values of the tractive-force coefficient we have in the following Table the train-weight, velocities, and working-expenses per tonne-km. of gross-load.

TABLE III.

Goods Trains: Flat Land Lines.

Number of Coupled Driving-axles.	1	2			3		
Tractive-force coefficient ...	·03	·04	·05	·06	·07	·08	·09
Weight of train Q (in tonnes) ...	190	273	357	440	523	607	690
Velocity v (in metres per sec.) ...	15	11·25	9	7·5	6·4	5·6	5
Working-expenses, *f*, per ton-km. gross-load, in pfennigs..	·444	·384	·352	·332	·319	·309	·361

In the formula by which the working-expenses given in the above Table are calculated no account is taken of the fact that the injurious frictional resistance of the working parts of the locomotive, together with its maintenance-charges, increases with the number of the coupled-axles; nor that with the increase in the weight of the trains and of their resulting increased length the expense of shunting and the consequent greater length of sidings increases the cost of stations. Finally, there is no account taken of the fact that with the consequent decrease of velocity of travel due to the increased expenditure of tractive-force, the accruing expense of train-staff and the interest of the capital expended in locomotives and vehicles per km. become greater. When all these items are taken into account the working-expenses would diminish faster with the increase of the tractive-force coefficient than the Table shews, and would quickly reach a lower limit below which with a further increase of the tractive-coefficient they would even increase. It is in recognition of these facts that flat land goods-engines have not usually more than 2 coupled-axles, thus ensuring that the traction-coefficient remains somewhere between ·05 and ·025.

For a line in mountainous country of which the gradient is $\cdot 025$, with a locomotive weight of 60 tonnes and for a tractive-coefficient z, the weight of the train will be

$$Q = \frac{z - \cdot 025 - \cdot 0036}{\cdot 025 + \cdot 0036} \times 60 = 2098\,z - 60$$

and the working-expenses per tonne-km. of gross-load will be, from Eqn. 4,

$$k = \cdot 15 + 2 \times \cdot 025 + \frac{32\,(\cdot 0036 + \cdot 025)}{60\,(z - \cdot 0036 - \cdot 025)} + \frac{\frac{25}{2} z\,(\cdot 0036 + \cdot 025)}{z - \cdot 0036 - \cdot 025} \text{ pf.}$$

or

$$k = \cdot 2 + \frac{\cdot 01526 + \cdot 3575\,z}{z - \cdot 0286} \text{ pfg.}$$

The following Table gives the train-weight, velocity, and working-expenses for various values of the traction-coefficient.

TABLE IV.

Goods Trains: Mountain Lines.

Number of Coupled Driving-axles.	2	3			4		
Tractive-force coefficient	·06	·07	·08	·09	·10	·11	·12
Weight of train (tonnes)	66	87	108	129	150	171	192
Velocity (metres per sec.)	7·5	6·4	5·6	5	4·5	4·1	3·75
Working-expenses per tonne-km., in pfennigs...	1·419	1·174	1·053	·973	·914	·871	·846

For the reasons already given the working-expenses will not decrease with the increase of the tractive-force coefficient exactly in the same degree as in the Table. In order to avoid a too low velocity on a mountain line having a grade of ·025, the tractive-force coefficient must be taken at about ·10 for goods-engines.

The traction-coefficient should according to the preceding discussion increase with the value of the ruling gradient; and may, taking everything into consideration, be suitably fixed, for goods-engines, at

$$z = \cdot 05 + 2\,s.$$

On mountain railways the weight of a goods-engine may, of course, be increased in order to haul heavier trains beyond the average weight of 60 tonnes. This increase in weight will not decrease the working-expenses to any extent worth notice, and the formula employed to calculate them will still be valid, since the expenses, B_0, depending on the performance of the locomotive, increase almost in the same ratio as the weight of the engine.

For the value of the traction-coefficient $z = \cdot 05 + 2\,s$ the weight of the goods-train on a line on which the ruling gradient is s is

$$Q = \frac{\cdot 0464 + s}{\cdot 0036 + s}\,60$$

And the maximum velocity up the grade is

$$v = \frac{\cdot 45}{\cdot 05 + 2\,s}$$

From which formula the following Table is calculated.

TABLE V.

Weight and Velocity of Goods Trains.

Ruling Gradient of line	·0036	·006	·100	·015	·025
Weight of train	417	327	249	198	150
Velocity up-hill in m/sec. ...	8	7·2	6·4	5·6	4·5

For **passenger-trains** lighter locomotives are employed of an average weight, including the tender, of 54 tonnes, and having a smaller traction-coefficient than goods-engines.

For a line of which the ruling gradient is equal to the resistance-coefficient of **passenger-trains**, *i.e.*, ·0055, with a traction-coefficient z the possible train-weight is

$$Q = \frac{z - \cdot 0055 - \cdot 0055}{\cdot 0055 + \cdot 0055} 54$$

or

$$Q = 491\, z - 54.$$

The working-expenses per tonne of the passenger-train are by Eqn. 3,

$$k = \cdot 405 + \frac{27\,(\cdot 0055 + \cdot 0055)}{54\,(z - \cdot 0055 - \cdot 0055)} + \frac{25.\, z. \cdot 0055}{z - \cdot 0055 - \cdot 0055}:$$

or

$$k = \cdot 405 + \frac{\cdot 0055 + \cdot 1375\, z}{z - \cdot 011}$$

The velocity, if the H. P. of the engine is put = ·006 L as for goods-trains, is

$$v = \frac{\cdot 45}{z}$$

The following Table gives the weight of train, velocity, and working-expenses for various values of the traction-coefficient.

TABLE VI.

Passenger Trains.

Number of Coupled Driving-axles.	2		3		
Traction-coefficient	·02	·03	·04	·05	·06
Weight of train (tonnes)	44	93	142	191	241
Maximum velocity up-hill (m/sec.)	22·5	15	11·3	9	7·5
Working-expenses per tonne of train-weight, in pfennigs ...	1·321	·968	·784	·717	·685

Since for passenger-trains it is not the haulage of a great weight of train but the attainment of great speed that is required, the traction-coefficient on a line of the assumed gradient must not be taken greater than about ·03· Consequently, one driving-axle would suffice, but usually in order to enable trains to be started rapidly 2-coupled-axle locomotives are used.

For a mountain line having a ruling gradient of ·025 the weight of the passenger-train haulable is

$$Q = 177\, z - 54$$

and the working-expenses per tonne-km. for such a train will be

$$k = \cdot 405 + \frac{\cdot 01525 + \cdot 38125\, z}{z - \cdot 0305}$$

In the following Table the train-weights, velocities up-hill, and working-expenses per tonne-km. of passenger-trains for various values of the traction-coefficient are given.

TABLE VII.

Passenger Trains: Mountain Lines.

Number of Coupled Driving-axles.	2		3		
Traction-coefficient	·05	·06	·07	·08	·09
Weight of train (tonnes)	34	52	70	87	105
Velocity up-hill (m/per sec.)	9	7·5	6·4	5·6	5
Working-expenses per tonne-km.	2·165	1·968	1·467	1·329	1·238

In order not to have too small a train-weight and at the same time to ensure that the speed shall not be too low, for mountain lines of grade ·085 the traction-coefficient may be taken at ·07.

According to the preceding the traction-coefficient for passenger-trains is in general to be taken at

$$z = \cdot 02 + 2\,s$$

whence the weight of the passenger-train will be

$$Q = \frac{\cdot 0145 + s}{\cdot 0055 + s}\,54$$

and the maximum velocity up-hill will be

$$v = \frac{\cdot 45}{\cdot 02 + 2\,s}$$

from which we obtain the following Table:

TABLE VII.

Ruling Gradient, s	·0036	·006	·010	·015	·025
Weight of train, Q	107	96	85	78	70
Attainable velocity up-hill	16·5	14·1	11·3	9	6·4

§ 13.

Approximate Formulæ for the Calculation of the Working-Expenses and Cost of the Train-Kilometre.

If the value of the tractive-force coefficient of a **goods-locomotive**

$$z = \cdot 05 + 2\,s$$

is inserted in the Eqn. 4 for the working-expenses of the tonne-km. of gross-load of goods-trains, and if further

$$f = \cdot 15, \quad e = 2, \quad B_0 = 32, \quad a = 25, \quad L = 60, \quad w = \cdot 0036;$$

then

$$k = \cdot 15 + 2\,s + \frac{\cdot 0417 + \cdot 6233\,s + 625\,(s_1 + c) + 25\,s\,(s_1 + c)}{\cdot 0464 + s}$$

To determine the working-expenses on "non-injurious" grades put

$$s_1 + c = w = \cdot 0036$$

in the above formula. The following Table is calculated according thereto.

TABLE IX.

Goods Trains.

Ruling Gradient.	Working-Expenses in pfennigs per tonne-km.				
	on "non-injurious" gradients.	on "injurious" gradients and curves, $s_1 + c =$			
s	s_0	·006	·010	·015	·025
1 : ∞	·288	—	—	—	—
·0036	·337	—	—	—	—
·006	·366	·402	—	—	—
·010	·410	·447	·509	—	—
·015	·458	·498	·563	·644	—
·025	·539	·581	·651	·739	·914

This Table shows that the working-expenses per tonne km. of gross-load of goods-trains can be represented with a satisfactory degree of exactitude by the approximate formula

$$k = \cdot 24 + 10\,s + 17\,(s_1 + c) \qquad \ldots \qquad \ldots \qquad (8)$$

in which for "non-injurious" grades

$$s_1 + c = w = \cdot 0036$$

is to be substituted.

If we disregard the working-expenses on a line level and straight throughout its whole length, which indeed are of no practical importance, then the maximum error introduced by using this approximate formula for grades up to ·025 is less than 2%.

For the working-expenses on the ruling gradient, viz. for $s_1 + c = s$, Eqn. 8, gives results identical with the values given in Table IX.

For **Passenger-traffic**, inserting the tractive-force coefficient of the passenger-locomotive

$$z = \cdot 02 + 2\,s$$

and putting $f = \cdot 405, \quad B_0 = 27, \quad a = 25, \quad L = 54, \quad w = \cdot 0055;$

then Eqn. 4 gives

$$k = \cdot 405 + \frac{\cdot 004125 + \cdot 6375\, s + \cdot 25\,(s_1 + c) + 25\, s\,(s_1 + c)}{\cdot 0145 + s}$$

The following Table is calculated from the above formula, in which $s_1 + c = w = \cdot 0055$ is to be inserted for "non-injurious" gradients.

TABLE X.

Passenger Trains.

Ruling Gradient.	Working-Expenses in pfennigs per tonne-km.			
	on "non-injurious" grades.	on "injurious" grades and on curves $s_1 + c =$		
s	s_0	·010	·015	·025
1 : ∞	·784	—	—	—
·0055	·893	—	—	—
·010	·945	1·037	—	—
·015	·986	1·081	1·187	—
·025	1·035	1·135	1·246	1·468

The Table shows that here also the working-expenses may be represented with sufficient exactitude by the approximate formula,

$$k = \cdot 73 + 8\, s + 22\,(s_1 + c) \qquad \ldots \qquad \ldots \quad (9)$$

in which for "non-injurious" grades is to be inserted

$$s_1 + c = w = \cdot 0055.$$

The maximum error involved in the above approximate formula is less than $1\frac{1}{3}\%$.

In the Traffic working-year 1885-86, the ratio of gross-load to paying-load on the Prussian State Railways—excluding weight of locomotive and tender,—namely, the **Load-coefficient**, b, for goods-traffic was $2\frac{1}{3}$; so that the working-expenses per paying tonne-km. amounted to

$$k = \cdot 56 + 23\tfrac{1}{3}\, s + 39\tfrac{2}{3}\,(s_1 + c) \qquad \ldots \qquad \ldots \quad (10)$$

The transport of each passenger is equivalent to the haulage of $1\frac{1}{3}$ tonnes gross-load; so that the working-expenses per passenger-km. are

$$k = \cdot 973 + 10\tfrac{2}{3}\, s + 29\tfrac{1}{3}\,(s_1 + c) \qquad \ldots \qquad \ldots \quad (11)$$

The figures in Tables IX and X multiplied by the train-weights give the cost of a train-km.

If in the Eqn. between the train-weight, tractive-force coefficient, and the ruling gradient, namely

$$Q = \frac{z - w - s}{w + s}\, L$$

z is put $= \cdot 05 + 2\, s$, $w = \cdot 0036$, and $L = 60$, then the weight of the goods-train is

$$Q = \frac{\cdot 0464 + s}{\cdot 0036 + s}\, 60 \quad \text{tonnes}:$$

and for

$$z = \cdot 02 + 2\, s, \qquad w = \cdot 0055, \qquad \text{and } L = 54,$$

the weight of a Passenger-train is

$$Q = \frac{\cdot 0145 + s}{\cdot 0085 + s} 54 \quad \text{tonnes.}$$

Employing the train-weights given by the above formulæ the Tables XI and XII are obtained.

TABLE XI.

Ruling Gradient.	Train-weight in tonnes.	Cost in pfennigs of a goods-km.				
		on "non-injurious" grades.	on "injurious" grades and in curves $s_1 + c =$			
s	Q	s_0	·006	·010	·015	·025
1 : ∞	773	223	—	—	—	—
·0036	437	140	—	—	—	—
·006	328	120	138	—	—	—
·010	249	102	111	126	—	—
·015	198	91	99	112	128	—
·025	150	81	87	98	111	137

TABLE XII.

Ruling Gradient.	Train-weight in tonnes.	Cost in pfennigs of passenger-km.			
		on "non-injurious" grades.	on "injurious" grades and in curves s_1 c =		
s	Q	s_0	·010	·015	·025
1 : ∞	142	111	—	—	—
·0055	98	87	—	—	—
·010	85	80	88	—	—
·015	78	76	84	92	—
·025	70	72	79	87	102

NOTE.—It must be borne in mind that the costs of the tonne-km., passenger-km., and train-km. given in Tables IX to XII, and in Eqns. 8 to 10 are simply the **actual** or **intrinsic** working-expenses of the section—as required for the economic location of the trace—**excluding the maintenance and station-expenses**; they are therefore considerably less than the so-called working-expense data which include these latter expenses.

A comparison of the expenses of goods- and passenger-trains shows that on a straight level line a goods-train would be twice as expensive to run as a passenger-train. But on undulating ground only 1¾ times, and on mountain lines of ·025 grade, only 1⅓ as costly.

The weight of the locomotive has, for the purpose of the calculation of expenses, been assumed to be 60 tonnes for goods-, and 54 tonnes for passenger-trains; on mountain lines, however, it will be much heavier. But the expenses per tonne-km., or passenger-km. are not thereby diminished, or at least only in an inconsiderable degree; since the train-expenses and the expenses dependent on the performance of the locomotive per tonne-km., or per passenger-km., remain the same; and the expenses independent of the performance of the locomotive increase almost proportionally with the weight of the locomotive, so that $\frac{B_0}{L}$ for different weights of locomotive remains the same.

Also, when as is the case on Local lines* the weight of the locomotive is less than the above, the two costs calculated on the basis of a locomotive weight of 60 and 54 tonnes respectively, per tonne-km. and per passenger-km., are almost exactly the same.

[* For the meaning of this term, see "The Commercial Trace," forming Part I of this Work, p. 59.—TR.]

The weight of the locomotive need not have appeared at all in the formula for the working-expenses if the constant quantity $\frac{B_0}{L}$ had been put $= b$. The expenses per train-km. increase, on the contrary almost in equal ratio with the weight of the locomotive. But since the working-expenses in Trace-problems should be calculated **not** from the number of train-kms. but perferably from the number of tonne-km. and passenger-km., it is permissible in every case to assume as a basis the weight of a goods-locomotive to be 60 tonnes, and that of a passenger-locomotive as 54 tonnes.

If the number of train-kms. has to be dealt with in calculations, then for different weights of locomotives, for example for 72 tonnes, such a train is to be treated as $\frac{72}{60} = 1{\cdot}2$ goods-trains, or as $\frac{72}{54} = 1{\cdot}33$ passenger-trains, as the case may be.

§ 14.

Determination of the Working-Expenses when the Traffic is not the same in both directions.

When the traffic preponderates in one direction—which for this reason will hereafter be called the Main direction—then the train-load in this direction must of necessity be greater than in the opposite direction, because—owing to the return-journeys of the locomotives and vehicles—the number of trains in both directions is necessarily the same.

If the paying-load carried in the main direction be T_1 the weight of a train carrying the paying-load be Q_1 and the gross-load of a train $b\,Q_1$ then if the paying-load carried in the reverse direction be T_2 and the paying train-load be Q_2 the gross train-load of a train in the reverse direction is

$$(b-1)\,Q_1 + Q_2$$

and putting $T_2 = r\,T_1$, that is $Q_2 = r\,Q_1$,
it becomes

$$(b + r - 1)\,Q_1$$

If the ruling gradient is in the main direction, then

$$b\,Q_1 = \frac{z - w - s}{w + s}\,L$$

Thus a stiffer gradient s_1 is permissible in the reverse direction, and its limit-value is given by the equation

$$(b + r - 1)\,Q_1 = \frac{z - w - s_1}{w + s}\,L$$

as

$$s_1 = \frac{z\,b\,s + (1 - r)\,(z - w - s)\,w}{z\,b - (1 - r)\,(z - w - s)} \qquad \dots \qquad \dots \qquad (12)$$

For example: if $b = 2\tfrac{1}{3}$, $z = \cdot 09$, $s = \cdot 02$, $w = \cdot 0036$, then in the opposite direction (to the main) a grade may be permitted

for	$r = \tfrac{3}{4}$,	of $s_1 = \cdot 0218$,
	$r = \tfrac{1}{2}$,	$s_1 = \cdot 0244$,
	$r = \tfrac{1}{4}$,	$s_1 = \cdot 0273$,
	$r = 0$,	$s_1 = \cdot 0309$,

But if there be a severer grade in the opposite direction than that given by the above Eqn. 12 the former is to be considered as the real ruling gradient.

If the ruling gradient lies in the main direction then the sum of the trains moving annually in both directions is

$$n = 2\,\frac{T_1}{Q_1} = \frac{2\,b\,T_1\,(w + s)}{L\,(z - w - s)}$$

or if the total traffic is put

$$T_1 + r\,T_1 = T$$

and consequently

$$T_1 = \frac{T}{1 + r}$$

then

$$n = \frac{2\,b\,T}{1 + r}\;\frac{w + s}{(z - w - s)}\,L \qquad \dots \qquad \dots \qquad (13)$$

On the other hand, if the ruling gradient lies in the reverse direction, then from

$$(b + r - 1)\, Q_1 = \frac{z - w - s}{w + s} L$$

is obtained

$$Q_1 = \frac{1}{b + r - 1} \cdot \frac{z - w - s}{w + s} L$$

Whence the number of trains per annum is

$$n_1 = 2\, \frac{b + r - 1}{1 + r}\; T\, \frac{w + s}{(z - w - s)\, L} \qquad \dots \qquad \dots \quad (14)$$

The working-expenses will now be determined for the case in which **the ruling gradient lies in the main direction** of the traffic. The various gradients forming the line are, for the purpose of the calculation of the locomotive-expenses, to be divided into 4 groups: viz.

(1) Those on which in descending **brakes are not used** and which **rise** in the main direction of the traffic, having a total length l_0 and a total height h_0, and on which there are curves of which the total central-angle amounts to α_0. On such grades the locomotive-expenses for a train in the main direction are

$$B_0\, l_0 + a\, (b\, Q_1 + L)\, (w\, l_0 + h_0 + \cdot 000018\, \alpha_0)$$

or, since necessarily,

$$(b\, Q_1 + L)\, (w + s) = z\, L$$

therefore

$$B_0\, l_0 + \frac{a\, z\, L}{w + s} (w\, l_0 + h_0 + \cdot 000018\, \alpha_0)$$

Whereas, for the opposite direction they are

$$B_0\, l_0 + a \left[(b + r - 1)\, Q_1 + L \right] (w\, l_0 - h_0 + \cdot 000018\, \alpha_0)$$

or

$$B_0\, l_0 + a\, (b\, Q_1 + L)\, (w\, l_0 - h_0 + \cdot 000018\, \alpha_0) - a\, (1 - r)\, Q_1\, (w\, l_0 - h_0 + \cdot 00008\, \alpha_0)$$

whence, inserting

$$Q_1 = \frac{z - w - s}{b\, (w + s)} L$$

we obtain

$$B_0\, l_0 + \frac{a\, z\, L}{w + s} \left(w\, l_0 - h_0 + \cdot 000018\, \alpha_0\right) - (1 - r)\, \frac{a\, (z - w - s)}{b\, (w + s)}\, L \left(w\, l_0 - h_0 + \cdot 000018\, \alpha_0\right)$$

Consequently, the total locomotive-expenses per train in both directions are

$$2\, B_0\, l_0 + \frac{2\, a\, z\, L}{w + s} (w\, l_0 + \cdot 000018\, \alpha_0) - (1 - r)\, \frac{a\, (z - w - s)}{b\, (w + s)}\, L\, (w\, l_0 - h_0 + \cdot 000018\, \alpha_0)$$

(2) Gradients on which **brakes are required in** descending and which **rise** in the main direction of traffic, of a total length l_1 and total height h_1 and in which the sum of the central angles of the curves is α_1.

The locomotive-expenses per train in the main direction are

$$B_0\, l_1 + a\, (b\, Q_1 + L)\, (w\, l_1 + h_1 + \cdot 000018\, \alpha_1)$$

or

$$B_0\, l_1 + \frac{a\, z\, L}{w + s} (w\, l_1 + h_1 + \cdot 000018\, \alpha_1)$$

and for a train in the opposite direction they are $B_0\, l_1$; so that the sum of the locomotive-expenses per train in both directions is

$$2\, B_0 l_1 + \frac{a\, z\, L}{w + s} (w l_1 + h_1 + \cdot 000018\, \alpha_1)$$

(3) Gradients on which **brakes are not** used in descending and which **fall** in the main direction of the traffic, of a total length of l_2 and a total fall of h_2 and on which the sum of the central-angles of curves is α_2.

The locomotive-expenses per train in the main direction are

$$B_0 l_2 + \frac{a z L}{w + s} (w l_2 - h_2 + \cdot 000018 \alpha_2):$$

and for a train in the opposite direction,

$$B_0 l_2 + a\Big[(b + r - 1) Q_1 + L (w l_2 + h_2 + \cdot 000018 \alpha_2)$$

or

$$B l_2 + \frac{a z L}{w + s}\Big(w l_2 + h_2 + \cdot 000018 \alpha_2\Big) - (1 - r) \frac{a (z - w - s)}{b (w + s)} L (w l_2 + h_2 + \cdot 000018 \alpha_2).$$

So that the total locomotive-expenses per train in both directions are

$$2 B_0 l_2 + \frac{2 a z L}{w + s}\Big(w l_2 + \cdot 000018 \alpha_2\Big) - (1 - r) \frac{a (z - w - s)}{b (w + s)} L (w l_2 + h_2 + \cdot 000018 \alpha_2).$$

(4) Grades on which brakes must be used in descending, and which fall in the direction of the greater traffic, of a total length of l_3 and a total fall of h_3 and having curves of total central-angle of α_3.

The locomotive-expenses per train in the principal direction of the traffic are

$$B_0 L_3$$

and in the opposite direction are

$$B_0 l_3 + a \Big[(b + r - 1) Q_1 + L\Big] \Big(w l_3 + h_3 + \cdot 000018 \alpha_3\Big)$$

Accordingly, bearing in mind the previous transformation made, the sum of the locomotive-expenses per train in both directions, is

$$2 B_0 l_3 + \frac{a z L}{w + s}\Big(w l_3 + h_3 + \cdot 000018 \alpha_3\Big) - (1 - r) \frac{a (z - w - s)}{b (w + s)} L (w l_3 + h_3 + \cdot 000018 \alpha_3).$$

From what has preceded the total locomotive-expenses for the whole length of the line $l = l_0 + l_1 + l_2 + l_3$, per train in both directions are

$$2 B_0 l + \frac{2 a z L}{w + s} \left\{w l_0 + w l_2 + \frac{w l_1}{2} + \frac{w l_3}{2} + \frac{h_1}{2} + \frac{h_3}{2} + \cdot 000018 \left(\alpha_0 + \alpha_2 + \frac{\alpha_1}{2} + \frac{\alpha_3}{2}\right)\right\}$$

$$- (1 - r) \frac{a (z - w - s) L}{b (w + s)} \left\{w l_0 + w l_2 + w l_3 + h_2 + h_3 - h_0 + \cdot 000018 \left(\alpha_0 + \alpha_2 + \alpha_3\right)\right\}$$

The train running-expenses and cost of brake-service for the less loaded train in the opposite direction hauling the same number of vehicles as the train running in the main direction are the same as for the train in the main direction, and thus together the total for each trains in both directions is,

$$2 (f + e s) l b Q_1$$

or

$$2 (f + e s) l \frac{z - w - s}{w + s} L$$

The number of trains to be hauled annually in both direction—from Eqn. 13— is

$$\frac{n}{2} = \frac{b T}{1 + r} \frac{w + s}{(z - w - s) L}$$

so that the working-expenses for the whole line, inverting the Trace-modulus p (§ 11), and putting

$$p_0 = w l_0 + w l_2 + w l_3 + h_2 + h_3 - h_0 + \cdot 000018 (\alpha_0 + \alpha_2 + \alpha_3)$$

are

$$K = \frac{2 b T}{1 + r} \left[\left(f + e s + \frac{B_0 (w + s)}{L (z - w - s)}\right) l + \frac{a z p}{z - w - s}\right] - \frac{1 - r}{1 + r} a T p_0$$

Since the working-expenses per tonne on the whole length of the line for an equal traffic in both directions are (§11)

$$k = \left(f + e s + \frac{B_0 (w + s)}{L (z - w - s)}\right) l + \frac{a z p}{z - w - s}$$

then the working-expenses for unequal traffic in the two directions are

$$K = \frac{2 b}{1 + r} T k - \frac{1 - r}{1 + r} a T p_0 \quad \dots \quad \dots \quad \dots \quad (15)$$

The second term in the above expression represents the saving in cost of tractive-force due to the smaller train-load in the opposite direction as compared with that in the main direction. When the traffic in the main direction does not greatly exceed that in the other, viz. if it be not more than double as great as in the opposite direction, this term hardly amounts to 2% of the first term, consequently it may be, as a rule, neglected, and the working-expenses for a traffic preponderating in one direction may therefore be represented by the simpler expression,

$$K = \frac{2 b}{1 + r} T k \quad \dots \quad \dots \quad \dots \quad (16)$$

If K were equal simply to $b\,T\,K$ the working-expenses in the direction of lesser traffic would be the same as if the train carried the same paying-load as in the main direction. But since the paying-load is smaller than this load in the ratio of $1 + r : 2$, the expenses $b\ T\ k$ for the traffic equal in both directions require to be multiplied by the factor $\frac{2}{1 + r}$.

If the difference of the volume of traffic in the two directions be small, the working-expenses may be calculated **on the assumption of an equal traffic in both directions, and the inequality of the traffic distribution can be taken account of by modifying the load-coefficient, namely by taking it as**

$$b_1 = \frac{2 b}{1 + r}$$

where b is the load-coefficient in the main direction of the traffic, and r the ratio of the paying-load in the opposite direction to that in the main direction.

But if the ruling gradient is in the subsidiary direction of the traffic then the working-expenses are to be found from the number of trains as given by Eqn. 14, viz.

$$K = \frac{2 (b + r - 1)}{1 + r} T k - \frac{(b + r - 1)(1 - r)}{b (1 + r)} a T p_0$$

or, neglecting the second term as being in most instances very small,

$$K = \frac{2 (b + r - 1)}{1 + r} T k \quad \dots \quad \dots \quad \dots \quad (17)$$

SECTION II.

DISCUSSION OF THE DETAILS OF THE TRACE.

SECTION II.

THE DISCUSSION OF THE BEST FORM OF THE DETAILS OF THE TRACE.

§ 15.

Introductory.

Unlike the Commercial Trace—which is assumed to lie in a **horizontal plane,** and on which therefore the working-expenses for the weight- and distance-unit may be assumed as everywhere the same—the Technical Trace is a series of polygonal lines **in space** the angles of which are rounded-off by curves. Here the working-expenses for the weight- and distance-unit are dependent on the gradients and curvature of the Trace, according to the principles developed in Section 1.

Guided by the **general principle of location**—namely, that the total transport-expenses, viz. **the interest on the capital sunk, the track maintenance, and the working-expenses** shall be **a minimum**—it is possible to derive from the dependance of the working-expenses on the grades and on curvature certain definite rules for the location of the Trace in plan and elevation; and to establish a Method which shall enable the Engineer to decide on the technical value of the Trace.

There are indeed certain Regulations extant and obligatory, such as the "Technische Vereinbarungen", viz., the Regulations of The Union of German Railway Administrations, which are based on multitudinous experiments and on experience extending over many years; nevertheless further special investigations are yet required before it is possible to formulate a complete body of Standard Rules, Regulations, and Methods of procedure.

The conditions determining the shape of the Trace are not limited solely to curvature and gradient. The width occupied by the line, and the position of the stations thereon are also highly important factors and make their influence felt in various ways. Accordingly, the following four groups of conditions may be distinguished as influencing the shape of the Trace.

(1) The **width occupied by the line,** under which is included the gauge, number of tracks, their distance apart, the width of formation, the inclination of the side slopes, the width of the side-drains, protection-clearings at side of line, service roads, etc.

(2) The **curvature**—namely the minimum permissible radius, and the normal radius which can be carried out without reducing the grade: also the widening of gauge and rail-superelevation in curves, the form and length of transition-curves, and the length of the intermediates between curves of contrary-flexure.

(3) The **gradients,** namely the choice of the best ruling gradient either with or without development of the length of the Trace, or when the whole line is divided up into separate working sections: also, the permissibility and the possible length of momentum grades, the reduction of grade in curves and in tunnels: the rounding-off of grade-intersections at the bottoms and summits of grades, and the avoidance of lost height.

(4) The **positions of stations** and especially their length, and the permissible gradients and curves in stations; also the determination of the best distance apart of stations.

The existing Regulations and the results of experience with regard to the above-cited conditions governing the position of the Trace, together with the requisite calculations and investigations for the choice of the same, are discussed in what follows.

§ 16.

The Width of the Track and its Details.

The total width occupied by the line and its component widths is determined by the width of gauge. Railways are now too widely spread over most civilized countries to permit of any change in the established standard width of gauge, viz., 4′ 8½″ being even thought of; accordingly, any general investigation of what **might** be the best width of gauge can have no practical value. Besides, such an investigation would be extremely difficult. The very circumstance that for every individual line the best width of gauge must be in each case different, varying with the amount of the cost of construction and the nature and volume of its traffic, and the fact that the demand for a standard or uniform gauge would involve the determination of one averaging most successfully all existing gauge-widths, clearly shows how impossible of solution of the problem is.

An easier problem is the determination in any given case of the dependance of the cost of construction on the gauge: although even here the authorities who have handled this question differ on individual points.

But it is very much more difficult to determine the influence of gauge on the first cost and maintenance of locomotives and vehicles which have to be expressed in the unit of performance of the locomotive and the unit of vehicle-load, respectively. Manifestly, these expenses would be a minimum for a certain gauge which, however, will not be the same for locomotives and for the different types of vehicles, and they would increase if the gauge were either decreased or increased. Regarding these intricate questions, only solvable by long and carefully carried-out experiments and observations, little exact knowledge has been obtained up to the present; and indeed little more is generally agreed upon than that the early broad-gauge of the G. W. R. in England of 2·2m. was too wide.

The problem of determining the other items of the working-expenses and the influence of gauge on the velocity, safety, and comfort of travelling is still more difficult and insoluble. Evidently, as the gauge increases the travel of a vehicle becomes steadier since the angle formed by lines drawn from the C. G. of the vehicle to the points of contact of the wheels with the sides of the rails, of which the magnitude is usually taken as a criterion of the steadiness of the vehicle's travel, becomes obtuser with every increase in gauge. Also, the inevitable differences in level of rails, assuming them to be the same for all widths of gauge, produce greater swayings of the vehicle in small gauges than in larger ones.

Fortunately, the advantages and disadvantages of an increase or decrease of gauge are so associated that fairly large deviations may be made above and below the distinctly best uniform or standard gauge without the total expenses of railway-working being increased thereby in any considerable degree. It is also indisputable that the gauge most in use, the so-called normal-gauge of 1435 mm (= 4′ 8½″), approaches very closely to the theoretically best general gauge. But, as already remarked, this question is now of no practical importance.

Another important question is this—Whether in any given case a deviation from the standard gauge in favour of a smaller or larger gauge be desirable? The governing considerations in this matter are discussed in § 18.

For the width of **Main** lines* the "Technische Vereinbarungen" of 1882** lay down the following:

" §. 4. The Normal gauge is 1435 m/m in the clear of the rails.

[* For definition see p. 59, "The Commercial Trace," forming Part I of this Work—Tr.]

[** The last edition of the "T. V." is of 1896—Tr.]

"§. 7. Tracks outside stations are not to be less than $3 \cdot 5^m$ apart from c to c.
"When additional tracks are added to the double-track the distance "from the latter, and from each other, is not to be less than 4^m. "For new lines a distance c to c of tracks of not less than 4^m is "recommended."

"§ 8. The width at formation is to be such that the distance of the intersection of "a line drawn through the underside of the rails with the plane of "side slope shall not be less than 2^m from the centre of the adjacent "track."

The Union of German Railway Administrations, distinguishes **Main** Lines, Subsidiary or **Secondary** Lines, and **Local** or Tertiary Lines.* Secondary Lines as regards track agree in all essentials with Main Lines, so that all locomotives and vehicles belonging to Main or Trunk Lines can travel over them. The velocity, however, is limited; and at no point is a velocity of 40 km/hr. to be exceeded. Local Lines—which may be either Normal- or narrow-gauge—handle public traffic, but it is mainly local; they are worked by steam traction and depend solely on adhesion. The maximum wheel-load is restricted to 5000 kg. and the velocity to 30 km/hr.

The Grundzüge or "Regulations for the Construction and Working of **Secondary** Lines" issued by the Verein in 1886 contain the same Regulations (§§ 4, 7) as regards gauge and distance apart of tracks as for Main Lines. But in § 8 the distance of the edge of the formation from the centre of the next adjacent track is reduced to $1 \cdot 75^m$; and in sharp curves and on high banks they recommend an increase of width.

By the other Grundzüge or "Regulations for the Construction and Working of Local or **Tertiary** Lines," also issued in the same year 1886, the **Normal**-gauge was fixed at $1 \cdot 435^m$, exactly as for Main and Secondary Lines; and they recommend that **narrow** gauges when employed on Local Lines be restricted to the widths of 1000^{mm} and 750^{mm} in the clear.

Normal-gauge tracks outside stations are not to be at a less distance apart than $3 \cdot 5^m$ when run over by vehicles of both Main and Secondary Lines. When this is not the case, and also for narrow-gauge lines, it is recommended that this distance be made 500^{mm} greater than the maximum outside width of the largest vehicle when loaded.

The width of the track substructure is to be such that the distance of the point of intersection of a straight line drawn through the underside of the rails with the plane of the slopes of bank—termed lthe "**crown-width**"—is for normal-gauge not less than 1500^{mm} from the middle of the next adjacent track, and for narrow-gauge line not less than the width of track. In sharp curves and on high banks an increase in these widths is recommended.

In the location of the trace the width of the earthwork at the bottom of the ballast, *i.e.*, **the Formation width,** is a ruling factor and in contradistinction to the crown-width, is termed the working-width of the line. Accordingly when locating the trace the depth of the ballast measured from the formation to the underside of the rails must be previously fixed-upon; and because banks are more easily drained than cuttings this depth to be less for banks than for cuttings, and for cuttings in earthy soil less than when they are in harder stuff.

In fixing the depth of ballast the rule laid down in § 10 of the "Technische Vereinbarungen" must be borne in mind, namely, that the ballast is to extend at least 200^{mm} below the underside of the sleepers. The depth of ballast should therefore be be taken on banks as a rule at 350 to 400^{mm}, and for cuttings at $\cdot 4$ to $\cdot 5^m$.

Ballast is usually laid on the formation with slopes of 1 to 1 or 1 to $1\frac{1}{4}$, with a horizontal berm of $\cdot 2^m$ or $\cdot 3^m$ from the foot of the ballast-slope up to the edge of the slopes of the

[* For further details regarding this classification, see Appendix to "The Commercial Trace" forming Part I of this Work—Tr.]

bank. Accordingly, the top-width of the ballast is somewhat less than the dimension laid down for the distance apart of the points of intersection of a straight line through the undersides of the rails with the bank-slopes produced.

The formation-width of banks with $1\frac{1}{2}$ slopes and $\cdot 35^m$ depth of ballast exceeds the top-width of the ballast by $2 \times 1\frac{1}{2} \times \cdot 35 = 1{\cdot}05^m$. In cuttings the formation-width is further increased by the width of the side-drains at formation level.

Assuming that the minimum bottom-width of side-drains in cuttings be $\cdot 3^m$, and their minimum depth below formation be $\cdot 3^m$, then the width of a side-drain at the height of the formation, for drain-slopes of $1 : 1\frac{1}{2}$, is $1{\cdot}2^m$. Accordingly, the formation-width of cutting for a depth of $\cdot 4^m$ of ballast is $2\ (1\frac{1}{2} \times \cdot 04 \times 1{\cdot}2) = 3{\cdot}6^m$ wider than the crown-width *i.e.*, at the height of the underside of rails between the points of intersection with slopes of bank produced.

But such a drain can only deal with small quantities of water, and accordingly side-drains are frequently made much wider and deeper, thus increasing the formation-width of the cutting.

In cuttings in hard ground with vertical sides the formation-width may, when necessary, be made smaller than for cuttings in earth, and the side-drains may then have the minimum dimensions.

The width of ground required for the construction of the line will be greater than the formation-width not only owing to the slopes of the banks and cuttings but also to provide width for the **drains** lying outside of the outer edge of slope—usually of a width of $\cdot 5^m$ to 1^m: also for the **clearings** strips in meadow and forest land for protection against fire, **service and other roads, snow fences**, and **protection walls** against stone avalanches, for **drains** on the hill-side of a cutting, and for **fence-ditches**, and finally, for **spoil-banks** and **borrow-pits.**

Works already carried out show, however, considerable differences not only in the average total-width of the requisite area of land but also in the dimensioning of the crown-width and formation-width. The Gotthard Railway may be cited as an example.

Hellwag[1] and **Dolezalek**[2] fixed $3{\cdot}6^m$ as the crown-width of ballast for a single-track line at the height underside of rails, and for two-track line $7{\cdot}1^m$ when the distance of the tracks is $3{\cdot}5^m$ from c to c. The distance of the point of intersection of a plane through the underside of rails with the planes of the bank-slopes is $4{\cdot}2^m$ for single-track, and $7{\cdot}7^m$ for double-track line.

The depth of ballast on banks is $\cdot 35^m$; in cuttings it is $\cdot 4^m$, and in rocky stuff $\cdot 5^m$. The formation-width banks carrying single-track is $5{\cdot}2^m$; for two tracks it is $8{\cdot}7^m$; and in cuttings for a single-track, $7{\cdot}9^m$, and for double-track $11{\cdot}4^m$; and the side-drains are $\cdot 35^m$ in bottom-width, $\cdot 3^m$ deep below the bottom of the ballast, and they have a width of $1{\cdot}25^m$ at the level of the top of the ballast for $1\frac{1}{2}$ slopes. For deep cuttings the side-drains are deepened and widened, and the formation-width increased to 9^m and $12{\cdot}5^m$ respectively.

The side-slopes for banks are usually $1\frac{1}{2}$ to 1; for cuttings, 1 to $1\frac{1}{2}$; in stiff material or rock, vertical.

Where the lateral space is limited the bank is formed of layers of rough broken stone having side slopes of 1 to 1 up to a max. height of 20^m, and for at least 1^m in depth they are topped with earth. Retaining walls of dry stone-work with a $\frac{2}{3}$ batter are quite frequently carried up to 10^m height and retaining walls of masonry are usually without batter.

Outside drains have a width of $\cdot 5^m$ to 1^m: protection-clearings in grass-land (against fire), a width of 8^m-10; in leaf-wood plantations, 15^m-20^m; in coniferous plantations, 20^m-25—all measured from the nearest adjacent track.

(1) Hellwag: Die Bahnachse und das Längenprofil der Gotthardbahn. Zurich. 1876.

(2) *Zeitschrift des Hannoverschen Architekten-und Ingenieur-Vereins.* 1882. s. 479.

§ 17.

Single- and Double-Track.

If it be certain that a single track will always suffice to handle the expected traffic then of course the trace is only worked-up on that basis—with the exception of the stations, which will under all circumstances have two tracks.

But if it appear certain that a second track will be necessary in the future, although perhaps only after a considerable time, then it is advantageous from the very commencement of operations to locate the trace for a 2-track line; and according to the estimate of the time after which the second track will become necessary to take measures for its construction.

Even when the second track will probably be necessary only after the lapse of 30 or 40 years, it will still be entirely justifiable to acquire the land for the same at the time of the original location. Land acquired for the second track need not exceed a strip of 4^m in width for the line between stations, which latter may be taken at $\frac{9}{10}$ ths of the whole length of the line: so that on an average there is required, 36 ares* per km. of road of which the purchase-price may be put at between 2,000 and 3,000 M.

At the end of 40 years, these sums at 4% compound-interest would become 9,600 and 14,400 M. respectively. And supposing that the land could be leased out for at least 1% of its cost-price, there would then be only a loss of 3% or, after 40 years the cost per km. would have increased to something between 6,500 and 9,750 M.

But it is probable that after the lapse of 40 years the land would sell at a higher price since, owing to its increased value, houses would quickly spring up in the immediate neighbourhood of the railway, or for other reasons.

Naturally, the advisability of the above procedure rests on estimates which have no very high degree of trustwortliness.

Further provision for the future construction of a second track may be made in the arrangement and treatment of the works of art on the line. Long tunnels, of course, will usually be laid out for double-track with the object of securing sufficient ventilation; and similarly, long-span bridges with iron superstructure are usually of double-track width with the object of fully resisting wind-pressure.

In the case of short tunnels, if it is probable that after n years a second track will become necessary then, for a rate of interest i, the calculation is made as follows.

If the cost of a unit of length of the tunnel for a single track at the time of the original construction be A_1, and for a double-track, A_2, and if the subsequent widening of the single-track tunnel to accommodate a double-track, or the construction of a second single-track tunnel parallel with the first, be A, then the immediate construction of a double-track tunnel is advisable if

$$(A_2 - A_1)(1 + i)^n < A$$

In the case of arched bridges, and iron-bridges with the main girders underneath the track, the substructure for the double-track will as a rule be advantageously constructed at the time of the actual building of the single-track; but the wing-walls should be built complete on one side only, and on the other side a retaining wall to be removed subsequently

[* 1 Are = 100 sq. metres—Tr.]

when the double line is built, and the abutments of this second half of the bridge are to be so built-up and stepped-off that they act as wings obliquely slanted off and forming a continuation of the abutments of the first half of the bridge, the slopes of the single-track bank being carried-up to them. The pecuniary propriety of such procedure is to be ascertained by means of the formula previously given in connexion with the construction of the tunnel.

For iron bridges in which the track runs **between the main girders** it is to be noted that the substructure of piers carrying two distinct single-track lines will be some $1{\cdot}5^m$ wider transversely than if the superstructure were that of a two-track line. If P be the extra cost, of the longer piers for two single-track superstructures over that of the piers of a two-track bridge; and if A_1 be the cost of a single-track superstructure, and A_2 the cost of a double-track superstructure; then by initially only building a single-track the saving as compared with a double-track superstructure is

$$A_2 - A_1 - P$$

This saving in the course of the n years after which the construction of the second track becomes necessary should exceed, if the policy is to justify itself, at compound-interest the outlay which will have to be incurred after n years; so that the condition which should be fulfilled is

$$(A_2 - A_1 - P)(1 + i)^n > A_1$$

For example: if the metallic superstructure of a bridge of 3 spans of 30^m cost $A_1 = 36{,}000$ M. and that of a double-track $A_2 = 71{,}000$ M., and if further, the superstructure for two single-track bridges cost $P = 8{,}000$ M. more than for a double-track bridge, then it would be the better policy to at once construct the two-track bridge if the second track were required subsequently in less than 7·4 years—since

$$(71{,}000 - 36{,}000 - 8{,}000)\,(1{\cdot}04)^{7{\cdot}4} > 36{,}000.$$

The earthwork, originally thrown-up for a single-track line, cannot be widened to accommodate a second track out of the initial saving $A_2 - A_1$ made by restricting the line to one track, since the roads and trucks tracks used in throwing-up or removing the earth, which without any considerable extra expense might have been utilized at the time for the widening of the bank, have to be re-contructed and equipped afresh; and the revetted work or the turfing on the slopes of the bank of the single-track line has to be taken off on one side and relaid after the widening for the second track; and precautionary measures must be taken to guard against the slipping down of the newly thrown-up bank.

Special difficulties occur when the line lies in sidelong ground because the hill- or inner-side of the line being in cutting and the valley-side in bank the revetting or turfing of the two slopes has to be postponed for the time-being and carried out latter on: and the track has to be slewed laterally. It is preferable therefore, both when initially constructing the single track and also when carrying out its later widening to the full 2-track section, instead of the usual balancing of cut and fill in the cross-section, to balance or equalize the quantities longitudinally—which may be done in the following manner.

The cross-section of the double track is not at any point fully completed but instead only the right-and left-hand parts of it so as to form a track for a single track line and that alternately for definite lengths. Subsequently when 2 tracks have become necessary, the alternate and incompleted portions of the original 2-track section are completed.

As the longitudinal haul of earth is more costly than moving it sideways, and since in the transition length from one track to the other, slewing of the track and redressing of one or of two slopes will be unavoidable, such supplementary widening will always be a considerable increase of expense. This method of procedure, in which the throwing-up of the right-hand track alternates with that of the left, should be carried out as largely as possible in the intermediate straights lying between reverse curves, and naturally demands a very detailed consideration. Under any circumstances the subsequent widening of

the formation-width for the second-track is an extra expense represented by

$$\Delta = m\ (A_2 - A_1)$$

which is only justifiable when and if the money saved viz. $(A_1 - \Delta)$ in the first instance by building a single track is greater at compound-interest for a term of n years within which one track suffices for the traffic requirements than the above expense; so that the earthwork should be thrown up at once for two tracks if

$$(1 + i)^n < m$$

For example: if $m = 1{\cdot}5$, as may be assumed as a rule, then at a rate of interest of $i = {\cdot}04$ the earthwork should be made at once for double-track if the second track would be required before the expiry of $n = 10$ years.

Thus, as the preceding discussion shows, there may advantageously be various stages of completeness in the building of the second track dependent on the length of time during which a single-track suffices, which stages lie between the simple acquisition of the subsequently requisite ground and the complete building to subgrade, further modified by the different methods and degrees of completeness in carrying out the works of art on the section.

§ 18.

Standard- and Narrow-Gauge Compared.

According to the Regulations ("T. V.") of the Union of German Railway Administrations Main lines and Secondary lines are under all circumstances to be built on the normal-gauge; and it is only in the case of the third class of lines, viz., Local lines which have a public traffic but mainly local that a choice between normal-gauge and narrow-gauges of 1000 $^{m}/_{m}$ or 750$^{m}/_{m}$ is permissible.

In comparing the Standard- with the Narrow-gauge it is necessary to assume that the number of trains, the paying-load carried by a single train, the velocity and, consequently, the type of rolling-stock, is the same for all cases.*

As inherent distinctive advantages of the **Narrow**-gauge are to be reckoned:—

(1) The saving in cost of construction due to the narrower width of the line.

(2) The saving in cost of construction arising from the employment of sharper curvature consequent on the closer conformity of the line to the shape of the ground.

(3) The ability, due to the use of sharper curves, to adapt the line more completely to the requirements of the traffic, enabling the line to penetrate more effectually into the heart of districts and also to approach more closely to industrial points.

On the other hand, the advantages peculiar to the **Standard** or Normal gauge are:—

(1) The avoidance of loading and unloading goods in passing from Main to Subsidiary lines:

(2) Its greater utility for military purposes and for certain special kinds of transport.

(3) The possibility of re-using old equipment, such as vehicles, turntables, etc., which have been rejected on lines where a higher degree of velocity obtains.

(4) The possibility of being able to transform the line at a small cost into a line of a higher order, when an increase of traffic demands it; for example, from a Secondary to a Main line.

The further advantage often claimed for the Standard-gauge of cheaper working is the not, for the reasons given in the sequel, here considered.

It is evident that no generally applicable conclusion is possible as to which gauge possesses the greater advantages: particular cases only can be adjudicated upon. As a guide to the calculations and decisions requisite to be made in such cases the following discussion will assist.

Reducing the gauge lessens the cost of land, earthwork, works of art, ballast, and sleepers. For the Narrow-gauge the rails cannot be made of less weight than for Standard track, because for equal velocities and for equal train-weights the weight of the locomotive must remain the same, although the wheel-pressure of the vehicles on the Narrow-gauge is somewhat smaller than on the Standard track. Strictly speaking, the rails for Narrow-gauge should even be increased in strength, because the oscillation of the vehicles arising from inequalities in the height of the rails is greater on Narrow-gauges than on the Standard.

* This has not always been done in the dispute of Standard *versus* Narrow-gauge which has been carried on for so many years; and the decision of this question, which indeed has been discussed *ad nauseam*, has frequently been much obscured and false issues raised from commercial and personal considerations.

Although the Verein has fixed as the minimum permissible crown-width of narrow-gauge railways at twice the width of gauge, this fact need not be considered in the comparison of different gauges, since an equal degree of safety in working is only possible when and if in the various gauges the distance of the ends of the sleepers, the edge of the ballast, and the sides of the earthwork from the outer rail-edge, are each the same in the two cases. When this is is so, the width of the land to be acquired, the width of the earthwork, the works of art, the width of the ballast, and the length of the (cross) sleepers is only smaller by the difference of gauge. Thus in the case of a gauge of 1^m as compared with the Standard-gauge, this difference amounts to $\cdot45^m$; and for the gauge of $\cdot75^m$ approximately to $\cdot7^m$.

The saving in cost of construction for a line of 1^m gauge as compared with the Standard-gauge may be estimated per km. as follows :—

(*a*) Saving in cost of land—a strip of $\cdot45^m$ wide, viz. $4\frac{1}{2}$ ares @ 60 M.	270 M.
(*b*) Saving in earthwork assuming an average difference in the height of the bottom of ballast from the surface of the ground of 1^m = 450 c b/m. @ 1 M.	450 „
(*c*) Saving in ballast with $\cdot3^m$ depth of ballast = $\cdot3 \times \cdot45 \times 1000 = 135$ c b/m @ 3 M.	405 „
(*d*) Saving in sleepers, $1111 \times \cdot45 = 500$ running metres, @ 1·5 M. ...	600 „
(*e*) Saving in land, earthwork, and in track in stations—an average per km. of road an addition of 10% of the items under (*a*) to (*d*). ...	173 „
(*f*) Saving in the works of art, say	302 „
Total =	2,200 M.

For a line of $\cdot75^m$ gauge the saving is greater in the ratio of ·6 to ·45, or say ... ... 3,800 M.

An additional saving the amount of which could only be ascertained after completely working-up the project of the line arises from the employment of the sharper curves possible on narrow-gauges.

According to the valuable theoretical investigations of **Bœdecker,*** curve-resistance remains constant if the wheel-base, wheel-radius, clearance in gauge, and curve-radius vary in the same proportion with the gauge. Of course such a proportionate reduction of the wheel-diameter with the decrease in gauge is obviously out of the question; but the wheel-diameter for the Standard gauge being between $\cdot9^m$ and $\cdot97^m$, for a metre-gauge line it is usually not less than $\cdot85^m$, and for a gauge of ·75, not less than $\cdot75^m$.

However, according to Bœdecker's investigations it appears that the diameter of wheel, within the above limits, is after all really only of quite minor influence on the magnitude of curve-resistance; sc that if the wheel-base is diminished proportinately with the gauge, it may be assumed that a curve of 50^m radius in a track of $\cdot75^m$ gauge, or a curve of 70^m radius, in a track of 1^m gauge, produces the same resistance as a curve of 100^m on a line of Standard-gauge.

Curve-resistance, (§ 7) on the Standard-gauge being taken as $c = \frac{1}{R}$ for a gauge of m may be expressed generally as $c = \frac{\cdot7m}{R}$; and consequently, for $m, = 1, c = \frac{\cdot7}{R}$; and for $m, = \cdot75, c = \frac{\cdot525}{R}$: where R is the radius of curve in metres.

[* Bœdecker: Rad und Schiene. Hannover; 1887, of which an English version by the present Translator is obtainable from Higginbotham, Madras.]

On Local (or tertiary) lines the disadvantages of sharp curves have now become greatly lessened by the use of movable axles now so generally and successfully employed, for example, on the Saxon narrow-gauge lines.

It will be readily seen that the use of smaller curve radii in difficult country may lead to much greater saving in the cost of construction than that directly arising from a reduction in width of gauge.

If the gauge be reduced while the trace and, consequently, the curve-radii, remain unchanged, there will of course be a decrease in the working-expenses due to the resulting decreased curve-resistance. On a narrow-gauge line of $\cdot 75^m$ gauge the curve-resistance will be less than on the Standard-gauge by the amount

$$\frac{\cdot 7\,(1\cdot 435 - \cdot 750)}{R} = \frac{\cdot 48}{R}$$

Thus in the haulage of a tonne on a curve of α^0 central angle and R^m radius, of which the length is given by $\cdot 0000175\,\alpha\,R$, there will be a saving in the work done in overcoming curve-resistance of

$$\frac{\cdot 48}{R}\cdot 0000175\,\alpha\,R = \cdot 000008\,\alpha.$$

Since the work done by a tonne of tractive-force on a length of 1 km. costs (§ 5) 25 pfennigs or ¼ M., there results a saving per tonne of $\cdot 000002\,\alpha$ M.

For example: suppose the sum of the central-angles average 50° per km. of length of line; then in consequence of the smaller curve-resistance on the narrow-gauge per tonne-km. there is a saving of ·0001 M. in working expenses; on "injurious" grades this is so only in one direction, but for "non-injurious" grades it is true of both directions. The saving however is an insignificant one, since assuming 200,000 tonnes gross-load is hauled over the line and that half the curves are on "non-injurious" grades, it only amounts to 15 M. per annum.

Of hardly more importance on the reduction of working-expenses is the smaller ratio (as compared with the Standard) of gross-weight to paying-load attainable on the narrow-gauge. If vehicles of various gauges be built on the same type and run at the same speeds then by narrowing the width of gauge hardly anything is saved in the weight of vehicle, expressed in terms of the unit of useful-load, in the under frame, viz., in the axles, wheels, bearings, and springs, while in the buffer and coupling gear there probably will be an actual increase of weight; whereas in the body of the vehicle a decrease in weight is attained by diminishing the wheel-base in the ratio of the gauge. Accordingly, for the whole vehicle, a narrowing of gauge would only improve the ratio of the gross-load by an insignificant amount. But this ratio is better in the narrow-gauge goods-vehicle from the fact that the Standard-gauge vehicle to enable it to travel on Main lines at the higher velocity must be built more solidly than the narrow-gauge vehicle which is confined to Local lines.

According to precise data* regarding the vehicles on the Royal Saxon Secondary Lines the vehicle-weight per 100 kg. useful-load of Standard-gauge covered goods-waggons varies from 61 to 95 kg.; but on the narrow-gauge of $\cdot 75^m$ from only 38 to 46 kg.; and for uncovered Standard-gauge goods-waggons, from 47 and 72 kg.; and on the narrow-gauge, from 37 to 44 kg. As regards passenger-vehicles, which are built much lighter both on the Standard-gauge Secondary lines and on Main lines, there is no decrease of weight on the narrow-gauge. Thus on the Saxon Secondary lines the weight per seat of the normal-gauge passenger-car is 39 to 174 kg. and of the narrow-gauge car, 152 to 172 kg.

Accepting these vehicle-weights, and assuming in each case an equal degree of utilization of the vehicle's capacity, it may be taken that in goods-traffic there is on an average per tonne of useful-load at least ⅓rd tonne less of vehicle-weight to be hauled on a line of $\cdot 75^m$

* By Geh. Finanzrath Köpcke and Finanzrath Strick, of Dresden.

gauge than on Standard-gauge. If the curve-resistance on the whole division averages ·006 of the load, then for the transport of a tonne of useful-load per km. of the narrow-gauge line there would be a saving of ·002 tonne-km., which corresponds to a money-saving of ·0005 M.

For example: on a line on which the goods-traffic amounted to 50,000 tonnes paying-load, there would be a saving of 25 M. per km., on a narrow-gauge line of ·75m gauge as compared with the cost on a Standard-gauge line.

But the saving in tractive-force on narrow-gauge lines arising from the decreased curve resistance and the smaller ratio of gross-load to useful-load is counter-balanced by an extra cost of train-service.

The cost-price of covered goods-waggons per 100 kg. of useful-load was on the Saxon Secondary lines on the Standard-gauge 22 to 49 M.; and on the narrow-gauge, 21 to 22 M. For Standard-gauge open goods-wagons it amounted to 16 to 23 M.; for narrow-gauge waggons, to 18 to 19 M.; and for a Standard-gauge passenger-car per seat it was 58 to 107 M.; and on the narrow-gauge, 97 to 122 M.

According to these figures the cost of the Standard-gauge goods-vehicle was somewhat less, while for passenger stock it was somewhat greater than for the narrow gauge; so that in general no noteworthy difference in these items of expense on the two gauges is recognisiable.

But the maintenance-expenses of the vehicles, for goods-traffic [as to which there is no comparative data available,] must doubtless be greater on the narrow-gauge than on the Standard; since the maintenance-expenses are dependent on the number of individual parts of the vehicle such as axles, wheels, bearings, springs, buffers, couplings, etc., of which more are required on the narrow than on the Standard-gauge for the transport of an equal amount of paying-load.

Since the expenses of maintenance and renewals of Main lines' goods-waggons, according to § 5 amount to ·185 pf. per paying-load tonne-km., and on Secondary lines of Standard-gauge will certainly not be less than ·12 pf. then if on narrow-gauge lines this expense be assumed to increase by half, the extra outlay for maintenance of vehicles per km. annually for a traffic of 50,000 tonnes paying-load would be

$$50{,}000 \times \cdot 06 = 30 \text{ M.}$$

Although the preceding discussion is neither wholly satisfactory nor exhaustive, nevertheless on the strength thereof it may be asserted that: **Assuming the same method of working the traffic, the working-expenses on Standard-and narrow-gauge differ very slightly from one another,** since what is saved on the narrow-gauge in traction is almost entirely lost in the extra cost of maintenance of its vehicles.

If the circumstances be taken into account that by adopting a narrow-gauge the advantage is lost of re-using much of the line-equipment and rolling-stock which has been rejected on Main lines but which is still serviceable for Local lines; that in times of an unusually heavy traffic getting out of difficulty by borrowing stock from neighbouring lines is impossible, and that consequently, it is imperative to maintain a full stock of rolling material; that the narrow-gauge lines have not for military purposes and for certain kinds of traffic the same suitability and capacity of work as the Standard-gauge; and finally, that all traffic either coming from or going to the Standard-gauge lines must be unloaded and reloaded: then the cases will be rare in which all these disadvantages, due to reducing the width of the track, are counterbalanced by the saving in cost of construction, which saving all-told only amounts to 5 or 10% of the whole capital-cost.

Amongst the capital disadvantages of the narrow-gauge, in most cases, is the necessity of **unloading and loading goods.** By the aid of suitable mechanical transhipping arrangements, shoots, etc., and sometimes by the transfer of the whole body of the vehicle, detached from its under-frame, from one gauge to the other, the expense of break of gauge has been often reduced to very small proportions. But under such conditions the interest on the capital and the maintenance-expenses of all the devices employed in the transfer-operations must not be left out of account nor under certain circumstances the loss of time,

depreciation of value of the goods due to the handling, and the necessity of a larger number of vehicles which such a transfer from one line to another involves.

Numerical Example: If for a branch-line of 25 km. length and $\cdot 75^m$ gauge the saving in construction-cost as compared with the Normal-gauge amounted to

$$25 \times 3{,}800 = 95{,}000 \text{ M.};$$

and if the working-expenses were the same as on a Standard-gauge line, then not more than the interest of that saving, *i.e.*

$$\cdot 04 \times 95{,}000 = 3{,}800 \text{ M.}$$

should be expended per annum in the transhipment of goods, if the narrow-gauge is to be worked at a profit. If the cost of transhipment, inclusive of all accessory expenses is taken at $\cdot 2$ M. per tonne, then not more than 19,000 tonnes should be so transhipped annually.

It is impossible to answer the question whether, the narrow-gauge is the more suitable for Local lines; and each individual case must be considered from the points of view above discussed. As a general rule, the narrow-gauge will only be advantageous where at its junction with the Standard-gauge the transhipment of merchandise is insignificant in amount in proportion to the length of the narrow-gauge line, or where at its contact with waterways a transhipment of goods would be necessary even if the line were of Standard-gauge; or finally, where by the adoption of sharper curves it is possible to adapt the line more closely to the ground, or to conform it to existing roads, or for the purpose of penetrating into densely populated localities.

But having decided upon a narrow-gauge it is very rarely worth while to halt at the gauge of 1^m: **in the majority of cases the gauge of $\cdot 75^m$ should be chosen**; since a gauge of 1^m possesses all the distinctive disadvantages of the narrow-gauge to almost the same extent as the $\cdot 75^m$ gauge, while the advantage due to saving in construction is relatively very much smaller than it is for a gauge of $\cdot 75^m$. But although a decrease in width of gauge below $\cdot 75^m$ may often be attended under special conditions with pre-eminent success, as, for instance, on the well-known Festiniog Railway (which has a width of only $\cdot 59^m$), still it is not desirable, owing to the increased difficulty of satisfactorily designing the rolling-stock; and on account of the greater and more violent swayings and impacts of vehicles and locomotives which occur on narrow-gauges.

Fig. 2.

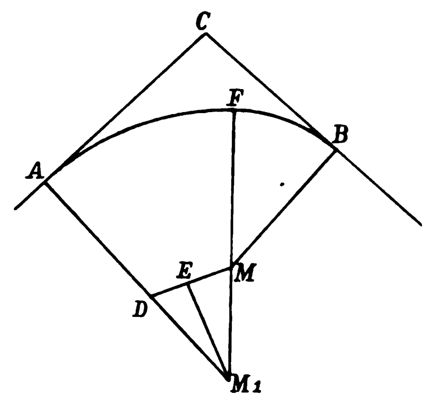


Fig. 3.

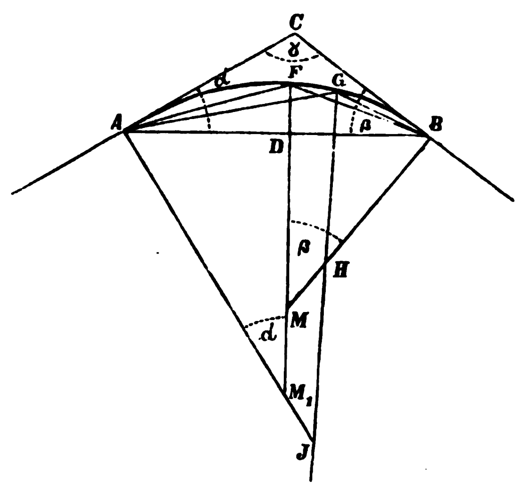
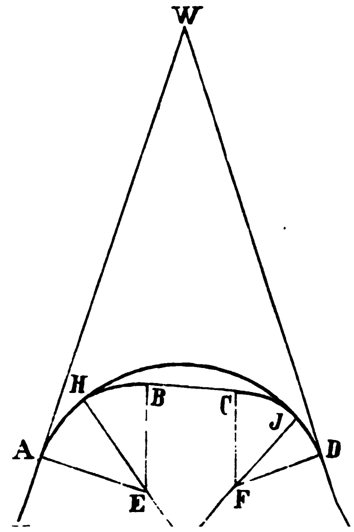


Fig. 4.



§ 19.

Curvature.

The circle is exclusively employed in Railway work, for the purpose of rounding-off the angular junctions of the string of straight lines forming the trace. On Roads, on the contrary, other curves which give a smoother, *i.e.*, more gradual junction with the straights are preferable as, for instance, the parabola.

The circle is necessarily employed in railway curves because the superelevation of the outer rail and the easement of gauge depend on the radius, which it is desirable should remain constant in amount for the whole or nearly the whole length of the curve.

The smoother transition from the curve to the tangent, and from the superelevation and enlarged gauge to the normal-track level and gauge is, as is well known, obtained by the use of **transition curves.***

If owing to the tangents being equal in length a circular curve does not fit the ground sufficiently exactly, it can be replaced by a compound curve formed of two circular arcs of unequal radii and two tangents of unequal length.

If the radius $B\,M$—**Fig. 2**—of the curve tangential to the line $C\,B$ is arbitrarily fixed at $r = M\,B$ within the permissible limits of radius, then the radius of the second circular curve starting from the tangent $A\,C$ is found by drawing from A perpendicular to $A\,C$, the line $A\,D = M\,B = r$ and from the middle E of the line joining D and M erecting a perpendicular of which the point of intersection M_1 with $A\,D$ produced is the centre of the circle tangential at the point A.

If it be required that the radii of both circles shall differ as little as possible from one another, then—**Fig. 3**—the angles $C\,A\,B = \alpha$ and $C\,B\,A = \beta$ are to be bisected by the lines $A\,F$ and $B\,F$, and the line $F\,D$ drawn perpendicularly to $A\,B$ from the point of intersection F of the bisecting lines to intersect $A\,M_1$ and $B\,M$, perpendiculars to the tangents $A\,C$ and $B\,C$ in M_1 and M, which are thus the centres of the circular curves of which the radii differ as little as possible from one another.

To demonstrate this, note that the angle $A\,F\,B$ or $A\,G\,B$ formed by the chords of the two curves, whatever the radius of the curve may be, is always equal to

$$180^\circ - \frac{\alpha + \beta}{2}$$

Therefore if a line $A\,G$ is drawn at an angle $\left(\frac{\alpha}{2} - x\right)$ with the line $A\,B$, a line $B\,G$ from B at an angle $\left(\frac{\beta}{2} + x\right)$ with $A\,B$ then the point of intersection, G, is the point of contact of the two circular arcs of the compound curve, of which the centres J and H are obtained by erecting perpendiculars in the middle of the chords $A\,G$ and $B\,G$.

Calling r the radius $H\,B$, and R the radius $A\,J$,

then
$$A\,G = 2\,R \sin\left(\frac{\alpha}{2} + x\right)$$

$$B\,G = 2\,r \sin\left(\frac{\beta}{2} - x\right)$$

therefore
$$\frac{r}{R} = \frac{B\,G}{A\,G} \quad \frac{\sin\left(\frac{\alpha}{2} + x\right)}{\sin\left(\frac{\beta}{2} - x\right)}$$

[* The advantages of which do not appear to be sufficiently well known in distinctively English practice, where it appears to be still considered "a fad" and "unpractical." See Appendix—Tr.]

or, since
$$\frac{BG}{AG} = \frac{\sin\left(\frac{\alpha}{2} - x\right)}{\sin\left(\frac{\beta}{2} + x\right)}$$

therefore
$$\frac{r}{R} = \frac{\sin\left(\frac{\alpha}{2} - x\right)\sin\left(\frac{\alpha}{2} + x\right)}{\sin\left(\frac{\beta}{2} + x\right)\sin\left(\frac{\beta}{2} - x\right)} = \frac{\sin^2\frac{\alpha}{2} - \sin^2 x}{\sin^2\frac{\beta}{2} - \sin^2 x}$$

which expression is a maximum for $x = o$. Q. E. D.

The values of r and R corresponding to the given condition, putting $A\,B = \rho$ and $A\,C\,B = \gamma$, are

$$r = \frac{c}{2}\,\frac{\sin\frac{\alpha}{2}}{\sin\frac{\beta}{2}\cos\frac{\gamma}{2}}$$

and

$$R = \frac{c}{2}\,\frac{\sin\frac{\beta}{2}}{\sin\frac{\alpha}{2}\cos\frac{\gamma}{2}}$$

If two curves spring from the **same** side of a short intermediate straight then, as laid down in § 17 of the "Technische Vereinbarungen," when the intermediate straight is less than 40^m the superelevation of the curves is to be carried-over into the straight. But as a general rule it would be preferable to replace such short intermediates by a third curve, thus producing a compound three-centred curve.

As shown in **Fig. 4** if R_1 is the radius assumed for the third or middle curve its centre G will be obtained by the intersection of an arc of radius $R_1 - R$ and centre E with an arc of radius $R_1 - r$ and centre F.

Sometimes it is desirable to produce the tangents $A\,K$ and $D\,L$ to their intersection in W and to describe a compound curve from two centres within this angle, $A\,W\,D$, according to the method already described.

If the two curves spring from the straight in **opposite** directions, then as laid down in the "Technische Vereinbarungen" the length of this intermediate straight between the superelevation inclines of the outer rail must be at least 10^m. This rule holds both for Secondary and Local lines.

On the Gotthard Railway* the length between the two superelevation-ramps of the outer rail in intermediate straights between reverse curves was fixed at 40^m. This length which holds for the sharpest curves also may be decreased for curves of greater radii. For curves of more than 2000^m radius the intermediate straight may be wholly omitted.

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* Helwag: Die Bahnachse und das Längenprofil der Gotthardbahn. Zurich: 1876.

§ 20.

Transition-Curves. *

Transition-curves depend on the amount of the superelevation and on the length or distance in which the superelevation tails out: accordingly rules are required for locating these curves.

In § 17 of the "Technische Vereinbarungen" it is laid down that

> "In curves the outer rail is to be superelevated above the inner one by an "amount, depending on the maximum velocity permitted on the section of the "road in question, such that the wheel-flanges shall attack the inner sides of the "rail as little as possible.
>
> "The superelevation of the outer rail shall have its full amount at the "commencement of the curve. The superelevation is to run out to zero in the "straight or in the parabolic transition-curve in a distance which must not be "less than 200 times the amount of the superelevation.

The above Regulations apply equally to Secondary lines and to Local lines.

It appears desirable to extend the length of the superelevation-ramps beyond what the "Technische Vereinbarungen" has fixed as the minimum distance. On the Gotthard Railway the length of these ramps, and consequently, the length of the transition-curves, on the mountain sections on grades of ·026, was fixed at 40^{m}; and consequently, with a superelevation of 100^{mm} in curves of 300^{m} radii, the superelevation-ramp was 400 times the superelevation For the approach lines on grades of ·010 the length of this ramp was made as much as 80^{m}, and consequently for a superelevation of 150^{mm} in curves of 300^{m} radius, it was $533\frac{1}{3}$ times the superelevation. In curves of greater radii the transition-curves were reduced in length, and in curves of 2000^{m} radius they were omitted altogether.

The superelevation required by the "Technische Vereinbarungen" to reduce the grinding action of the flanges on the inner-sides of the rails is a quantity, h, which has the same ratio to the gauge as the centrifugal force C to the weight Q of the vehicle, namely

$$h = \frac{C}{Q} m$$

Or since for a velocity of v m/sec. and for radius R^{m}

$$C = \frac{Q}{g} \frac{v^2}{R}$$

therefore $$h = \frac{m v^2}{g R}$$

Putting the width of gauge, $m = 1{\cdot}5^{m}$, acceleration of gravity $g = 10$, (approx.)

then $$h = \frac{{\cdot}15 v^2}{R}$$

For the maximum velocity of 90 kg/hr. or 25 m/sec., the superelevation would be $\frac{94}{R}$: for lines having no express service and a max. velocity of 60 km/hr. or $16\frac{2}{3}$ m/sec. it would be

$$h = \frac{40}{R}$$

and on Local lines and for velocity of 20 km/hr. or $5\frac{5}{9}$ m/sec.

$$h = \frac{4{\cdot}5}{R}$$

and generally, $$h = \frac{c}{R}.$$

[* See Talbot: "The Railway Spiral:" and Crandall: "The Transition Curve—by offsets and deflexion angles" New York: Wiley. 1899. These are probably the best works extant in English on the subject. See also "The Engineering News," New York, *passim* for several years past.—Tr.]

Usually the superelevation is run out on a straight ascending ramp or incline on the outer rail, the inner rail being unaltered in position. But it is preferable to raise the outer rail, as is done on the Gotthard Railway, by only the half superelevation and to depress the inner rail by the same amount. The advantage of this is that during motion the height of the centre of gravity of the vehicle remains unchanged.

If there be no transition-curve the superelevation is carried out wholly in the adjoining tangent, so that at the commencement of the curve there shall be the full amount of superelevation. But when a transition-curve is employed half the superelevation occurs at the original initial point of the curve.

The form of the transition-curve is determined by the condition that the radius ρ shall be everywhere and at each point that corresponding to the actual rail-superelevation at the point as given by the expression

$$z = \frac{c}{\rho}$$

and accordingly at the initial point of the ramp $\rho = \infty$, and at the junction of the transition with the circular curve, $\rho = R$.

If in—**Fig. 5**—C be the centre of the circular curve $B\ D$ of radius $C\ B = C\ D = R$, and $A\ B$ the tangent to the same, then in order to insert the transition-curve $E\ J\ P\ D$ the tangent must be displaced outwards by a certain amount $A\ E = B\ K = G\ F = U$ (the so-called "shift"). Taking this new position of the tangent as the axis of x, and neglecting the difference in length between arc and chord—which in the present case is perfectly permissible— then at the point P of the transition-curve the superelevation is

$$z = \frac{x}{l}\,h$$

and to determine the curve radius at this point P the condition

$$\frac{c}{\rho} = z = \frac{x}{l}\,h$$

must be fulfilled, viz.

$$\frac{1}{\rho} = \frac{h}{c\,l}\,x.$$

For brevity put

$$\frac{h}{cl} = \frac{1}{q}; \text{ then } \frac{1}{\rho} = \frac{x}{q}.$$

Since for Main lines the coefficient c for rail-superelevation generally lies between 30 and 60, and the gradient of the superelevation-ramp between $\frac{1}{200}$ and $\frac{1}{400}$, q will be between 6000 and 24000 for Main lines, and for Local lines between 1000 and 2000.

Putting $\frac{1}{\rho}$ as approximately $= \frac{d^2 y}{d x^2}$

then

$$\frac{d^2 y}{d x^2} = \frac{x}{q}$$

from which, after twice integrating and putting the constants $= o$, is obtained

$$y = \frac{x^3}{6\,q} \qquad \dots \qquad \dots \qquad \dots \quad (18)$$

This is the equation to a cubic parabola, and is admittedly only an approximately correct form of transition-curve, but is sufficiently correct for all practical purposes.

The end-ordinate of the transition-curve $y_1 = D\ F$ is thus

$$y_1 = \frac{1}{6\,q}\,l^3$$

whereas the ordinate of the circular curve at this point referred to the original position of the tangent $A\ B\ G$ is

$$y_2 = D\,G,$$

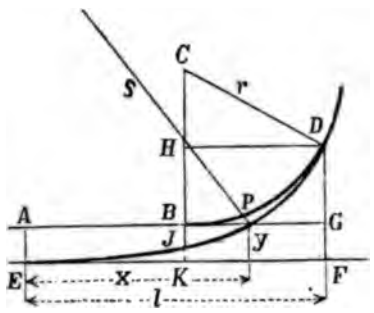


Fig. 5.

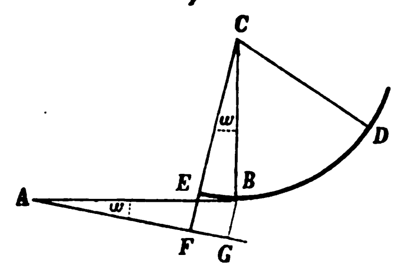


Fig. 6.

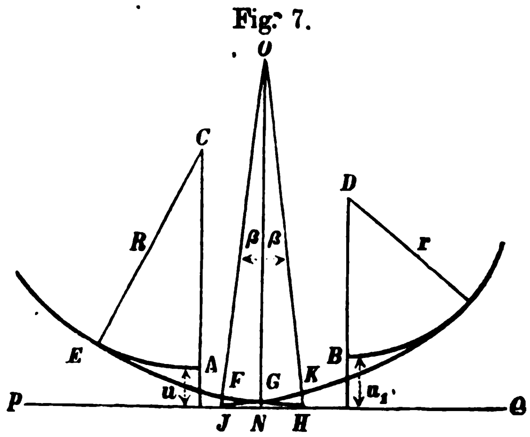


Fig. 7.

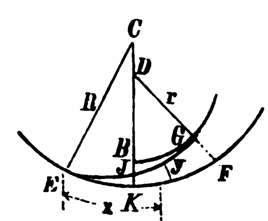


Fig. 8.

and since $B\,G = H\,D = \frac{l}{2}$, we have sufficiently accurately

$$y_2 = \frac{l^2}{8\,R}$$

and since $\frac{1}{R} = \frac{l}{q}$, the above may be written

$$y_2 = \frac{l^3}{8\,q}$$

Thus in order to lay-in the transition-curve the original tangent must be shifted outwards by an amount

$$u = y_1 - y_2$$

or

$$u = \frac{l^3}{24\,q} \qquad \ldots \qquad \ldots \qquad \ldots \quad (19)$$

or since

$$l = \frac{q}{R}$$

$$u = \frac{q^2}{24\,R^3} \qquad \ldots \qquad \ldots \qquad \ldots \quad (20)$$

The ordinate $J\,K$ of the transition-curve at the original initial point of the circular curve is given for $x = \frac{l}{2}$, viz.

$$J\,K = \frac{l^3}{48\,q} = \frac{u}{2}$$

in other words, the transition-curve opposite to the original initial point of the circle lies half-way between the circle's tangent and the transition-curve.

For example—

let $s = 50$, $\frac{h}{l} = \frac{1}{300}$, and thus $q = 15000$,

the equation to the transition-curve is $y = \frac{x^3}{90000}$:

whence for	$x = 10^m$	$y = {\cdot}011^m$
	$= 20^m$	$= {\cdot}089^m$
	$= 30^m$	$= {\cdot}300^m$
	$= 40^m$	$= {\cdot}711^m$
	$= 50^m$	$= 1{\cdot}389^m$
	$= 60^m$	$= 2{\cdot}40^m$

The "shift" outwards of the tangent is $\frac{l^3}{360000}$.

Thus for a circular arc of 500^m radius,

when $l = 300 \times \frac{50}{500} = 30^m$ ... $u = {\cdot}075^m$

For a circular curve of 375^m radius

where $l = 300 \times \frac{50}{375} = 40^m$ ... $u = {\cdot}178^m$

For a circular arc of 300^m

$l = 300 \times \frac{50}{300} = 50^m$, $u = {\cdot}347^m$.

And for a circular curve of 250^m

or $l = 300 \times \frac{50}{250} = 60^m$ ... ... $u = {\cdot}600^m$

Instead of moving the tangent into a position parallel to its original one it may be sometimes preferable to rotate it about some point in itself.

Suppose the tangent $A\,B$—**Fig. 6**—be rotated about the point A, then the initial point of the curve moves through the angle ϖ from B to E, *i.e.*, through the distance $R \sin \varpi$. $E\,F$ is then u and therefore

$$B\,G = u + R\,(1 - \cos \varpi)$$

or approximately,

$$B\,G = u + \frac{R \sin^2 \varpi}{2}.$$

The angular rotation ϖ must satisfy the equation

$$\sin\varpi = \frac{BG}{AB}$$

or, putting $AB = \lambda$,

$$\sin\varpi = \frac{u}{\lambda} + \frac{R}{2\lambda}\sin^2\varpi$$

whence

$$\sin\varpi = \frac{\lambda}{R}\left(1 - \sqrt{1 - \frac{2uR}{\lambda^2}}\right)$$

for which may be substituted in most cases—if λ be not too small,

$$\sin\varpi = \frac{u}{\lambda} \qquad \ldots \qquad \ldots \qquad \ldots \quad (21)$$

Whence the "shift" of the curve's initial point from B to E is

$$\frac{uR}{\lambda}$$

Instead of either a lateral translation or a rotation about a point in itself the position of the tangent may remain unchanged, the curve's centre being moved inwards on the bisector of the central-angle α of the circular arc by the amount

$$\frac{u}{\cos\frac{\alpha}{2}}$$

causing a displacement of the initial point of the curve of

$$u\tan\frac{\alpha}{2}.$$

Finally, while maintaining the positions of the tangent and of the centre of the curve unchanged, the length of the radius may be diminished to allow of the insertion of a transition-curve. Decreasing the radius R by the quantity u has no noteworthy effect on the form of the transition-curve.

The length of the transition-curve

$$l = \frac{q}{R}$$

increases with a decrease of the radius R, so that there are certain cases of small radii and small central-angles in which the length of the circular curve is not sufficiently long to completely effect the transition, since there must be a piece of the circular-curve between the ends of the transition curves in order that the superelevation-ramps of the outer rail may not meet in a point.

In such cases there is nothing but to increase the length of the radius of the circular curve, which can easily be done with short curves.

For instance: if, the coefficient $q = 15000$, and the central-angle $\alpha = 8°$, then for a radius of 300m the length of the circular arc would be 42m, whereas the length required by the transition-curve—the two halves of which take the place of the circular-curve—is 50m.

By an increase of the curve-radius to 350m the length of the circular arc becomes 49m and that of the transition curve 42·9m, so that now between the apexes of the superelevation-ramps there is a piece of circular arc of 6·1m. Increasing the curve-radius from 300 to 350m would, were the position of the tangent unchanged, only result in a displacement of the apex of the curve by ·135m inwards, and hence would not present any practical difficulties.

Between reverse-curves a somewhat longer intermediate-straight should be arranged for when locating to provide for the insertion of transition-curves, say $1\frac{1}{2}$ times as long as the average length of the two transition-curves.

If the intermediate straight is to remain unchanged in position as a tangent to the two circular-curves then either the centres of both must be shifted, or their radii must be decreased. It is generally simpler to rotate the short intermediate-straight through an angle ϖ which is given by the expression

$$\lambda\sin\varpi = u + u_1 + (R + R_1)(1 - \cos\varpi)$$

which may be replaced by

$$\lambda \sin \varpi = u + u_1 + \left(\frac{R + R_1}{2}\right) \sin^3 \varpi$$

or in most cases sufficiently accurately by

$$\sin \varpi = \frac{u + u_1}{\lambda} \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

where λ is the length of the intermediate-straight between the original initial points of the circular-curves, u and u_1 are the requisite shifts of the tangents of the two circular-curves, and R, R_1 the radii of the said curves.

The initial points of the circular-curves owing to the rotation of the intermediate-straight approach each other by the distances $R \sin \varpi$ and $R_1 \sin \varpi$—**Fig 6.**

For example—

let $R = 300^m$, $R_1 = 250^m$ $q = 15000$, $u = \cdot 347$, $u_1 = \cdot 600$, $\lambda = 75$,

then $\sin \omega = \cdot 126$,

and the decrease in length of the intermediate-straight is

$$250 \times \cdot 126 + 300 \times \cdot 126 = 6{\cdot}93^m$$

so that between the initial points of the 50^m and 60^m transition-curves there is only a straight of length

$$75 - \frac{50 + 60}{2} - 6{\cdot}93 = 13{\cdot}07^m$$

If two 'curves' of radii R and r be on the the same side of an intermediate-straight of but small length the transition-curves may intersect each other at an obtuse angle, as shown in **Fig. 7.**

But this angle may be replaced by a curve in which the rail-superelevation is constant. Thus, the centres of the circular-curves C and D are first moved inwards from the intermediate $P\,Q$ by the amounts respectively of

$$u = \frac{q^2}{24\,R^3} \quad \text{and } u_1 = \frac{q^2}{24\,r^3}$$

The point of the intersection, G, of the two transition-curves then falls in the middle between the initial points J and H, which points are distant from each other, $J\,H = x$.

The radii of the transition-curves are equal at the curve-points F and K of which the abscissœ are J and H, namely

$$\rho = \frac{q}{x}.$$

Also the angle of the tangents at these points and therefore, the angle of the curve-radius with the vertical is the same, viz,

$$\tan \beta = \frac{dy}{dx} = \frac{x^2}{2q}.$$

The projection of the transition-curve radius on the axis of abscissœ, *i.e.* $\rho \sin \beta$, can with sufficient exactitude be written

$$\rho \tan \beta = \frac{q}{x} \cdot \frac{x^2}{2q} = \frac{x}{2}.$$

Accordingly, the curve-radii, ρ, of both curve-points F and K meet in a point O from which a circular arc may be described meeting the transition-curves tangentially in the points F and K. On this connecting arc the superelevation-ramps, which do not extend to its end, are joined up by a length on which the superelevation is constant.

In proportion as the intermediate-straight is made shorter the point F, in which the junction-curve joining the two transition-curves begins, moves nearer to the point E in which the transition-curve meets the circular-curve of radius R, until in the limiting position the two points coincide, and the centre O of the junction-curve coincides with the centre C of the circular-curve, and its radius becomes R. There then remains only a single transition-curve

forming the transition from the circular arc of radius R to that of radius r, and thus the transition-curve between the two circles becomes a compound-curve.

The length of the transition-curve is then

$$q\left(\frac{1}{r}-\frac{1}{R}\right).$$

The point of commencement B of the smaller circular-curve of centre D and radius r—**Fig. 8**—must then be sprung back, as it were, towards the end K of the larger circle of centre C and radius R by the amount $B\ K$, or

$$u=\frac{q^2}{24}\left(\frac{1}{r}-\frac{1}{R}\right)^3 \quad \ldots \quad \ldots \quad .. \quad (23)$$

and the half $E\ J$ of the transition-curve $E\ J\ G$ then replaces the curve of the larger radius, and its half $J\ G$ replaces the arc of the smaller radius. The ordinates of the transition-curve are—from Eqn. 18—

$$y=\frac{x^3}{6q}$$

measured inwards from the larger curve on which, with E as origin, the abscissæ are laid-off. At the transition-point of both circles, the transition-curve at the point J is equally distant from the two circles.

For example:

if $\quad r = 300^m,\ R = 1000^m, \quad q = 15000;$

then $\quad n = \frac{15000^2}{24}\left(\frac{1}{300}-\frac{1}{1000}\right)^3 = \cdot 119^m$

and the length of the transition-curve is

$$15000\left(\frac{1}{300}-\frac{1}{1000}\right) = 35^m.$$

The ordinates, to be laid-off inwards from the circle of radius 1000^m, on which latter the abscissæ are to be measured, are—from Eqn. 18—

$$\begin{aligned} y &= \cdot 011^m, \text{ for } x = 10^m \\ y &= \cdot 060^m, \quad x = 17\tfrac{1}{2}^m \\ y &= \cdot 174^m, \quad x = 25^m \\ y &= \cdot 477^m, \quad x = 35^m. \end{aligned}$$

Helmert in his work "Railway Transition-curves"* treats exhaustively of the calculation and laying out of transition-curves for all possible cases; thus he gives for a case where the circular-arcs of the compound-curve meet tangentially with one another ten methods of inserting transition-curves. But the transition-curves thus obtained are of little value for practical use, since they are very long and made up of three different parts. Either the transition-curve springs from the circular-curve of greater radius outwards, with an increase of the curve radius and decrease of the rail-superelevation, passes into a circular-curve with larger radius than R and with constant height of superelevation, and from this again with decrease of curve-radius and increase of superelevation into the circular-arc of radius r; or, the transition-curve springs from the circle of radius r inwards with decrease of radius and increase of rail elevation, then passes into an arc of smaller radius than r and with constant rail-superelevation, and from this again with increasing radius and decreasing super-elevation into the arc or of radius R.

Manifestly, for practical use the most suitable transition-curve is one which leads from the circle of smaller radius to that of the larger one with a continuous increase of its radius, which is neither smaller than r nor greater than R; and with a rectilinear and decreasing superelevation longitudinally. This, the most simple and most effective arrangement, is attained if the initial and final points of both circles are moved towards each other in the direction of their radii in the manner indicated in Eqn. 23.

* Aachen, 1872, bei J. A. Meyer.

Fig. 9.

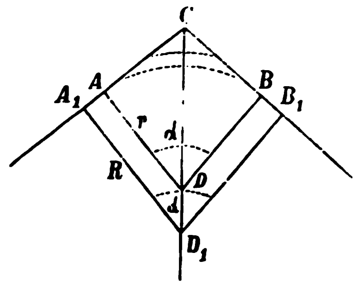


Fig. 10.

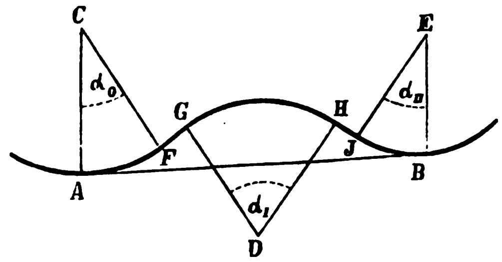
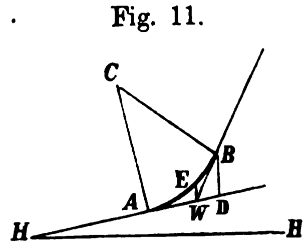


Fig. 11.



§ 21.

The Choice of Curve-Radius.

As regards the magnitude of the curve-radius the "Technische Vereinbarungen" lay down, § 3, that:—

> "The radii of curves in the line outside stations are to be as large as "possible. Radii under 300^{m} are only exceptionally permissible. Curves of "less than 180^{m} are in **no** case permissible. On steep grades the curves, if "any, are to be as flat as possible and change of grade should be effected as "far as practicable in the straight."

The above Regulation applies also to Secondary lines whereas the "Regulations for the Construction and Working of Local lines" lay down that

> "The radius of curves on normal track, on which Main-line vehicles exclusively run, shall not be less than 150^{m}, and in **no** case less than 100^{m}: "and on lines of narrow-gauge the curve-radius shall be of a length suited to "the width of gauge and the rolling-stock, and as a rule for 1^{m} gauge shall be "not less than 70^{m}; and for a $\cdot 75^{m}$ gauge the radius must not be less than "50^{m}.
>
> "On Normal-gauge lines having **special** types of rolling-stock a smaller "radius suitable to such stock may be employed."

In order to determine the working-expenses due to curvature it is necessary to know whether the curves occur on "injurious" or "non-injurious" grades since, as was shown in § 10, the increase in working-expenses due to curves on "injurious" grades, where they only occasion an increased consumption of steam in one direction are only half as large as on "non-injurious" grades under equal conditions.

If the radius r of a curve of central-angle α be increased to R, the length of the curve is increased from αr to αR, but the work done in overcoming the curve-resistance is constant, and is $\frac{c}{r} \alpha r = \frac{c}{R} \alpha R = c \alpha$.

The trace, however, is shortened—see **Fig. 9**—by

$$\lambda = (R - r)\left(2 \tan\frac{\alpha}{2} - \alpha\right)$$

If the maintenance-expenses per km. $= U$ the working-expenses per paying-load tonne-km. are, according to Eqn. 10, in pfennigs,

$$k = \cdot 56 + 23\tfrac{1}{3}\, s + 39\tfrac{1}{3}\, s_1$$

and per passenger tonne-km.—from Eqn. 11 — they are

$$k = \cdot 973 + 10\tfrac{2}{3}\, s + 29\tfrac{1}{3}\, s_1$$

in which s is the ruling gradient of the line, and s_1 the gradient of the particular section of the line under examination.

If T tonnes of paying-load and P passengers are carried annually then the maintenance- and working-expenses on the flatter curve from A_1 to B_1 of length l_1 and grade s_1 is, neglecting curve-resistance,

$$S_1 = l_1\left[U + T(\cdot 56 + 23\tfrac{1}{3}\, s + 39\tfrac{2}{3}\, s_1) + P(\cdot 973 + 10\tfrac{2}{3}\, s + 29\tfrac{1}{3}\, s_1)\right]$$

And if the difference of height between A_1 and B_1 is h, then $l_1\, s_1 = h$, and

$$S_1 = l_1\left[U + T(\cdot 56 + 23\tfrac{1}{3}\, s) + P(\cdot 973 + 10\tfrac{2}{3}\, s)\right] + (39\tfrac{2}{3}\, T + 29\tfrac{1}{3}\, P)\, h.$$

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The same items of cost on the length $A_1\ A\ B\ B_1$—**Fig. 9**.—with the sharper curve are

$$S_2 = l_2 \left[U + T\,(\cdot 56 + 23\tfrac{1}{3}\,s) + P\,(\cdot 973 + 10\tfrac{2}{3}\,s)\right] + (39\tfrac{2}{3}\,T + 29\tfrac{1}{3}\,P)\,h.$$

Consequently, the saving due to the sharper curve is

$$E = (l_2 - l_1)\left[U + T(\cdot 56 + 23\tfrac{1}{3}s) + P(\cdot 973 + 10\tfrac{2}{3}s)\right] \quad \ldots \quad (24)$$

or, inserting the previously-given value for the shortening, $\lambda = l_2 - l_1$

$$E = (R - r)\left(2\tan\frac{\alpha}{2} - \alpha\right)\left[U + T(\cdot 56 + 23\tfrac{1}{3}s) + P(\cdot 973 + 10\tfrac{2}{3}s)\right] \quad (25)$$

This saving is thus entirely independent of the grades of the section. So long as the saving is larger than the interest on the extra capital-outlay in flattening the curve this flattening is advantageous.

For example: suppose $U = 3{,}000$ M. $= 300{,}000$ pfennigs, $T = 600{,}000$ tonnes, $P = 400{,}000$ passengers, $s = \cdot 01$, $\alpha = 60^\circ = 1\cdot 047$ in circular measure, $\tan\dfrac{\alpha}{2} = \cdot 577$

then the saving in Marks would be

$$E = 1\cdot 29\,(R - r)$$

By employing a radius $R = 400^m$ instead of $r = 300^m$, thus shortening the line by $10\cdot 7^m$, there would be a saving in maintenance- and working-expenses of 129 M. If the capital-cost of flattening of the curve were not greater than $\dfrac{129}{\cdot 04} = 3{,}225$ M. then this flatter curve of 400^m would be the better investment.

If the central-angle of the above curve were 90° instead of 60°, and its length consequently $1\tfrac{1}{2}$ times greater, then the saving by increasing the radius would be 4 times as great. But were the central-angle 30° instead of 60°, and the curve thus half as long, then the saving would only be $\tfrac{1}{8}$th.

Consequently, it is more advantageous to **flatten curves of large central-angle** than it is to flatten those of small central-angle; and further, the heavier the traffic or the larger the ruling gradient the more imperative the flattening.

In the foregoing the circumstance that the **wear of rails and wheel-tires** increases very largely as the curve radius is diminished has not been taken account of because at present available data are wanting. However, in view of this fact the advantage of flattening curvature is considerably greater than that arising from shortening the line.

On mountain railways, where development of the line will usually be unavoidable, it is only the saving in the wear of rails and wheel-tires that has any weight in diminishing curve-radius, since the shortening of the trace has to be made up for in other parts of the line.

Since in hilly sections the tractive-force of the locomotive is fully worked-up to, the ruling gradient of the straights must in curves be diminished by the amount of the curve-resistance, $c = \dfrac{1}{r}$.

In any curve of length $\lambda = \alpha\, r$ the height surmounted is

$$h_1 = \alpha r\left(s - \frac{1}{r}\right) = \alpha r s - \alpha$$
$$= \lambda s - \alpha$$

And since $\lambda\, s = h$ is the height that would be surmounted if the length of the curve were so much straight line, the height h_1 surmounted in the curve is consequently less than that surmounted in an equal distance on the straight by the amount of the central-angle expressed in circular measure.

If the gradient, s, of the straight be diminished in curves by the amount of the curve-resistance then the length of trace requisite for the surmounting of a given height is independent of the curve-radii. If α be the sum of the central-angles of all the curves expressed in circular measure, then the length of the trace is increased by $\frac{\alpha}{s}$ metres; or if the sum of the central-angles is $\alpha°$, by $\cdot 000018 \frac{\alpha}{s}$ km. The curve radii are therefore simply to be so chosen that the sum of the interest of the construction-cost of the line and of the cost of wear of rails and wheel-tires shall be a minimum.

But both in location and in the working of the line it is a nuisance to have changes of grade at the commencement and end of curves; accordingly in *flat* curves the gradient of the straight is frequently continued unaltered into the curve.

In this case to obtain the ruling gradient the resistance of the sharpest curve in which the gradient of the straight is continued unchanged is to be added to the latter gradient.

The radius of this curve may be termed the **normal** or **ruling radius.**

In curves of smaller radii the gradient is simply reduced by the amount of the difference of resistance in the curve and in curves of the ruling radius.

For example: suppose a gradient of ·025 were carried out in curves of 500^m radius (lowest limit), then since the curve-resistance $c = \frac{1}{500} = \cdot 002$, the ruling gradient would consequently be

$$s = \cdot 025 + \cdot 002 = \cdot 027.$$

In sharp curves, for example of 400^m radius, ruling gradient would be reduced to

$$\cdot 027 - \frac{1}{400} = \cdot 0245;^*$$

and in curves of 300^m to

$$\cdot 027 - \frac{1}{300} = \cdot 0227.$$

However, the advantage arising from diminishing the number of changes of grade in the line which the use of a normal radius gives is purchased at a quite disproportionate cost.

If the trace can be located without prolonging its length then the adoption of a normal radius of r_0^m increases the ruling gradient by $\frac{1}{r_0}$, and consequently—according to the formula of § 13 for the working-expenses, for T tonnes of paying-load and P passengers per annum per km. of the section—increases the working-expenses by

$$V = \frac{1}{100}\left(29\tfrac{1}{3}\,T + 10\tfrac{2}{3}\,P\right)\frac{1}{r_0}\,\text{M}.$$

Thus for example: if $T = 400000$, $P = 300000$, $r_0 = 500$,

$$V = 300 \text{ M. per km.}$$

The adoption of a normal radius if a given height h has to be surmounted by lengthening of the line is still more costly.

If s_1 be the stiffest gradient ever in a curve of the normal radius r_0 then the ruling gradient is

$$s = s_1 + \frac{1}{r_0}$$

In the straights or "tangents" of which the total length is l_1 a height $h_1 = s_1\, l_1$ is surmounted; whereas had the ruling gradient s_1 been used therein the height surmounted would have been $h = s\, l_1$.

---

* Otherwise thus: curve of 500^m, resist. = ·002, gradient = ·025
„ „ 400^m, „ = ·0025 „ = ·0005
Diff. = ·0005. ∴ gradient = ·0245.

In a curve of radius r_1 greater than the normal radius r_0 and of which the central-angle in degrees is α_1 the height surmounted is, in km.

$$h_2 = \cdot 000018\,\alpha_1\,r_1\,s_1$$

whereas if the gradient $s - \frac{1}{r_1}$ be used a height might be surmounted of

$$h_3 = 000018\,\alpha_1\,r_1\left(s - \frac{1}{r_1}\right)$$

Thus in this curve there would be an extra height surmounted of

$$\cdot 000018\,\alpha_1\,r_1\,(s - s_1) - \cdot 000018\,\alpha_1$$

or putting the length of this curve $\cdot 000018\,\alpha_1\,r_1 = l_2$, the additional height gained would be

$$l_2\,(s - s_1) - \cdot 000018\,\alpha_1$$

In the tangents the additional height could be attained of

$$l_1\,(s - s_1).$$

Putting α for the sum of the central-angles of all curves having a greater radius than the normal radius, and l, for the total length of all these curves and of all the straights then abandoning the employment of a normal radius an additional height could be attained of

$$h_0 = l\,(s - s_1) - \cdot 000018\,\alpha.$$

As the height to be surmounted is given, the development-length might be diminished by an amount, corresponding to this additional height, of

$$\lambda = \frac{h_0}{s}$$

or by

$$\lambda = l\left(1 - \frac{s_1}{s}\right) - \cdot 000018\,\frac{\alpha}{s}$$

For this length of line the annual interest on capital-cost, maintenance of way, and the annual working-expenses depending on the height of the ascent might be saved. The saving for T tonnes of paying-load and P passengers in M. per km. amounts to

$$\frac{T}{100}\left(\cdot 56 + 23\tfrac{1}{3}\,s\right) + \frac{P}{100}\left(\cdot 973 + 10\tfrac{2}{3}\,s\right)$$

Accordingly, if instead of adopting a normal radius in all curves the ruling gradient were diminished by the amount of the curve-resistance the total saving would be

$$E = \left[Ai + U + \frac{T}{100}\left(\cdot 56 + 23\tfrac{1}{3}\,s\right) + \frac{P}{100}\left(\cdot 973 + 10\tfrac{2}{3}\,s\right)\right]\left[l\left(1 - \frac{s_1}{s}\right) - \cdot 000018\,\frac{\alpha}{s}\right] \quad (26)$$

For example: suppose a gradient $s_1 = \cdot 025$ be carried unaltered through curves of 500m radius—thus giving a ruling gradient of

$$s = \cdot 025 + \frac{1}{500} = \cdot 27$$

Then if the total length l of the straights and curves of radii greater than 500m = 15 km., the sum of the central-angles of these curves = 400°, and the goods traffic = 40,000 tonnes paying-load and 300,000 passenger, also the km.-cost of construction A = 300,000 M., the maintenance per km., U = 3,000 M., and the rate of interest $i = \cdot 04$—we obtain — Eqn. 26—

$$E = 19{,}884 \text{ M.}$$

Accordingly, on the basis of the above calculation, it may be asserted that: **The practice of employing a normal radius is wrong; instead, the maximum grade fixed for straights should be reduced in curves by the amount of the curve-resistance.**

However, it must here be remarked that the above investigation is not quite exact for very flat curves, because in these curves the resistance is less than $\frac{1}{R}$. For curves of

radii greater than **1000^m** or 1200^m the increase in **train-resistance is scarcely appreciable**; so that these curves and all flatter ones should be treated in the calculations as simply straight lines.

Curves should always be avoided where possible, and when employed their use should be justified by a saving in the cost of construction.

Where **development** of the line is necessary there is an additional disadvantage arising from the presence of curves, in addition to the wear of rails and wheel-tires which cannot be exactly numerically determined, viz. the increase in train-resistance and consequent increase in working-expenses. If by flattening the inclination of the straights to each other, (and so diminishing the lateral width occupied by the location,) the central-angle of a curve of radius r and, consequently, its length can be diminished by α° and $\cdot 000018\, \alpha\, r$ km., respectively, then in the transport of a tonne there is saved the work represented by

$$\cdot 000018\, \alpha\, r \frac{1}{r} = \cdot 000018\, \alpha \text{ tonne-km.}$$

And since the locomotive-expenses for the performance of a tonne-km. of work are $\frac{1}{4}$ M., the saving in working-expenses is

$$\cdot 0000045\, \alpha \text{ M.}$$

This saving accrues only in the up-hill journey on the sections of the line having "injurious" grades—which latter must always occur in a line artificially developed; and therefore for the journeys in both directions is about

$$\cdot 00000225\, \alpha \text{ M. per tonne.}$$

For the transport of a million tonnes—including the weight of the locomotive—the annual saving for a reduction of α° in the sum of the central-angles of the curves is $2{\cdot}25\, \alpha$ M.

On lines which are located **without development,** whenever a curve is avoided there is beside the decrease in the train-resistance the further advantage of a shorter line. Thus comparing the straight line $A\,B$ of length l_1—**Fig. 10.**—with the curved line $A\,F\,G\,H\,J\,B$ of which the length l_2 is greater by λ than the straight $A\,B$, there is, firstly, the saving in maintenance and in working-expenses independent of curve-resistance and of ascending grades given by Eqn. 24, and, secondly, the advantage due to getting rid of the curves $A\,F$, $G\,H$, and $J\,B$, of which the sum of central-angles $\alpha_0 + \alpha_1 + \alpha_2 = \alpha$.

If the sum of these angles is expressed in degrees then the curve-resistance in tonne-kms. for a total gross-load Q tonnes is

$$\cdot 000018\, \alpha\, Q.$$

Since the performance of a tonne-km. by the locomotive costs $\frac{1}{4}$ M. then by the curves there is a saving of

$$\cdot 0000045\, \alpha\, Q \text{ M.}$$

If the section of the line is on "non-injurious" grades then this saving is gained for the traffic in both directions; whereas for sections on "injurious" gradients this is only so on the up-journey, and therefore the saving is only the half.

The total saving in maintenance and working-expenses due to the cutting-out of curves is therefore

$$E = \lambda \left[U + \frac{T}{100} (\cdot 56 + 23\tfrac{1}{3} s) + \frac{P}{100} (\cdot 973 + 10\tfrac{2}{3} s) \right] + \cdot 0000045\, \alpha\, Q \ldots \qquad \ldots (27)$$

and when the section is on an "injurious" gradient the last term is to be halved.

For example: Let $s = \cdot 0027$, $T = 400{,}000$, $P = 300{,}000$, $u = 3000$, Q, including weight of engine $= 2200000$, $\lambda = \cdot 03$ km., $\alpha = 100°$,

then $E = 841$ M.

The line occupying less ground laterally would thus be preferable even if the construction-cost of carrying it out were increased by

$$\frac{841}{\cdot 04} = 21{,}000 \text{ M.}$$

§ 22

The Vertical Elevation or Grading of the Trace.

For the grading of the trace the "Technische Vereinbarungen" lay down the following Regulations which hold equally for Secondary lines.

§ "2. The maximum gradient in **Main** lines shall as a rule not exceed 1 : 40. "The change of grade is to be effected during construction by as flat "curves as possible. Between anticlinal or between synclinal grades of 1 : "200 and above, an approximately level piece, equal if possible to the "length of a goods-train, is to be inserted.

§ "9. The crown-width at the height of the under-sides of the rails shall,— "except for the line in dyked land,—as a rule be laid at least 600^{mm} above "the highest-known flood-level. The under-side of the ballast shall under "all circumstances be so situated as to be completely drained.

§ "53. Stations shall, as a rule, be on straights and on the level; and in no "case—excepting sorting sidings, branch-tracks, switches, and fly-sidings— "on a stiffer grade than 1 : 400.

"The length of stations shall be made sufficient to accommodate of the longest "train in use on the line. Where very long trains have to cross each other, "the end switches may be laid out on grades stiffer than 1 : 400."

For **Local** lines the "Grundzüge" of 1887 lay down as follows :—

§ "2—Grades as a rule are not to exceed 1 : 40 : any greater than 1 : 25 are to "be discouraged.

"Changes of grade are to be effected by circular arcs of not too small radius. "When it is practicable without difficulty such a radius should be at least "5000^{m}, and as a general rule 1500^{m} should be the minimum. Only at "level-crossings in stations is a radius of 1000^{m} permissible. Between "reverse grades intermediate lengths of level are desirable but not imperative.

§ "11— It is recommended that the crown-width of the road at the height of "the under-side of the rails be placed above the ordinary flood-level; it "may be placed without any hesitation below extra-ordinary and rarely-"occurring flood-levels.

§ "32. It is recommended that the gradients in stations—except in the sidings "be not greater than 1 : 400 : for smaller intermediate stations and flag-"stations stiffer grades are permissible."

The degree of the longitudinal inclination of the trace is to be expressed in the form either of a vulgar fraction or of a decimal fraction, or in per cents., or in millimetres per running metre: so that, for example, a grade may be indicated either as $\frac{1}{40}$, ·025, $2\frac{1}{2}\%$, $25\,{}^{m}/_{m}$, or as $250\permil$.

The trace is to be located both in plan and in elevation as a series of straight lines connected at the apexes by curves as laid down by the Regulations.

If the grade s_1 passes into the grade s_2 at the point W—**Fig. 11**—then if R be the radius of the connecting curve, and making $A\ W = B\ W = t$,

$$B\ D = (s_2 - s_1)\ t, \quad \text{and } 2\ R.\ B\ D = 4\ t^2$$

whence
$$t = \frac{R}{2}\ (s_2 - s_1).$$

At the apex of the angle the amount of the rise $E\,W$ is sufficiently accurately given by

$$\frac{B\,D}{4} = \frac{s_2 - s_1}{4}\,t = \frac{R}{8}\,(s_2 - s_1)^2$$

Thus if the rounding-off curve-radius $R = 10000^m$, and the gradients $s_1 = \cdot 005$, $s_2 = \cdot 025$, then the length of the tangent or "tangential distance" of the rounding-off curve be would be $t = 100^m$, and $E\,W$ would be $= \cdot 5^m$.

When so large a radius for the rounding-off curve of the grades is adopted and when the difference in the grades is great, the rounding-off of the gradients must be done at the time of throwing up the earthwork. But if the change from one grade to another be less sharp then even with a large radius the rounding-off can be carried-out in the platelaying.

When grades are short the features of the ground can be more closely conformed to, and thereby the construction-cost diminished: but, on the other hand, the working of the traffic will be more difficult, since every change of grade, *if the velocity is to remain constant*, makes it necessary either to vary the admission of steam, or to vary the firing, or to employ brakes.

The minimum permissible distance between a change of grade is, accordingly, dependent on the *speed*, since the regulating of the locomotive tractive-force would not be more difficult on a Local line, at a 5^m speed over a length of grade of 500^m than on a line on which the velocity was 14^m in a distance between grade-changes of 1400^m, or than on a line with express service of 20^m velocity over a grade-length of 2000^m.

It was formerly the opinion that the length of the maximum grade on mountain lines should for safety of working be limited to a definite figure. On the Semmering Incline this was fixed for a grade of $25^m/_m$ at 3160^m, which distance was always to be succeeded by a level or slightly rising piece of 500^m. But nowadays, there is no hesitation in making gradients of much greater lengths, although the presence of stations frequently causes a break in the ascent. On the Gotthard Line on grades of $26^m/_m$ the stations occur at distances apart of 8 or 9 km.

While the grading of the trace is made in general consonance with the rules laid down in the "Technische Vereinbarungen" it is at the same time dependent on a variety of circumstances amongst which **the cost of works to sub-grade, the hydraulic conditions, the presence of roads, the position of the stations, the working of the traffic** have to be given their appropriate weight.

The cost of earthwork is usually, if not always, cheapest when the fills and cuts are made to balance each other. If there be a deficiency of earth of volume Q cbm. on a Section of part the line where the bottom area of all the banks is D ares, and the original upper-surface of all the cuttings is E ares, and where therefore recourse would have to be had to side-cuttings, then by lowering the trace in the bank-sections through an average distance of $\frac{Q}{D}$ cm. the necessity of having recourse to side-cuttings for earth is obviated, and the resulting equalization of the cuts with the fillsmakes a saving in cost by the amount of the side-cutting. Such a lowering of the trace over a section of the line, however, is not always possible. If it were desired to lower the trace uniformly throughout its whole length so as to balance the banks with the cuttings then the amount of lowering would be

$$x = \frac{Q}{D + E}\ \text{centimetres.}$$

If the expense of the borrow-pits per cbm.—inclusive of the labour in getting the earth and the cost of the land—were a, and the cost of the haul, inclusive of cost of excavation of the earth $= b$, then by lowering the trace x $^c/_m$ there would be saved a sum

$$K = Q\,a - E\,x\,b$$

or inserting the value of x

$$K = Q\left(a - \frac{E}{D+E}b\right) \quad \dots \quad \dots \quad \dots \quad (28)$$

As a rule the areas D and E are nearly equal, and therefore $\frac{E}{D+E}$ will approach $\frac{1}{2}$, and b will not be less than $2a$; whence the lowering of the trace sufficiently to produce a balance between the cuts and fills will in the majority of cases result in a saving.

If, on the other hand, the elevation of a trace is such that the earth from cuttings is in excess by Q cb. m. and cannot be made use of in banks and must therefore be led to spoil, then if the whole trace were so raised as to balance the earthwork in cuts and fills there would be a saving of

$$K_1 = Q\left(a - \frac{D}{D+E}b\right)$$

Omitting rare and exceptional cases in which a long haul is imperative in order to balance cuts and fills, or in which the earth from cuts can only be partially used in banks either because it is unsuited for the purpose or because it it more profitably employed, for example, as sand for mortar, ballast, stone, etc., or finally, when wide and deep pits are employed as fences along the banks, the elevation of the trace should be graded so as to balance cuts and fills.

Deep cuttings are, as to cost, advantageously replaced by **tunnels.** If h be the depth of the cutting, b its formation-width at the height of the bottom of the ballast, $1:m$ the slope of the sides a the price per cb. m. of earth including price of land and labour in getting and haulage, then the cutting per running-metre costs

$$a\,(b\,h + m\,h^2)$$

On the other hand, if a tunnel for a length l costs A per running-metre and its two portals P, then the cost per running-metre will be $\frac{P}{l} + A$. Equating the cost of cutting and tunnel the limiting depth is obtained beyond which a tunnel becomes cheaper than a cutting, viz.

$$h = -\frac{b}{2m} + \sqrt{\frac{b^2}{4m^2} + \frac{P}{a\,m\,l} + \frac{A}{a\,m}} \quad \dots \quad \dots \quad (29)$$

For example: if for a 2-track line $A = 1{,}200$ M., $P = 20{,}000$ M., $a = 1$ M, $l = 200^m$, $b = 12^m$, and $m = 1\frac{1}{2}$; then $h = 25{\cdot}7^m$. At this depth of cutting tunnelling should be commenced, but the length of the tunnel will not be obtained exactly until the value of h has been again determined, and accordingly the calculation may have to be repeated several times, inserting each time the exacter value of l.

But independent of all questions of cost, tunnels may be necessary as protections against landships, avalanches, cascades, falling stones, etc.

High banks are advantageously as regards cost replaced by **viaducts**: the limiting height of which lies—according to the local conditions and the type of building—between 20^m and 30^m. But when the price of land is high, the limit is much lower; and in general for a single-track line the limit is lower than for a double-track line.

There also are other grounds independent of the cost of construction which may justify the building of a viaduct: for instance, in the neighbourhood of fortresses military considerations might preponderate or, when within towns, those of esthetics.

As regards the **hydraulic conditions,** the "Technische Vereinbarungen" for Main lines lays down that the height of the crown-width should be at least $\cdot 6^m$ above the highest flood-level; whereas for Secondary lines no limit is fixed, but it is only required that the crown-width of bank be not lower than the highest flood-level. For Local lines it is considered sufficient if the crown-width lies above the highest flood-level usually occurring, thus rendering it always possible that the bridge may be flooded by extra-ordinary but rarely-occurring floods.

The height of the trace as fixed by these Regulations suffices also for the smaller waterways, which as **open culverts** spanned by the rails may, as necessity arises, be increased to openings of two or more spans. Where requisite such openings may be built as **syphons.**

Iron bridges require less height than stone bridges to span the same openings. The under-side of the girders should be at least 1^m to $1{\cdot}5^m$ above flood-level in the case of rapidly-flowing streams, and this space may be reduced to $\cdot 6^m$ for less-rapidly running streams, and for canals to $\cdot 5^m$ or in case of necessity even to $\cdot 3^m$. In the case of navigable waters when the masts and funnels of vessels can be lowered, a clear height, according to the type of vessel, of 3^m to 4^m above the highest navigable water-level is required. Where such heights of bridges as the above would cause considerable expense, or when the lowering of masts and funnels is out of the question, then the question of movable bridges presents itself. A clear height of 30^m is sufficient for the passage of the largest sea-going vessels.

The distance in height between top of rail and the under-side of girder may be limited to $\cdot 55^m$ or $\cdot 6^m$ in the case of small spans when the rails rest directly on cross-girders of $\cdot 4^m$ to $\cdot 45^m$ depth, or between the sides of a trough-grider over the top edge of which the rail-head projects some $\cdot 04^m$. Even with the largest gauge and with the ordinary fastening of the rail to the cross-sleepers, for iron bridges a height between the top of rail and the lower side of girder of $\cdot 65^m$, is sufficient and for double-track one of $1{\cdot}15^m$.

For **stone bridges** the springings of the arches are usually placed above the highest flood-level. With semicircular arches it is permissible to place the springings of arch about $\frac{1}{8}$th the span below high flood-level. In locating the trace it is necessary to consider the distance from the top of rail to the crown of the extracts of the arch and the thickness. The former, assuming a depth of ballast of $\cdot 40^m$ and a protection of two layers of bricks laid flat, is to be made at least $\cdot 7^m$ in thickness, and the thickness of arch is determined from the formula

$$d = \cdot 25^m + \left(\cdot 025 + \cdot 0034 \frac{w}{f}\right) w \quad \ldots \quad \ldots \quad (30)$$

where w = the span, and f = the rise of arch. The above formula is for ashlar: for brick or rough masonry it should be somewhat larger.*

A rise of $\frac{1}{6}$th or at most of $\frac{1}{8}$th of the span should not be exceeded except under the compulsion of necessity, because arches of small rise require strong and costly abutments.

Thus for a bridge 20^m span and $\frac{1}{6}$th rise a thickness or depth at the crown of $1{\cdot}16^m$ would be necessary, giving from the flood-level at the springing up to the rail-top a height of

$$\cdot 7 + 1{\cdot}16 + 3{\cdot}33 = 5{\cdot}2^m:$$

whereas for a single-track iron bridge a depth of the structure between top of rail and high water would suffice of

$$\cdot 65 + 1{\cdot}0 = 1{\cdot}65^m:$$

In **iron arch-bridges** when the height of structure has to be restricted the under-sides of the cross-girders may be placed at the height of the crown of the intrados, thus requiring between this and the top of the rail a distance of only $\cdot 65^m$: but, on the other hand, the springing must be placed some $\cdot 6^m$ to 1^m above high water; so that in comparison with stone bridges it is not possible to reduce the height of the structure by very much.

The ramps or approach-inclines requisite at either end of a bridge may be carried up to the crown of the first span in the case of both iron bridges and stone bridges: as has been done, for example, at the bridge over the Rhine at Coblenz.

* Conf. von Kaven: Kurze Anleitung zum Projektiren von Eisenbahnen, p. 61. Achen; 1878. Verlag von J. A. Meyer.

The gradient for these approach-ramps is to be determined according to § 24: or, where a momentum grade may be used, according to § 26.

Roads as a rule affect the grading of the trace in a less degree than water, since the elevation of the roads to be crossed by the line can often be altered more advantageously by rebuilding, and at a smaller cost, than by making a change in the height of our trace already fixed from other considerations.

Level-crossings on Main lines are only permissible for roads of minor importance: on Local lines this—the cheapest form of road-crossing—is always to be aimed at.

When the requisite headway between road and rail is not obtainable for the over- or under-passage of a road it must be determined whether the already projected height of the trace or the existing height of the actual road shall be altered or not. When the difference of heights is small it is a question whether it would not be better to carry the road either over or under, bearing in mind that according to the Regulations regarding moving dimensions, for normal-gauge lines, a clear height of headway of 4·8^{m} is required; on lines of metre-gauge, one of 3·75^{m}; and for lines of ·75^{m} gauge, 3·1^{m}: while for common roads, according to the degree of their importance, a clear headway of from 3^{m} to 5^{m} is required.

If the road be carried under the line, a space of 3·6^{m} to 5·6^{m} is required between the surface of the road and rail-level; whereas if the road be carried over the line the minimum distance on normal two-track line must be 6^{m} to 6·2^{m}, for normal single-track, 5·5^{m}; for a narrow-gauge of 1^{m}, 4·5^{m}: and for a line of ·75^{m} gauge, 3·8^{m}.

The best gradient of the approach-ramps, *i.e.* that for which the sum of the construction-cost and working-expenses is a minimum, lies on a level ground according to the volume of the traffic, between ·025 and ·012, in easy country between ·03 and ·015; in hilly ground between ·035 and ·020; and in mountains between ·040 and ·025. When the approach-ramps have to be artificially lengthened the best grade will be greater, and

in easy country it will lie	between		·020 and ·030
„ hilly ground	„	„	·028 and ·040
„ mountains	„	„	·034 and ·050*

The grades of the approach-inclines to carry the line of **Cart or roads** are to be determined according to § 24. But in most cases the extra cost of construction and working which will result from such a change in the grades of the trace will be greater than the extra outlay in construction and working for a change in the elevation and grades of the high road to be crossed, so that as a rule rebuilding the road will be the preferable course.

When the projected line crosses an already existing **railway** it frequently happens that it is more advantageous to rebuild the latter than to alter the existing projected trace.

For example: if a single-track line having a crown-width of 4^{m} is to cross an existing double-track line the latter being at ground-level and having a width at the base of 10^{m}, and if from § 24 it appeared that the best grade for the approach-ramps of the projected line were ·005, then for an ascent of 6^{m} the ramps on both sides would with slopes of 1½ require 72,000 cbm. of earth. On the other hand, were the existing line sunk 2·5^{m} at the crossing point on the same grades of ·005 then the projected line passing over it and only some 3·5^{m} higher would require only 18,200 cbm. of material which might be entirely supplied from the equally large excavation required to sink the existing line. The saving in cost is thus so considerable, viz. a fourth, that the rebuilding of the road in actual service, difficult as it might be, would be certainly justified.

The cost of construction of **stations** is often of great influence on the height of the trace, because the elevation of stations should be within narrow limits fixed mainly with refrence to the approach-roads. Although it may not be always attainable it is nevertheless desirable that the line should fall both ways from stations, because the starting and stopping of trains is thereby facilitated.

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* Conf. "Theorie des Trassirens," Heft III: Technische Trassirung der Strassen: (Apparently never published. TR.) or Launhardt; Die Steigungsverhältnisse der Strassen. Hannover. Schmorl & von Seefeld. 1880.

In mountainous regions level stretches for the **sites of stations** are only procurable at a great cost; so that as a general rule—recognised as perfectly allowable—the station is laid out on grade 1: 400. By so doing in a station 400^m long, a height of 1^m is attained, and therefore whenever the line is developed on a grade of s the length of the whole line is made shorter by $\frac{1^m}{s}$ than would be the case were the station on the level.

With a grade $s = 0{\cdot}25$ the line is shortened 40^m and if, for example, the rate of interest on the construction-cost, working- and maintenance-expenses per km. per annum be 25,000 M. there is an annual saving of 1,000 M. for which sum the existence of a grade in the station can certainly be tolerated.

For the rounding-off of the grade-intersections at the entrance to stations on severe ascending grades curves of a radius of from 3000^m to 5000^m have to suffice in order to avoid a too great length of curve.

Finally, the question of **working-expences** is of prime importance in the grading of the trace. For all lines **the most advantageoous ruling gradient** as dictated by the conditions of the traffic and the configuration of the ground must be chosen—regarding the choice of which see the following Sections. Within the limits fixed by the ruling gradient the gradients for the individual sections of the line are to be determined in accordance with the principles developed in § 27. In exceptional cases for moderate lengths, grades exceeding the ruling gradient may be employed and such grades, termed **momentum grades**, are surmounted by drawing on the stored-up energy of the train.

On the other hand, the **ruling gradient** must be diminished in **curves** by the amount of the curve-resistance, as already discussed in detail in § 21. In **tunnels**, also, the maximum grade must be reduced, because the frictional adhesion of the driving-wheels on the rails is always less in tunnels owing to the rails being in a state of continual dampness, and also because the train-resistance, due to the friction against the walls of the tunnel, of the volume of air put into movement by the motion of the train is greater than outside.

As to the amount of this increase of train-resistance in tunnels, experiments *ad hoc* and information relative thereto is wanting, so far as is known to the writer. The decrease of the tunnel grade should be made for the lowest frictional **adhesion** on the rails and proportionately to the gradient, and thus on steep grades the reduction made will be greater than on flatter ones; whereas, in view of the increased **train-resistance**, the reduction should increase proportionally to the velocity, and thus on flat grades it will be greater than on steeper ones.

Consequently, for all cases where tunnel grades have to reduced, it is well to fix a constant amount of $2^m/_m$ to $3^m/_m$ per running-metre.

In short curves and in short tunnels the reduction of gradient may be omitted if it can be permissibly assumed that they will be operated as momentum grades.

A reduction of the ruling gradient is desirable also for those sections of line which are normally exposed to strong side winds; such cases occur frequently where a valley is crossed.

§ 23.

The Optimum Ruling Gradient.

When the first and provisional fixing of the plan and elevation of the trace has been made in accordance as far as practicable with the principles laid down in §§ 19 and 22, it remains to determine whether the stiffest gradient occurring in the line is permissible or not.

To decide this very important question we proceed as follows. All the gradients are divided into two groups, one of which will contain all those gradients which may at once be pronounced offhand as not too steep; the other, all the remaining ones which are likely on examination to turn out too severe and which consequently will have to be flattened.

The total height, h, surmounted by all the grades in the second group is determined, and since an equal volume of traffic is assumed in both directions, it is immaterial in which of the two directions the individual gradients rise. If these sections be flattened by lengthening the line to a gradient s which becomes the ruling gradient for the whole line, the latter must be lengthened by the amount $\frac{h}{s}$. But since on curves there must be a reduction of the grade by the amount of the curve-resistance, $c = \frac{1}{R}$, the height to be surmounted in a curve of R^m radius and central-angle α^0—in other words, in a length of $\cdot000018\ \alpha R$ kilometres—is less by $\cdot000018\ \alpha$ as compared with an equal length of level-straight. If α be the sum in degrees of the central-angles of all the curves then in order to rise through a height h we require a length of line of

$$l_1 = \frac{h + \cdot000018\,\alpha}{s}$$

or, putting

$$h_1 = (h + \cdot000018\,\alpha)$$

$$l_1 = \frac{h_1}{s}$$

If the capital-cost per km. be A, the annual maintenance-expenses per km. U, then the interest of the capital and the expenses of maintenance for the whole length of the section to be lengthened is

$$K_0 = (A\,i + U)\frac{h_1}{s}$$

In calculating the working-expenses a passenger is to be taken, as is quite customary in calculations of this kind, as equal to a tonne of paying-load, and it is likewise quite permissible, since the volume of the future traffic can at best be only guessed at.

If T is the (assumed) sum of the tonnes of paying-load and of passengers; b the load-coefficient, then, according to § 10, the working-expenses for the length $\frac{h_1}{s}$ on the ruling gradient s amount to

$$K_1 = \left(f + e\,s + \frac{\frac{1}{2}\,(B_1 + B_0)\,(w + s)}{(z - w - s)\,L}\right)\frac{b\,T\,h_1}{s}$$

For the first group of gradients which remain unaltered after the first projection of the trace the construction and the maintenance-expenses do not come into consideration. The working-expenses, however, do so, since these are dependent on the size of the ruling gradient which we are now seeking.

Having determined according to § 11 the equivalent grade s_2 for this first group of which the total length is l, the working-expenses thereon are given by the expression

$$K_2 = \left[f + e s + \frac{B_0 (w + s)}{(z - w - s) L} + \frac{\frac{1}{2} (B_1 + B_0) \ (w + s_2)}{(z - w - s) L} \right] b T l$$

The total sum of the construction- and working-expenses for the ruling gradient s, viz.

$$K = K_0 + K_1 + K_2$$

if $\frac{l}{h_1}$ is put $= m$, is therefore

$$K = \frac{h_1 b T}{L} \left[(A i + U) \frac{L}{b T} \cdot \frac{1}{s} + \frac{f L}{s} + \frac{\frac{1}{2} (B_1 + B_0) \ (w + s)}{(z - w - s) s} + e m L s \right.$$

$$\left. + \frac{m B_0 (w + s) + \frac{m}{2} (B_1 - B_0) \ (w + s_2)}{z - w - s} \right] + h_1 b T (e + m f).$$

Putting $(A i + U + f b T) \frac{L}{b T} = J$, and arranging in powers of s we obtain

$$K = \frac{h_1 b T}{L} \left[J (z - w) + \frac{B_1 + B_0}{2} w + \left[\frac{B_1 + B_0}{2} - J + \frac{m}{2} (B_1 + B_0) w \right. \right.$$

$$\left. \left. + \frac{m}{2} (B_1 - B_0) s_2 \right] s + \left[m B_0 + e m L (z - w) \right] s^2 - e m L s^3 \right] \frac{1}{(z - w) s - s^2} + h_1 b T (e + m f).$$

Neglecting the small term $e m L s^3$, and putting

$$J (z - w) + \frac{(B_1 + B_0)}{2} w = D,$$

$$\frac{B_1 + B_0}{2} - J + \frac{m}{2} (B_1 + B_0) w + \frac{m}{2} (B_1 - B_0) s_2 = E,$$

and

$$m B_0 + e m L (z - w) = G,$$

we obtain

$$K = \frac{h_1 b T}{L} \left[\frac{D + E s + G s^2}{(z - w) s - s^2} \right] + h_1 b T (e + m f).$$

Differentiating with respect to s, and equating to 0, we obtain for the best ruling gradient

$$s^2 \left[E + G (z - w) \right] + 2 D s - D (z - w) = 0$$

whence, putting

$$E + G (z - w) = N,$$

then

$$s = - \frac{D}{N} + \sqrt{\frac{D^2}{N^2} + \frac{D}{N} (z - w)}$$

or

$$s = \frac{1}{N} \left(- D + \sqrt{D (D + N (z - w))} \right).$$

If further $D + N (z - w)$ is replaced by F,

then

$$s = \frac{1}{N} \left(- D + \sqrt{D F} \right)$$

Noting that

$$N = \frac{F - D}{z - w}$$

we can write

$$s = \frac{(z - w) \ (- D + \sqrt{D F})}{F - D}$$

or

$$s = \frac{(z - w) \ (\sqrt{F} - \sqrt{D}) \sqrt{D}}{(\sqrt{F} - \sqrt{D}) \ (\sqrt{D} + \sqrt{F})}$$

and so

$$s = \frac{z - w}{1 + \sqrt{\frac{F}{D}}}$$

Inserting step by step in the above the full values of the abbreviated expressions then

since $$F = D + N(z - w)$$

and $$N = E + G(z - w)$$

we obtain

$$s = \frac{z - w}{1 + \sqrt{\dfrac{D + E(z - w) + G(z - w)^2}{D}}}$$

For the numerator under the radix after inserting the values of D, E, G, we obtain

$$F = J(z - w) + \frac{B_1 + B_0}{2} w + \frac{B_1 + B_0}{2}(z - w) - J(z - w) + \frac{m}{2}(B_1 + B_0)(z - w) w$$

$$+ \frac{m}{2}(B_1 + B_0)(z - w) s_2 + m B_0 (z - w)^2 + e m L (z - w)^3$$

$$= \frac{B_1 + B_0}{2} z + B_0 m z (z - w) + \frac{B_1 - B_0}{2} m (z - w)(w + s_2) + e m L (z - w)^3.$$

Noting that

$$B_1 - B_0 = a z L,$$

and putting $\frac{B_1 + B_0}{2}$ the mean of the expense of running a light engine and of one working at full tractive-power, per km. $= B$ then the numerator under the radix becomes

$$F = z\left[(B + m(z - w) B_0 + \frac{a L}{2}(w + s_2) \frac{e L}{z}(z - w)^2\right]$$

The denominator under the root, after inserting the expressions for D, J, and $\frac{B_1 + B_0}{2} = B$, becomes

$$D = B w + (A i + U + f b T) \frac{L(z - w)}{b T}$$

In the above we may, for brevity, insert the number, n, of the trains which on a straight and level line could haul annually the gross-load $b\,T$. The weight of the train would be

$$Q = \frac{z - w}{w} L$$

and thus $$n = \frac{b T}{Q} = \frac{b T w}{(z - w) L}$$

Inserting these values, then

$$D = w\left[B + \frac{1}{n}(A i + U + f b T)\right]$$

Inserting the values found for F and D in the expression derived for the best ruling gradient we obtain, finally,

$$s = \frac{z - w}{1 + \sqrt{\dfrac{z}{w} \cdot \dfrac{B + m(z - w)\left\{B_0 + \frac{a}{2} L (w - s_2) + \frac{e L}{z}(z - w)^2\right\}}{B + \frac{1}{n}\left(A i + U + b f T\right)}}} \quad \ldots \quad (31)$$

This formula shows that the greater the kilometric-construction-cost $Ai + U$ the stiffer must be the gradient of the lengthened trace; whereas the heavier the traffic T and, consequently, the greater the number of trains n, the flatter it must be; which, indeed, is self-evident.

This expression shows, further, that the greater m is the flatter must the gradient be made and, consequently, the longer must be the conterminous or approach-sections worked in common with the mountain sections—in comparison with the height to be surmounted.

This also is evident from the fact that the gradient of the developed line is the ruling gradient for the working of the conterminous or approach-sections, and the cost of this working diminishes with the lessening of the ruling gradient.

It will not be quite so evident why, as the expression shows, the gradient of the developed line must be steeper if the lower or approach-sections have a greater equivalent

ascent s_2; because at the first glance it will be at once assumed that in the developed line the ascents should be made steeper when there already exist severe grades in the conterminous or approach-sections.

But we must bear in mind that the decrease of the working-expenses on the approach-sections resulting from a reduction of the ruling gradient, is larger the greater these very working-expenses arising from the grades in the non-mountain or approach-sections are. As the decrease of the working-expenses, due to a reduction of the ruling gradient, and so on non-mountain-sections with large equivalent grades, is greater than when the equivalent grades are smaller, so in the first case a smaller ruling gradient is required.

Finally, the formula shows that the most advantageous gradient varies directly with the locomotive tractive-coefficient which may have been taken as a basis. This is the only figure as to which there is any freedom of choice in any particular case when the construction-cost, volume of traffic, length of the approach-section and its equivalent grade are given. With the increase of the tractive-coefficient, with which the optimum value of the ruling gradient increases, the total traffic-expenses diminish; consequently, also, the sum of the interest on the construction capital, the maintenance- and the working-expenses. We must not, however, exceed a certain optimum figure in fixing the tractive-coefficient; which figure is dependent on several circumstances not taken account of in the formula—as has already been pointed out in § 12. With the increase of this coefficient the maximum attainable velocity decreases; such increase when a certain definite limit is exceeded is disadvantageous to the traffic, since the cost per km. of the train-staff, and the interest on the capital-cost of rolling-stock increases. Also, the coupling of locomotive-axles necessary to obtain a greater tractive-coefficient increases the frictional resistance of the working parts of the locomotive and their maintenance-cost.

Accordingly, the best gradient can only be determined when the optimum value of the tractive-coefficient has been fixed upon.

The degree of influence which the various circumstances affecting the determination of the best value of the ruling gradient have on the result will be more clearly seen if we give a numerical example assuming several different values.

Suppose that in order to cross a watershed a height of 200m has to be surmounted on one side and on the other a height of 190m; and suppose that there occur in the inclines on both sides curves of a total of 550° of central-angle. Then the height $h_1 = \cdot 2 + \cdot 19 + \cdot 000018 \times 550 = \cdot 4$. If the approach sections leading up from the plains to the passage over the watershed have an equivalent grade $s_1 = \cdot 006$ and length $l = 60$ km. then the ratio m is $\frac{60}{\cdot 4} = 150$.

Let the cost of construction of the mountain sections be 320,000 M. and the yearly maintenance-expenses of the same be 3,000 M. per km., so that

$$Ai + U = 320{,}000 \times \cdot 04 + 3{,}000 = 15{,}800 \text{ M.} = 1{,}580{,}000 \text{ pf.}$$

Let the sum of the tonnes of goods and passengers to be hauled be $T = 600{,}000$, and the load-coefficient be assumed as $b = 2\frac{1}{3}$, then $b\,T = 1{,}400{,}000$. If, following a former calculation, we take $w = \cdot 0036$, $B_0 = 32$, $a = 25$, $e = 2$, $f = \cdot 15$ in pfennige, and assume the weight of the locomotive L as 60 tonnes then we obtain from the formula above deduced, for the best ruling gradient for the surmounting of the watershed,

for $z = \cdot 08$, the value of s as	$\cdot 0200$
$z = \cdot 09$	$s = \cdot 0212$
$z = \cdot 10$	$s = \cdot 0225$
$z = \cdot 11$	$s = \cdot 0239$
$z = \cdot 12$	$s = \cdot 0250$

For the above several tractive-coefficients we obtain then the weight of the train, $Q = \frac{z - w - s}{w + s} L$; the number of the trains requisite to deal with the volume of traffic $b\,T$ per annum $= \frac{b\,T}{Q}$; the velocity, assuming the locomotive to be of 360 $H.\ P$, $v = \frac{45 \cdot Q}{z}$ m. per second; and finally, the total working-expenses as given in the following Table, in which the capital-cost of the approach-sections of 60 km. in length are taken at 10 million M. and consequently the interest on this capital-cost and the maintenance-cost at

$$60 \times 3{,}000 + \cdot 04 \times 10{,}000{,}000 = 580{,}000 \text{ M.}$$

TABLE XIII.

Best Gradient and Total Traffic-Expenses for different Traction-Coefficients.

Tractive-coefficient = z	·08	·09	·10	·11	·12
Best gradient = s	·0200	·0212	·0225	·0239	·0250
Weight of a train = Q	144	158	170	180	192
Number of train per an.	9722	8861	3235	7778	7292
Velocity = v	5·63	5·00	4·50	4·09	3·75
	M.	M.	M.	M.	M.
Construction-cost of approach-section ...	580,000	580,000	580,000	580,000	580,000
Construction-cost of mountain-section ...	300,000	283,000	266,700	251,000	240,000
Working-expenses on approach-section ...	540,400	521,600	508,500	500,300	490,900
Working-expenses on mountain-section ...	249,200	233,300	220,200	209,400	200,700
Total cost and working-expenses per annum..	1,670,200	1,617,900	1,575,400	1,550,700	1,511,600

From the foregoing it is seen that the tractive-coefficient in the present Example should not exceed ·09, or at most ·10; and this can be attained only by the use of a 3-coupled locomotive. Although the tractive-coefficient might be increased from ·10 to ·12 and, as seen from to the above Table, some 4°/₀ saving on the sum of the construction-cost and working-expenses might result, nevertheless, with the 4-coupled locomotive which in that case would be necessary the cost of equipment and maintenance would increase and, in addition, there would be an increase in the fractional resistances of the moving parts of the engine, and a decrease of the velocity. Thus, ultimately, there would be an increase in expenses which would certainly exceed the calculated saving of 4°/₀.

Since it is very difficult, indeed hardly practicable, to exactly determine and to give due weight to all the factors which enter into the choice of the tractive-coefficient, the value of the most advantageous ruling gradient can only be determined within certain limits which, however, are tolerably narrow. In the Example just calculated there can be no doubt that the tractive-coefficient must not be less than 09 nor greater than ·11; and consequently, that the ruling gradient will lie between ·021 and ·024.

It is further to be noted that it is not at all necessary that the best ruling gradient, as above found, should be strictly adhered to, since a deviation therefrom of 10 °/₀ does not perceptibly increase the total traffic-expenses. The following Table shows that by raising the ruling gradient from ·0225 to ·030, namely 1⅓-fold, the total traffic-expenses are increased by only some 2°/₀ ; and that by a flattening of the same from ·0225 to ·015, or by ⅔, the amount of these expenses increases about 5 °/₀.

To judge of the effect that the amount of the traffic has on the optimum ruling gradient the statement will suffice that if all other conditions remain the same,

then for $T = 400{,}000$, s must $=$ ·0251
$= 600{,}000$, s $=$ ·0225
$= 800{,}000$, s $=$ ·0207

TABLE XIV.

Construction and working-expenses for the hypothetical line, take as an Example, on the basis of a tractive-coefficient of ·1 for various values of the ruling gradient, in units of 1,000 M.

Ruling Gradient.	Interest on the capital cost and of the cost of maintenance: Of approach-sections.	Interest on the capital cost and of the cost of maintenance: Of the Mountain sections.	Working-Expenses on the: Approach-sections.	Working-Expenses on the: Mountain sections.	Total of the expenses of construction and working.
·015	580	422	377	220	1599
·020	580	317	440	207	1534
·021	580	302	441	206	1549
·022	580	288	453	206	1527
·023	580	275	465	206	1526
·024	580	264	477	205	1526
·025	580	253	489	205	1527
·030	580	211	555	208	1554

Supposing—all other conditions remaining the same—that the kilometric construction-cost of the mountain-sections was 200,000 M. instead of 320,000 M. as has been assumed in the foregoing Example; then the most advantageous ruling gradient would be s = ·02 instead of s = ·0225; and if it were 400,000 M. then s would be = ·0241.

The influence of the magnitude of the 'equivalent-gradient' of the approach or foot-inclines is insignificant in its effect on the optimum ruling gradient; for if that equivalent gradient were s = ·008 instead of s = ·006, then the ruling gradient would be s = ·0223, instead of s = ·0225.

On the other hand, if the equivalent gradient has its smallest possible value viz. that of the coefficient of resistance—as would be the case were the line perfectly level—then the value of the most favourable ruling-gradient would be s = ·0229, instead of the calculated value s = ·0225, corresponding to an equivalent gradient of s = ·006.

Of much greater importance is the influence of the length of the foot or approach-incline. If the ratio m of its length to the height to be surmounted = 300, then the ruling gradient should be s = ·0176; which for m = 150, was found to be ·0225. On the other hand, were m = 75 only, then s would be = ·0279; and in the limiting case for m = 0 s would have to be = ·0425.

In the investigation which led to the formula for the most favourable ruling gradient the kilometric-cost of construction A was assumed to be independent of the gradient; whereas in reality the construction-cost per unit of length is not found to be the same for different gradients. Consequently, having calculated the optimum ruling gradient we must then examine whether it can really be carried out on the ground at the cost of construction provisionally assumed; and the ruling gradient must then be recalculated with a corrected value of the kilometric construction-cost.

The gradient actually corresponding to the construction-cost having at last been determined the following cases may arise :—

1. The value found for the best ruling gradient is less than some of the gradients of the foot-incline. When that is so the lengths containing these grades are to be cut-out of the whole length l of the these sections, and the height h, surmounted by them is to be added to the total height of the watershed crossing. The calculation for the determination of the optimum gradient is then to be repeated with the corrected and smaller ratio m, leading to a somewhat greater value of the gradient.

2. The optimum gradient when found is steeper than any of the approach-section gradients, but yet so flat that in order to surmount the watershed it can only be had by increasing the length of the line. In that case the problem is solved, and the location is accordingly determined.

3. The optimum gradient when found differs so little from the general inclination of the valley that by means of increased earthwork, etc., it is constructionally practicable without any development of the line. In that case the investigation of the optimum ruling gradient must be further carried out according to the method of § 24.

4. The optimum gradient is steeper than that of the section under examination. If then it is not possible to shorten this section, thus approximating it to the optimum grade, of course the flatter grades must be retained. But we must still examine in the manner explained in § 24 whether a further flattening of these grades, without at the same time increasing the length of the line, be not desirable.

5. The lengths of the approach or foot-inclines, as also the lengths of the actual mountain-section-inclines, are so great that it appears advisable to beak up the trains working on the approach or foot-inclines before entering the mountain-sections; in that case it is necessary to divide the whole line into separate working-divisions, beaking up the trains coming from the

lower section into smaller trains on the mountain-section, or working them undivided with additional locomotives. In that case the investigation of the most favourable ruling gradient is to be further carried out as explained in § 25.

6. The altitude to be surmounted is so small that it would be undesirable to make its gradient the ruling one for the whole remaining length of the location; in this case it is preferable to surmount the height on a steep gradient by means of a partial consumption of the stored-up energy of the train—namely, to work it as a momentum grade.

This latter case is to be examined as explained in § 26.

§ 24.

The Optimum Ruling Gradient without lengthening of Line.

When it is possible to fix the gradients of a line without lengthening the trace at any point, that is, without altering the length of the whole line, the height to be surmounted and the curves remaining unchanged, whatever the gradients may be, then the magnitude of the equivalent grade also remains unchanged.

In this case the simple formulæ given in § 13 for the working-expenses may be employed to determine the value of the ruling gradient, according to which for a ruling gradient s, an equivalent gradient s_2, a length of trace of l, and a traffic of P passengers and T tonnes of paying-load, the working-expenses are, in pfennige,

$$K = P\,(\cdot 967 + 10\tfrac{2}{3}\,s + 29\tfrac{1}{3}\,s_2)\,l + T\,(\cdot 56 + 23\tfrac{1}{3}\,s + 39\tfrac{2}{3}\,s_2)\,l \quad \ldots \quad (32)$$

This simple formula for the working-expenses could not have been employed in the preceding Section in discussing the best gradient for a developed trace because it is based on the assumption that the tractive-force coefficient

for goods-trains is

$$z = \cdot 05 + 2\,s$$

and that for passenger-trains is

$$z = \cdot 02 + 2\,s;$$

which is only correct for a value of s up to $\cdot 025$, or at most up to $\cdot 03$. In the present case where the optimum value of the ruling gradient is to be found without lengthening the line we are always far below that limit; but in the case where the line is for technical reasons lengthened this limit might be easily exceeded when the above relation between the tractive-force coefficient and the ruling gradient is employed. It was therefore necessary to undertake the general investigation of the preceding Section without any assumption of such a relation of the tractive-force coefficient.

In the present case the part of the working-expenses dependent on the magnitude of the ruling gradient, is

$$K_1 = (10\tfrac{2}{3}\,P + 23\tfrac{1}{3}\,T)\,s\,l \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

Exploying this formula, the optimum value of the ruling gradient will usually be found only by trial and error—viz., by determining the construction-expenses for a series of values of the ruling gradient and adding to them part dependent on s of the working-expenses.

Only in exceptional cases is it possible to express the construction-cost as a function of s and to determine the best value of s directly by differentiation.

For example: if a single-track Local line has to be carried over an existing railway at a height of 6m, then for a crown-width of 5m, and slopes of $1\tfrac{1}{2}$, the area of the cross-section of the ramp at the highest point is 84 qm., and at the middle 28·5 qm., and thus the mean area would be

$$= \tfrac{1}{6}\,(84 + 4 \times 28{\cdot}5 + 0) = 33 \text{ qm.}$$

For a gradient s the total length of the approach inclines on both sides would be $= \frac{12}{s}$; and consequently, the cubic content of the same $= \frac{396}{s}$ cbm. and @ 1 M. per cbm., the total cost of the earthwork $= \frac{96}{s}$ M. which at 4°/° interest corresponds to an annual sum of $\frac{16}{s}$ M.

If the annual volume of traffic on the 25 km. of line were $T = 80{,}000$, and $P = 30{,}000$, then the working-expenses dependent on s are

$$K_1 = \left(10\tfrac{2}{3} \times 30{,}000 + 23\tfrac{1}{3} \times 80{,}000\right)\frac{25}{100}\,s = 546{,}666\,s \text{ M.}$$

The part of the construction-cost and working-expenses dependent on s would then be

$$S = \frac{16}{s} + 546{,}666\, s$$

whence, differentiating with respect to s, the best value of the ruling gradient is

$$s = \sqrt{\frac{16}{546666}} = \cdot 0054$$

Of course this gradient, which thus becomes the ruling gradient for the whole line, is only to be given to the inclines if there is no steeper one in the other divisions of the line. If steeper grades occur then all of them would have to be simultaneously flattened when determining the best gradient for the inclines.

Of course, under all circumstances, the optimum value of the ruling gradient obtained by simply increasing the earthwork and without lengthening the line will be much smaller than if the trace were lengthened.

§ 25.

The Optimum Ruling Gradient when Assistant Engines are employed, or the Trains divided.

If steep grades are not scattered about on separate lengths of the line, but are bunched together forming a continuous length then, in most cases, it will not be possible to run unbroken-up trains over the whole line, because this would render necessary either too flat grades on the hilly sections, or too light trains on the contiguous, lower or valley, sections. But it would equally undesirable to break up the traffic at the foot of the mountain section and to make up entirely new trains. The trains coming from the flatter valley sections should be despatched over the mountain sections with the least possible change in their composition; and this may be done by hauling them by a single locomotive of greater tractive-force or by two locomotives, or by what comes to the same thing, despatching two or more smaller trains each hauled by a locomotive.

In the calculations connected with this subject it may be assumed that when heavier locomotives are employed the part B_0 of the cost of a locomotive-km. which is independent of the tractive-power increases proportionally with the weight of the locomotive: so that, for example, this amount B_0—which for a locomotive weighing 60 tonnes, inclusive of the tender, figures out to 32 pf.—would increase for a locomotive weighing 72 tonnes to $1{\cdot}2 \times 32 = 38{\cdot}4$. This assumption is justified by the higher cost-price of the locomotive and its greater maintenance-expenses, by the greater rail-wear, and by the greater outlay on staff which, owing to the smaller speed of these heavy locomotives, accrues per km.

Consequently, as has been noticed already at the end of § 13, a train of weight $Q = 150$ tonnes hauled by a locomotive of $L_1 = 72$ tonnes is 1·2 times dearer than a train of $Q = 125$ tonnes drawn by a locomotive of $L = 60$ tonnes.

To enable the same weight of locomotive $L = 60$ tonnes and the same value of $B_0 = 32$ pf., to be always employed in calculations the actual weight Q_1 of the trains hauled on the mountain sections by locomotives of weight L_1 is expressed in terms of a unit-weight Q corresponding to, *i.e.*, hauled by, a locomotive of weight $L = 60$ tonnes.

For example: if a train of $Q_1 = 300$ tonnes is hauled on a mountain section by two engines each of 72 tonnes weight, then in calculation in place of this single train there would be employed 2·4 trains each of $Q = 125$ tonnes and an engine of $L = 60$ tonnes. The train-load hauled on the approach-sections by a locomotive of 60 tonnes is then expressed as a $t\,Q$ so that were the said train of 300 tonnes hauled on the approach-sections by a locomotive of 60 tonnes t would be 2·4.

It is not always possible to so arrange the separately-worked mountain sections that they shall be wholly on the ruling gradient s; but in addition to the developed line on which the height h_1 is surmounted on the grade s there will be sections of flatter grades having a total length of l and the equivalent grade s_2.

In order to determine the optimum ruling gradient we obtain therefore, as in § 23, the construction- and maintenance-costs of the lengthened line, viz.,

$$K_0 = (A\,i + U)\,\frac{h_1}{s}$$

Also, the working-expenses of the whole mountain section, viz.,

$$K_1 = \left[f + e\,s + \frac{\frac{1}{2}(B_1 + B_0)\,(w + s)}{(z - w - s)\,L}\right]\frac{b\,T h_1}{s}$$

and

$$K_2 = \left[f + e\,s + \frac{B_0\,(w + s) + \frac{1}{2}\,(B_1 - B_0)\,(w + s_2)}{(z - w - s)\,L}\right] b\,T\,l$$

and we must add thereto the working-expenses of the approach-sections which are worked with a train-weight of $t\,Q$.

If the length of these latter sections be l_1 and their equivalent grade s_1 and the maximum grade thereon be s_3 then those working-expenses are

$$K_3 = \left(f + e\,s_3 + \frac{B_0 + \frac{a}{2}(t\,Q + L)\,(w + s_1)}{t\,Q}\right) b\,T\,l_1$$

or since

$$Q = \frac{z - w - s}{w + s}\,L$$

then

$$K_3 = \left(f + e\,s_3 + \frac{B_0\,(w + s)}{t\,L\,(z - w - s)} + \frac{\frac{a\,z}{2}\,(w - s)}{z - w - s}\right) b\,T\,l_1$$

If now the sum of the costs

$$K = K_0 + K_1 + K_2 + K_3$$

be arranged in powers of s and differentiated with respect to s, and if further we put $\frac{l}{h_1} = m$, $\frac{l_1}{h_1} = m_1$, $\frac{B_1 + B_0}{2} = B$, the kilometric locomotive-expenses $B_0 + \frac{aL}{2}\,(w + s_{11}) = B_2$, and $B_0 + \frac{aL}{2}\,(w + s_1) = B_3$, and the tractive-force required to haul on level road, viz. $\frac{b\,T\,w}{(z - w)\,L} = n$, then after the same transformations as in § 23, we obtain

$$s = \frac{z - w}{1 + \sqrt{\frac{z}{w} \cdot \frac{B + m\,(z - w)\left(B_2 + \frac{e\,l_1}{z}\,(z - w)^2\right) + \frac{m_1}{t}\,(z - w)\,B_3}{B + \frac{1}{n}\,(A\,i + U + f\,b\,T)}}} \quad \ldots \quad (34)$$

To illustrate by an example: let the line be taken on which the working is uniform—not broken up into different working divisions—and on which $s = {\cdot}0225$ was found in § 23 as the best value for a tractive-force coefficient $z = {\cdot}1$. In that example $L = 60$, $T = 600{,}000$ tonnes, $b = 2\frac{1}{3}$, $w = {\cdot}0036$, $s_2 = {\cdot}006$, $i = {\cdot}04$; and let the expenses be expressed in pfennige; $B_0 = 32$, $a = B = 25$, $B_0 + \frac{a\,z\,L}{2} = 32 + \frac{1}{2} \times 25 \times {\cdot}1 \times 60 = 107$, $e = 2$, $A = 32000000$, $U = 300000$, $f = {\cdot}15$; and finally, the number of trains on the level

$$n = \frac{2\frac{1}{3} \times 600000 \times {\cdot}0036}{({\cdot}1 - {\cdot}0036)\,60} = 871$$

If now instead of an ordinary continuous service on the mountain section the trains are hauled either undivided by two locomotives, or in two halves by two locomotives, then the ratio t in the formula will $= 2$. If on the mountain section worked by two locomotives there occur a length of 6 km. on an equivalent grade $s_2 = {\cdot}006$, then $m = \frac{6}{{\cdot}4} = 15$; and there remains a length of $60 - 6 = 54$ km. which can be worked with one locomotive, so that $m_1 = \frac{54}{{\cdot}4} = 135$, and $s_1 = {\cdot}006$.

Accordingly, the best value of the ruling gradient is

$$s = {\cdot}0287.$$

The trains on the mountain railway will then have a weight of

$$Q = \frac{{\cdot}1 - {\cdot}0036 - {\cdot}0287}{{\cdot}0036 + {\cdot}0287} \times 60 = 12[illegible] \text{ tonnes,}$$

and on the approach-sections, 252 tonnes. If the ruling gradient in these latter is ·010 with curves of 500m radius then the requisite maximum tractive force is

$$s = (252 + 60)\ (\cdot 01 + \cdot 002 + \cdot 0036) = 4867 \text{ tonnes.}$$

But if on the mountain section two locomotives each weighing 72 tonnes are employed to haul the train then the ratio $t = \dfrac{2 \times 72}{60} = 2\cdot 4$, and the most favourable ruling gradient is

$$s = \cdot 0298$$

The weight of the train would in that case would be

$$t\,Q = \frac{\cdot 1 - \cdot 0036 - \cdot 0298}{\cdot 0036 + \cdot 0298} \times 2 \times 72 = 288 \text{ tonnes}$$

and the maximum tractive-force requisite on the lower or approach-sections would be

$$Z = (288 + 60)\,(\cdot 01 + \cdot 002 + \cdot 0036) = 5\cdot 429 \text{ tonnes}$$

which is quite within the capacity of a locomotive weighing 60 tonnes and having a tractive-force coefficient of $\frac{1}{10}$. Having thus found the most favourable gradient for the separate working of the mountain-section it remains to investigate whether a subdivision of the whole line into separate working-sections would be advantageous, and if so, what increase in engine-power on the separate sections is most desirable.

When the train on the mountain-section is divided into halves each of which is drawn by a locomotive of 60 tonnes weight and having a tractive-force coefficient of $z = \cdot 1$; or when on the approach-sections the train drawn by such a locomotive is re-enforced on its arrival at the mountain-sections by a second similar locomotive employed either as a hauler or as a pusher, then the expenses for the best gradient, s, figure-out as follows:

(1)	Interest on the capital cost, plus maintenance-expenses of the 60 km. section having an equivalent grade s_2 of ·006 (see § 23)	580,000 M.
(2)	Interest on the capital cost, plus the maintenance-expenses of the lengthened line on a grade of $s = \cdot 0287$ to attain an elevation of ·4 km. ...	220,000 „
(3)	Working-expenses on said length	207,000 „
(4)	Working-expenses on the 6 km. length having an equivalent grade of $s_2 = \cdot 006$, which is worked in common with the mountain-section ...	54,000 „
(5)	Working-expenses on the 54 km. length worked with *one* locomotive ...	337,000 „
	Total...	1,398,000 M.

In the other case, where two locomotives of 72 tonnes are employed on the mountain-section to haul the train which on the conterminous approach or lower sections is drawn by one locomotive of 60 tonnes weight, the above items of cost in the order above given are—

$$580 + 201 + 209 + 56 + 336 = 1{,}392{,}000 \text{ M.}$$

Thus of the two methods of working here described it is manifest that in the present instance the first should have the preference: since the small saving arising in the second case of 6,000 M. per annum (or of not quite ½°/₀) cannot counterbalance the disadvantage—of which no account has been taken in these calculations—that specially constructed locomotives differing entirely in type from the standard ones used on the adjoining lowland sections would be required, and consequently a much larger and more expensive park of locomotives would have to be maintained, and one which would never be fully utilized.

But making the mountain-section a separate working length has in comparison with the undivided and continuous working of the whole line—as a reference to § 23 shows—the very considerable advantage of saving of 1526 – 1398 = 128,000 M. per annum. Now although this advantage would be lessened by the necessity of more extensive station arrangements at both ends of the mountain-section owing to the longer halts of the trains at those stations and by the greater number of locomotives utilized which could not be fully worked up to their capacity, there still remains a very considerable balance of advantage.

However, there is no need in this case to adhere too scrupulously to the calculated optimum gradient, since *small* deviations therefrom do not increase the total expenses in any considerable degree.

The formula given in the preceding for the best gradient, and that in § 23 for the uniform working, is to be used simply as a starting-point or basis of operations, since the

assumption on which the formula is based, viz., that the kilometric-cost of construction, (*i.e.* cost per km.) is the same for all gradients, is only partially true. Besides, the assumption that the working-expenses per passenger-km. are the same as those of the paying-load-km. is not absolutely also true. For these reasons, having determined the best gradient from the above formula, estimates must then be made with various grades of values assumed nearly equal to the calculated value, and for each one the construction and working-expenses both for goods- and passenger-traffic must be worked out in order to discover that grade for which the total transport-expenses is a minimum.

§ 26.

Momentum Grades.

Severe and short inclines which the tractive-force of the locomotive is insufficient to overcome can be surmounted by a draft on the energy of the train, or as the expression is, can be "rushed."

If the velocity with which the train arrives at the foot of the grade $= v_1$ and if the train at the top of the grade is to have a velocity $= v_0$, then the energy available to carry the train up the incline is

$$\beta \frac{Q + L}{2g}\left(v_1^2 - v_0^2\right)$$

wherein β is a coefficient which includes the additional energy due to the rotation of the wheels and axles. As a fairly accurate mean value β may be taken as 1·08.

The train-resistance, diminishing as the velocity falls, on such an incline on a grade s_1 is

$$(Q + L)\ (a + b v^2 + s_1)$$

in which from § 7—p. 17,

$$a = \cdot 00273 : \text{ and } b = \cdot 0000131.$$

Subtracting the tractive-force exerted by the locomotive $z\,L$, the amount of work performed by the train's energy on a length $d\,l$, is

$$\Big[(Q + L)\ (a + b v^2 + s_1) - z\,L\Big]\,d\,l.$$

Since the kinetic energy of the train lost during a fall in velocity of $d\,v$ is

$$\beta \frac{(Q + L)}{g} v\,d\,v$$

we have the equation

$$\Big[(Q + L)\ (a + b v^2 + s_1) - z\,L\Big]\,d\,l = \frac{\beta\,(Q + L)}{g} v\,d\,v$$

whence

$$d\,l = \frac{\beta}{g}\ \frac{v\,d\,v}{a + b v^2 + s_1 - \dfrac{z\,L}{Q + L}}$$

If the ruling gradient be s then we have the equation

$$(Q + L)\ (w + s) = z\,L$$

and therefore

$$d\,l = \frac{\beta}{g} \cdot \frac{v\,d\,v}{a + b v^2 + s_1 - w - s}$$

or, since

$$w = a + b v_1^2$$

$$d\,l = \frac{\beta}{g} \cdot \frac{v\,d\,v}{b\,(v^2 - v_1^2) + s_1 - s}$$

Integrating the above we obtain

$$l = \frac{\beta}{2\,bg} \tan nt\ \frac{s_1 - s}{b\,(v_0^2 - v_1^2) + s_1 - s} \qquad \ldots \qquad \ldots \qquad (35)$$

which is the length of the incline on a grade $s_1 > s$ which can be surmounted during the fall of the train's velocity thereon from v_1 to v_0

For example: putting $g = 9{\cdot}81$, $\beta = 1{\cdot}08$, $b = \cdot 0000131$, $v_1 = 7$, $v_0 = 3$, then for a ruling gradient of, say $s = \cdot 0036$, the length of an incline on a grade s_1 which can be surmounted solely by the train's momentum is

$$l = 4202 \tan nt \frac{s_1 - \cdot 0036}{s_1 - \cdot 004124}.$$

The following Table gives the Lengths of Momentum Inclines for various grades and also for the ruling gradients of ·0036; ·01, ·025

TABLE XV.

Gradient of Momentum Incline.	Length of Momentum Grade in metres for a Ruling Gradient of		
	·0036	·010	·025
·05	48	·55	89
·025	104	149	—
·010	369	—	—
·005	1970	—	—

But for all practical purposes the length of the momentum incline can be calculated with quite sufficient accuracy in a much simpler manner, if the decrease of the coefficient of resistance with the velocity be disregarded, taking it in fact as constant.

Thus from the equation

$$l \left[(Q + L)\ (w + s_1) - z\,L\right] = \beta \frac{Q + L}{2\,g} (v_1^2 - v_0^2)$$

we obtain directly

$$l = \frac{\beta}{2\,g} \cdot \frac{v_1^2 - v_0^2}{\left(w + s - \frac{z\,L}{Q + L}\right)}$$

or inserting the ruling gradient given by

$$(Q + L)\ (w + s) = z\,L$$

we obtain the simpler formula

$$l = \frac{\beta}{2\,g} \cdot \frac{v_1^2 - v_0^2}{s_1 - s} \qquad \ldots \quad \ldots \quad \ldots \quad (36)$$

The following Table is calculated from the above formula and shows a sufficiently close coincidence with the more exact figures of Table XV for all practical purposes.

TABLE XVI.

Grade of Momentum Incline.	Length of Momentum Incline in metres for a Ruling Gradient of		
	·0036	·010	·025
·05	47	55	88
·025	103	147	—
·010	344	—	—
·005	1,573	—	—

The height surmounted on the momentum incline, *viz.* $h = l\ s_1$—from the simpler formula just given for l—is

$$h = \beta . \ \frac{v_1^2 - v_0^2}{2\,g} . \ \frac{s_1}{s_1 - s}$$

If this height is to be surmounted without momentum, viz., on the ruling gradient s, then the incline must be of the length

$$l_1 = \frac{h}{s} = \frac{\beta}{g} \frac{(v_1^2 - v_0^2)}{2} \ \frac{s_1}{s\,(s_1 - s)}.$$

Fig. 12.

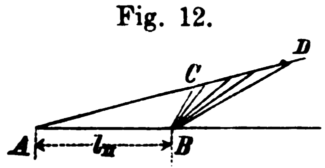


Fig. 13.

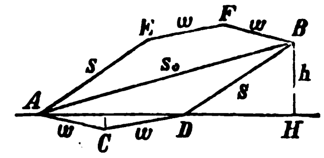


Fig. 14.

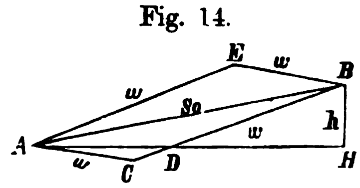
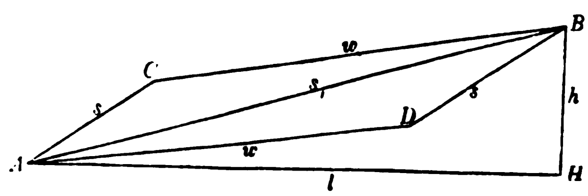


Fig. 15.



Thus the momentum incline may be changed into a ruling gradient by lengthening it by

$$l_2 = l_1 - l$$

or

$$l_2 = \frac{\beta}{2g} \cdot \frac{\left(v_1^2 - v_0^2\right)}{(s_1 - s)} - \frac{\beta}{2g} \cdot \frac{\left(v_1^2 - v_0^2\right)}{(s_1 - s)} \cdot \frac{s_1}{s}$$

whence

$$l_2 = \frac{v_1^2 - v_0^2}{2g} \cdot \frac{\beta}{s} \qquad \ldots \qquad \ldots \qquad \ldots \quad (37)$$

The amount of the lengthening required to transform a momentum grade into a ruling gradient is thus inversely proportional to the ruling gradient, but otherwise is the same for any other grade of the momentum incline.

If the distance l_2 be marked off from the foot of the momentum incline—**Fig. 12**—from B towards A, and the sloping line $A\ C\ D$ be drawn on the ruling gradient, then any momentum incline $B\ C$ or $B\ D$ starting from B may be obtained by continuing it upwards to its intersection with the ruling gradient $A\ C\ D$.

Numerical example: putting $v_1 = 7$, $v_0 = 3$, $\beta = 1{\cdot}08$, $g = 9{\cdot}81$,

then		$l_2 = \frac{2{\cdot}2}{s}$
whence for	$s = {\cdot}0036$,	$l_2 = 611^m$
	$s = {\cdot}010$,	$l_2 = 200^m$
	$s = {\cdot}025$,	$l_2 = 88^m$

At the foot of the momentum incline the line would have to be raised in any case by $\frac{\beta}{g} \cdot \frac{v_1^2 - v_0^2}{2g}$ if the ascent were made on the ruling gradient instead of on the momentum grade: thus for $v_1 = 7$, and $v_0 = 3$, the rise $= 2{\cdot}2^m$. This is the height through which the train must fall in order to acquire the energy lost in ascending the incline.

The working of a momentum incline, since the kinetic energy lost in the ascent is subsequently regained, requires precisely the same quantity of steam as in surmounting the same elevation on the ruling gradient and is, in this respect, not disadvantageous.

But the loss of time and the disturbance in the working of the line which momentum inclines may cause, unless the speed of the train and the firing be carefully regulated, make it desirable to avoid momentum grades as much as possible.

It may be added that steep and consequently short momentum inclines hardly permit of the rounding-off so desirable at the junction of synclinal gradients.*

[* Vide Appendix for a further discussion.—Tr.]

§ 27.

Lost Height.

In the working of traffic on Cart roads any deviation from a uniform gradient causes an increase in the working-expenses.* This is not so in the case of a railway.

On a railway between two points at any vertical distance apart the uniform gradient is under all circumstances the best one *provided that it is the ruling gradient,* since the working is the cheaper the flatter the ruling gradient.

But if, on the other hand—the ruling gradient, being determined or given by other sections of the line—the gradients between the two points A and B (which have a horizontal distance l and a height h) are to be fixed solely in the interests of the traffic-working and without regard to the cost of construction, then there are two cases to be distinguished, namely when the uniform gradient $\frac{h}{l}$ is a "non-injurious" one, and therefore less than w; and secondly when it is an "injurious one," namely greater than w.

First case—Suppose the point B—**Fig. 13**—is distant horizontally l from A and vertically h above it. If, further, the uniform gradient rising from A to B, viz. $\frac{h}{l} = s$, is less than w, then the engine-power requisite to haul a train of weight Q from A to B on this uniform grade is, for the ascent,

$$(Q + L)\ (w + s_0)\ l = (Q + L)\ (w\,l + h).$$

If instead of the through uniform gradient $A\ B$, there be one, say, from B down to D, h below B, and two others from A and D at the inclination w continued downwards to their intersection C, then the required tractive-power on the discontinuous line of ascent, $A\ C\ D\ B$,

if $$D\ B = l, \text{ and } A\ C = C\ D = \frac{l_2}{2}$$

will be— on the gradient $A\ C = 0$

$$\text{,,} \quad \text{,,} \quad C\ D = (Q + L)\ (w + w)\ \frac{l_2}{2}$$

$$\text{,,} \quad \text{,,} \quad D\ B = (Q + L)\ (w + s)\ l_1$$

and consequently, since $l_1 + l_2 = l$, and $s\ l_1 = h$, the whole $= (Q + L)\ (w\ l + h)$. Thus the working-expenses for the ascent on the discontinuous grade $A\ C\ D\ B$ are equal to those on the continuous grade $A\ B$. The expenses would also be the same for an incline rising from A on the ruling gradient up to E, h above A, and then from E to F for half the distance $C\ B$, on the gradient w and falling with this same gradient from F down again to B.

On any chain of lines drawn within the parallel 6-sided figure $A\ E\ F\ B\ C\ D$, which in the direction A to B has no stiffer fall than w, nor a stiffer rise than s, but which otherwise may have any shape whatever, the working-expenses in the **ascent** from A to B are the same as those along the continuous uniform grade $A\ B$.

For the **descent,** the condition of things is different. The locomotive tractive-force necessary to take a train of weight Q down a uniform grade from B to A is

$$(Q + L)\ (w\ l - h).$$

* See "Theorie des Trassirens," Heft III: (Apparently never published. Tr.) or Launhardt: "Die Neigungs-Verhältnisse der Strassen." Hannover. Schmorl u. v. Seefeld. 1880.

On any discontinuous line of descent which only differs from the line $A\ C\ D\ B$—Fig. 13—in that the line $D\ B$ is not on the ruling gradient s but on some other and "injurious" gradient s_1 less than s and greater than w, the descents on the lines $B\ D$ and $D\ C$ require no tractive-force; whereas in the section $A\ C$, of which the length is $\frac{1}{2}\ (l - CH)$ $= \frac{1}{2}\left(l - \frac{h}{s},\right)$the tractive-power required is

$$(Q + L)\ \frac{w + w}{2}\left(l - \frac{h}{s_1}\right) = (Q + L)\left(w\,l - \frac{w\,h}{s_1}\right)$$

It is thus greater than that on the descent on the line $B\ A$ except when $s_1 = w$, as represented in **Fig. 14.** In that case, instead of the broken line $C\ D\ B$ of Fig. 13—in which $C\ D$ is on the gradient w and $D\ B$ on the ruling gradient s—we have the straight line $C\ D\ B$, of Fig. 14, on the gradient w. The working-expenses for the descent from B to A will be the same as on a broken line $A\ E\ B$ of which the limbs ascend from A and B respectively and are continued till they intersect in E. Generally, on any series of lines drawn within the parallelogram $A\ E\ B\ C$ which nowhere has any gradient stiffer than w and, consequently, has everywhere "non-injurious" grades, the working-expenses of the descent are not greater than they are on the uniform gradient $A\ B$.

For the descent, the parallelogram within which the gradients may be moved without causing an increase in the working-expense, viz., without "**lost height**" occurring, is one the sides of which are inclined to the horizontal at the resistance-coefficient w.

In the **Second case**, when the uniform grade from A to B is greater than w, viz. is an "injurious" grade, the conditions are changed.

On the upward journey the performance of the locomotive in the parallel 6-sided figure of Fig. 13 (the sides of which have inclinations equal to the *ruling gradient* and to the *resistance-coefficient*) is, for all lines which in the direction A to B have no greater descents than w and no greater ascents than s, equal to $(Q + L)\ (wl + h)$—just as on the uniform ascent from A to B.

On the other hand, for the descents on the uniform grade, on which the train runs down without steam and with brakes applied, no tractive-force is required.

Consequently, in descents, only such lines on which no steam is used, namely those which are on "injurious" gradients, are equivalent to the uniform gradient. All such lines will lie within a parallelogram—**Fig. 15**—of which the sides $A\ C$ and $D\ B$ are on the ruling gradient, and $A\ D$ and $C\ B$ on a gradient equal to the resistance-coefficient. All lines lying inside this parallelogram and ascending in the direction from A to B on gradients steeper than w but less than s—and thus wholly "injurious" ascents—are not more expensive to work than the uniform ascent, and have therefore no "lost altitude."

The conclusions reached in the preceding investigation may be formulated in the following principle :—

A trace has no "lost height" on any given section of it when it consists either wholly of "non-injurious" gradients indifferently in which direction they rise; or when it consists wholly of "injurious" grades all rising in the same direction.

Whether the traffic is equal in both directions or whether it preponderates in one direction is quite immaterial. Only in the case—rarely occurring in railway working—where the traffic is all up-hill would the limits inside of which the trace can be laid down without "lost ascents" be wider than the above.*

[*Vide Appendix A.—Tr.]

§ 28.

The Distance Apart of Stations.

The distance apart of stations, which in the interests of the traffic should not be great, is determined by the expense and the loss of time which is occasioned by the stopping and starting of trains, the halts at stations, and by the cost of building and maintaining stations.

In order to decide whether the establishment of any particular station is justified or not, it is firstly necessary to know the increase in the traffic which it would bring to the railway and the resulting increase in net-returns. Accordingly the cost of construction, the annual expenditure on salaries and on the material up-keep of the station, and the cost of the stopping of trains, has to be determined.

This latter expense, which will now be determined, is made up of the expenses of stopping (braking) the arriving trains, of starting the departing trains, and of the halt of the trains at the stations, and in making this calculation, the loss of time in slowing-down and in again getting-up steam must be added to the duration of the halt.

The energy of a train moving with the velocity v—as already explained in § 26—is

$$M = \frac{\beta}{2g}(Q + L)\, v^2$$

If Q = weight of train, L = weight of locomotive (both in tonnes), v = the velocity in m/secs. $g = 9{\cdot}81$, then the energy of the train is

$$M = 55\,(Q + L)\, v^2$$

For a goods-train for which	$Q = 350$, $L = 60$, and $v = 7$,
	$M = 1100000$ m kg.
For a passenger-train when	$Q = 118$, $L = 54$, and $v = 13$,
	$M = 1600000$ m kg.
and for an express-train	$Q = 68$, $L = 54$, and $v = 18$,
	$M = 2000000$ m kg.

If a train moving with a speed v up a grade s_1 experiences a resistance $(Q + L)\,u$ due to the application of the brakes then, if the coefficient of resistance $w = a + b\,v^2$ there will be performed on a length $d\,l$ work represented by

$$d\,W = (Q + L)\left(u + s_1 + a + b\,v^2\right) d\,l$$

and the kinetic energy of the train is diminished by

$$d\,M = \frac{\beta}{g}(Q + L)\, v\, d\,v.$$

Equating the above two expressions we obtain

$$d\,l = \frac{\beta\, v\, d\,v}{g\,(u + s_1 + a + b\,v^2)}$$

Integrating this between the limits o and v, the length of run before coming to a standstill is obtained, *viz*,

$$l = \frac{\beta}{2gb}\log n\,\frac{u + s_1 + a + b\,v^2}{u + s_1 + a} \quad \ldots \quad \ldots \quad (38)$$

The time of slowing-down—since $\frac{dl}{v} = d\,t$—is given by

$$d\,t = \frac{\beta\, d\,v}{g\,(u + s_1 + a + b\,v^2)}$$

namely

$$t = \frac{\beta}{g\sqrt{(u + s_1 + a)\,b}}\ \text{arc tan}\ v\sqrt{\frac{b}{u + s_1 + a}} \quad \ldots \quad (39)$$

If, § 7, we put $a = {\cdot}00273$, $b = {\cdot}0000131$, then from the above equations (38, 39) a goods-train moving at 7 m/sec. after shutting-off steam but *without applying brakes* will come to a standstill after an interval of 247 seconds and after a run of 886^{m}. Similarly, a passenger-

train moving at a velocity of 13 m/sec. will come to a stop after an interval of 399 seconds having run 2492^m; and an express-train moving at a velocity of 18 m/sec. after the lapse of 489 seconds and having run $?939^m$.

The goods-train would with its initial velocity unchanged have covered the distance of 886^m in 127 seconds; so that if it is brought to rest by simply shutting-off steam and without the application of brakes there would be a loss of time in the journey of $247 - 127 = 120$ seconds.

For a passenger-train the loss of time under the same circumstances would be 207 seconds, and for the express-train 268 seconds.

For goods-trains in flat land the brake-coefficient u may be put at ·08; accordingly on a horizontal line the train would be brought to rest *after applying the brakes* in a length of 245^m and in 72 seconds.

For passenger- and express-trains in which all the axles except those of the locomotive are braked the braking-coefficient u may be taken as ·075; so that a passenger-train moving at a velocity of 13 m/sec. could be pulled up by the brakes after 18 seconds in a distance of 118^m, and an express-train moving at 18^m velocity, after 26 seconds in a length of 223^m.

But when entering stations, the efficiency of brakes—which can be much increased by the adoption of the contre-vapeur system—is as a rule inferior to this.

The distance in which the initial velocity is reduced to 3^m may on an average be assumed for goods trains at 400^m, for passenger-trains at 500^m, and for express-trains at 800^m.

In calculating the length of run and the time required to **pull-up a train** for any given value of the brake-coefficient there is no necessity for practical purposes to regard the dependence of the coefficient of resistance on the velocity. A perfectly sufficient degree of accuracy is obtained by the assumption of a constant resistance-coefficient corresponding to an average velocity, just as in the case of the momentum grade in § 26.

If the mean value of the coefficient of resistance be w then for a brake-coefficient u, on a length l, we have work done

$$W = (Q + L)\ (u + w + s_1)\ l$$

whence the energy consumed is

$$M = (Q + L)\,\frac{\beta v^2}{2g}$$

Consequently, equating the above,

$$l = \frac{\beta v^2}{2g\,(u + w + s_1)}$$

and inserting $\beta = 1{\cdot}08$, $g = 9{\cdot}81$,

$$l = \frac{{\cdot}055\ v^2}{u + w + s_1} \qquad \ldots \qquad \ldots \qquad \ldots \quad (40)$$

Equating the work done on the length $d\,l$ with the consumption of energy, we have

$$(u + w + s_1)\ d\,l = \frac{\beta\, v\, d\,v}{g}$$

or since $v\,d\,t = d\,l$,

$$d\,t = \frac{\beta\, d\,v}{g\,(u + w + s_1)}$$

whence integrating between the limits o and v,

$$t = \frac{\beta\, v}{g\,(u + w + s_1)}$$

or, in terms of Eqn. 40,

$$t = \frac{2\,l}{v} \quad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (41)$$

Now were the velocity constant, the distance l would be travelled over in the time $\frac{l}{v}$. Consequently, the time occupied in pulling-up the train is equal to the loss in time occasioned by the pulling-up of the train.

The distance required in which **to start and get-up speed**—assuming that the locomotive works with its full power $z\,L$—is determined as follows:

While the train is describing the length $d\,l$ there is communicated to the train an additional quantity of energy producing the increase in velocity $d\,v$; consequently

$$z\,L\,d\,l = (Q + L)\,(a + b\,v^2 + s_1)\,d\,l + \beta\frac{(Q + L)}{g}\,v\,d\,v$$

whence

$$d\,l = \frac{\beta\,(Q + L)}{g}\cdot\frac{v\,d\,v}{z\,L - (Q + L)\,(a + b\,v^2 + s_1)}$$

and integrating between the limits v and o,

$$l = \frac{\beta}{2\,g\,b}\,\text{logn}\,\frac{z\,L - (Q + L)\,(a + s_1)}{z\,L - (Q + L)\,(a + b\,v^2 + s_1)} \quad \dots \quad (42)$$

The time in which this distance is travelled-over is

$$d\,t = \beta\frac{Q + L}{g}\cdot\frac{d\,v}{z\,L - (Q + L)\,(a + b\,v^2 + s_1)}$$

and integrating between the limits v and o

$$t = \frac{\beta\,(Q + L)}{2\,g\,p\,q}\,\text{logn}\,\frac{p + v\,q}{p - v\,q} \quad \dots \quad \dots \quad \dots \quad (43)$$

where $\quad p = \sqrt{z\,L - (Q + L)\,(a + s_1)}, \quad$ and $q = \sqrt{b\,(Q + L)}$

For example: for a goods-train, if $Q = 350{,}000$ kg $\quad L = 60{,}000, \; z = \cdot06$
$g = 9{\cdot}81 \quad \beta = 1{\cdot}08 \quad a = \cdot00273$
$b = \cdot0000131 \quad s_1 = 0$

then the time required to attain a velocity of 7^m is 132 secs. and the distance in which this is done is 472^m.

Now this distance would be traversed in 67 secs. at a velocity of 7^m: whence the loss of time due to starting and getting-up speed is

$$132 - 67 = 65 \text{ secs.}$$

For a passenger-train of weight $Q = 118000$ kg., $L = 54000$, $z = \cdot04$ for one driving-axle, the velocity 13^m on the level will be attained in 157 secs. in a distance of 1062^m, and the loss of time in getting-up velocity will be 75 secs.

If the passenger locomotive has 2 coupled driving-axles, so that the tractive-coefficient $z = \cdot06$, then it would attain a velocity of 13^m after the lapse of 86 secs. in a distance of 616^m.

For an express-train, when $Q = 68{,}000$, $L = 54{,}000$, $z = \cdot04$ the time required to attain a velocity of 18^m would be 132 secs. and the distance 1244^m.

And here again for practical purposes the dependence of the coefficient of resistance upon the velocity may be ignored and a mean resistance-coefficient w assumed for the whole period of the getting-up of steam.

Accordingly,

$$z\,L\,d\,l = (Q + L)\,(w + s_1)\,d\,l + \beta\,\frac{Q + L}{g}\,v\,d\,v$$

whence

$$l = \frac{\beta}{2\,g}\cdot\frac{(Q + L)\,v^2}{z\,L - (Q + L)\,(w + s_1)} \quad \dots \quad \dots \quad (44)$$

and, substituting $d\,l = v\,d\,t$, the time consumed is

$$t = \frac{\beta}{g}\cdot\frac{(Q + L)}{z\,L - (Q + L)\,(w + s_1)}$$

or

$$t = \frac{2\,l}{v} \quad \dots \quad \dots \quad \dots \quad (45)$$

Since with the final velocity v, the distance l would be travelled-over in half this time the time lost in starting and acquiring velocity is equal to the time occupied therein.

According to the above calculation the loss of time in the slowing-down of the train and the getting under way again, as compared with the time of the unbroken journey, may be put on an average at 2 mins. Reckoning the halt at stations by passenger-trains at 1 min. and by goods at 5 min. then the total loss of time which calling at a station occasions is for passenger-trains 3 mins. and for goods-trains 7 mins.

If from the cost of train-expenses and locomotive expenses B_0, deduced in §§ 4 and 5, we deduct the outlay on lubrication and the maintenance and renewal of rolling-stock the remainder is the working-expenses dependent on the time. These per goods-train-km. are found to be 18·7 pf. for the train-service and 23·6 pf. traction-expenses, in all 42·3 pf. Per passenger-train-km. these items are train-service 22·3, and traction 18·8 pf. in all 41·1 pf.

Accordingly the expenses of a goods-train for a halt of 1 minute, its speed being 7^m, are

$$\frac{42{\cdot}3 \times 7 \times 60}{1000} = 17{\cdot}8 \text{ pf.}$$

and for a passenger-train—on the assumption that 16 million train-kms. are made by express-trains, and 53 million train-kms. by the ordinary passenger-trains and where accordingly the speed may be taken at 14^m—the cost is

$$\frac{41{\cdot}4 \times 14 \times 60}{1000} = 34{\cdot}8 \text{ pf.}$$

Since the cost of doing a tonne-km., viz. a million metre-kgs., by the locomotive has been evaluated at 25 pf., the cost of starting and getting-under way of a goods-train having an energy of 1100000 m-kg. is $1{\cdot}1 \times 25 = 27{\cdot}5$ pf.; and that of a passenger-train—taking the energy for both express- and passenger-trains as on the average 1700000 m-kg.—is $1{\cdot}7 \times 25 = 42{\cdot}5$ pf.

A part of the energy stored in the train is made use of when entering stations to carry the train in after shutting off steam. But this saving in steam may be disregarded because in the estimate of cost, the cost of braking, of the wear and tear of the brake-mechanism, of the wheel-tires, and of the rails, are likewise omitted.

Accordingly, the cost of stopping a goods-train is

$$27{\cdot}5 + 7 \times 17{\cdot}8 = 152{\cdot}1 \text{ pf.}$$

and of a passenger-train

$$42{\cdot}5 + 3 \times 34{\cdot}8 = 146{\cdot}9 \text{ pf.}$$

so that, generally, the expense of halting of a train at a station may be fixed at $1\frac{1}{2}$ M.

It may be possibly objected against the determination of the above sum that the few minutes loss of time which the halting of a train at a station causes does not increase the wages of train-staff, nor the quantity of the rolling-stock employed, nor the interest on its capital-cost; and that, in short, a halt occasions no extra expense. Certainly no very considerable increase of expense occurs if a single extra passenger leaves or enters the train; but it is not the single case but sum the of the cases that has to be considered—not the differential but the integral.

Having disposed of the above calculations, the question whether any proposed station is desirable or not is to be decided in the following manner.

For example: suppose that for the station at a place having 800 inhabitants the interest on the capital-cost and the annual maintenance-expenses of the whole station-equipment amount to 2,000 M., and the pay of the staff to 3,000 M. per annum; and assume that 3 trains in each direction stop at the station so that the cost of the halting of the 6 trains are $6 \times 365 \times 1\frac{1}{2} = 3{,}285$ M., then the total cost occasioned by the presence of the station amounts to 8,285 M.

It must now be investigated whether the additional business brought by the station to the whole system justifies such an expenditure.

According to Part I of this work § 17, every individual newly drawn into the railway traffic produces on an average, in a district of average economic importance an increase of net-earnings for the whole railway of 17·6 M. Leaving out of consideration the influence of the hinterland—since of course, the presence of the station in question will withdraw a portion of their hinterland traffic from the neighbouring stations—the establishment of a station produces an increase of net-earnings of $17{\cdot}6 \times 800 = 13{,}080$ M.; and thus, after deducting the expenses of the station installation, yields $13{,}080 - 8285 = 4{,}795$ M.

If the locality had 400 inhabitants, and the station were simply a flag-or temporary passenger-station, then in that case the annual outlay on persons and things might be reduced to 1,000 M. to which the cost of the stopping of 2 daily trains in each direction, viz., $4 \times 365 \times 1\frac{1}{2} = 2{,}190$ M. would have to be added, and the total cost would thus be 3,190 M.

As on an average the distance travelled per annum per head of the inhabitants is 285 km. and the net-gain per passenger-km. is to be taken as ·02 M. the establishment of a flag-station would yield an increase in net-gain of $400 \times \cdot 02 \times 285 = 2{,}280$ M.

Consequently, the creation of such a halting-place would from a purely business point of view be unjustifiable.

But since the public benefit drived from railways is considerably greater than $2\frac{1}{2}$ times the net-gain—see Part I.—therefore **on State Railways, stations and flag-stations may and should be fixed at much smaller distances apart than on Company's lines.**

It hardly needs to be explicitly stated that the above investigation as to the propriety of the establishment of intermediate stations can make no claim to great accuracy.

§ 29.

Virtual Length.

There are certain problems connected with railways in discussing which we do not need to know the absolute amount of the working-expenses—but merely whether the working-expenses of a line are higher or lower than those on another line with which it is compared, or the multiple the working-expenses of one line are of another line.

This occurs, for example, when it is a question of determining amongst several lines connecting two points or terminals the line on which the traffic can be carried most cheaply.

And also when the freight-rate is not to be fixed proportionately to the length of the projected line but in proportion to the working-expenses it is only necessary to know their relative and not their absolute amount.

In the solution of such problems the conception of **virtual length** has been employed.

In determining the virtual length of a line the simplest seems to be to select the working-expenses on a straight level line as the unit. But for practicalpurposes it is more convenient to choose for the unit (as has been proposed by **Schübler**) the working-expenses on a flat land or undulatory line of which the limit of the "non-injurious" grade is the ruling gradient.

If the unit working-expenses per km. on such a flat-land line are k_1 then the working-expenses on a section of length l on a grade s_1 for a curve-resistance c and for a ruling gradient s, are according to § 10

$$k\,l = \left[\alpha + \beta s + \gamma (s_1 + c)\right] l$$

then the virtual length λ of the section—*i.e.* **the length of a flat-land line which would have the same working-expenses as the line under examination**—is found from the equation

$$\lambda\,k_1 = \left[\alpha + \beta s + \gamma (s_1 + c)\right] l$$

namely

$$\lambda = \left[\frac{\alpha}{k_1} + \frac{\beta}{k_1} s + \frac{\gamma}{k_1} (s_1 + c)\right] l$$

or, if

$$\frac{\alpha}{k_1} = \alpha_1, \quad \frac{\beta}{k_1} = \beta_1, \quad \text{and} \quad \frac{\gamma}{k_1} = \gamma_1$$

then

$$\lambda = \left[\alpha_1 + \beta_1 s + \gamma_1 (s_1 + c)\right] l$$

Thus the virtual length of a section is not only dependent on its gradient s_1 and on its curve-resistance c, but it is also affected by the ruling gradient s of the whole line.

The term within the brackets

$$\alpha_1 + \beta_1 s + \gamma (s_1 + c) = \phi$$

is termed the **virtual coefficient.**

From § 10 the working-expenses per tonne-km. gross-load were determined as

$$k_1 = \cdot 24 + 10\, s + 17\,(s_1 + c)$$

and for a flat-land line, where $s = \cdot 0036$, $s_1 = \cdot 0036$, $c = o$,

$$k_1 = \cdot 337 \text{ pf.}$$

Accordingly, the virtual length for goods-traffic is

$$\lambda = \cdot 71 + 30\, s + 50\,(s_1 + c) \qquad \ldots \qquad \ldots \quad (46)$$

The following Table gives the Virtual Lengths as obtained by dividing the working-expenses given in Table IX by $k_1 = \cdot 337$, and the result is an almost perfect coincidence with Formula 46.

TABLE XVII.

Virtual Lengths for Goods Traffic.

Ruling Gradient.	"Non-injurious" gradient.	Virtual Lengths for "injurious" grades of			
		·006	·010	·015	·025
·0036	1·00	—	—	—	—
·006	1·09	1·19	—	—	—
·010	1·22	1·33	1·51	—	—
·015	1·36	—	1·67	1·91	—
·025	1·60	—	—	2·19	2·71

The working-expenses per tonne-km. of passenger-trains on a line of which the ruling gradient is $s = \cdot 0036$, are from Eqn. 11—in which the limit-value of the "non-injurious" gradient is to be substituted for s_1—

$$k_1 = \cdot 725 + 8 \times \cdot 0036 + 22 \times \cdot 0055 = \cdot 875.$$

If the figures of Table X be divided by this quantity then we obtain the virtual lengths for passenger-traffic as given in the following Table.

TABLE XVIII.

Virtual Lengths for Passenger Traffic.

Ruling Gradient.	"Non-injurious" gradient.	Virtual Lengths for "injurious" grades.		
		·010	·015	·025
·0036	1·00	—	—	—
·006	1·02	—	—	—
·010	1·08	1·18	—	—
·015	1·12	1·23	1·35	—
·025	1·18	1·29	1·42	1·67

By dividing Eqn. 11 by the working-expenses $k_1 = \cdot 875$—the requisite figure for the passenger tonne-km. on a flat land line—the virtual length for passenger-traffic is obtained, viz.

$$\lambda = \cdot 83 + 9\,s + 25\,(s_1 + c) \quad \ldots \quad \ldots \quad (47)$$

which gives a completely satisfactory coincidence with the figures of Table XVIII.

Tables XVII and XVIII shew what has been already stated in § 10, viz. that **gradients have very slight influence on passenger-traffic working-expenses as compared with their effect on goods-traffic.**

For goods-traffic the working-expenses on a mountain railway having a ruling gradient of ·025 on sections with "non-injurious" grades are 1·6 times as large as on flat-land line: whereas for passenger-traffic they are only 1·18 times. On a grade of ·025 the working-expenses for goods-traffic are 2·71 times as large as those on a level line; whereas for a passenger-traffic they are only 1·67.

The virtual length of any given trace can be very simply determined if we first calculate the equivalent grade, s_2, of the line—by the method of § 11.

For example: suppose there are two alternative lines A and B of a projected railway between two points, of which the line A has a length of 60 km., a ruling gradient of s = ·015, and an equivalent grade s_2 = ·010. The line B has a length of 50 km., a ruling gradient s = ·025, and an equivalent grade s_2 = ·015. From Table XVII—or from Eqn. 46—the virtual length of the line A for goods traffic would be

$$\lambda = 60 \times 1{\cdot}67 = 100{\cdot}2$$

and that the line B_1

$$\lambda = 50 \times 2{\cdot}19 = 109{\cdot}5$$

For passenger-traffic, the virtual length according to Table XVIII—or from Eqn. 47—for the line A would be

$$\lambda = 60 \times 1{\cdot}23 = 73{\cdot}8$$

and for the line B

$$\lambda = 50 \times 1{\cdot}42 = 71{\cdot}0.$$

Consequently the line A for goods-traffic is cheaper in working than B, but for passenger-traffic its working would be more expensive. From Table IX, the actual working-expenses for goods-traffic on A per tonne are

$$60 \times {\cdot}563 = 33{\cdot}78 \text{ pf.}$$

and for the line B

$$50 \times {\cdot}739 = 36{\cdot}95 \text{ pf.}$$

And from Table X, on the line A per tonne of passenger-trains they are

$$60 \times 1{\cdot}076 = 64{\cdot}56 \text{ pf.}$$

and on the line B

$$50 \times 1{\cdot}241 = 62{\cdot}05 \text{ pf.}$$

This example discloses the important fact that in weighing the merits of several differently circumstanced lines connecting two points **a choice of any one line on the ground; of both goods- and passenger-traffic may under certain circumstances be impossible.** However, for passenger-traffic the working-expenses are not the sole basis for choice since it is often the case that owing to the greater speed possible on the line with the flatter grades it may be the preferable one in spite of the fact that the working-expenses are somewhat greater.

The foregoing discussion also shows that in determining which of several lines connecting two localities has the smallest working-expenses the employment of the virtual length is in no way simpler than directly calculating the working-expenses.

When it is not a question of comparing the working-expenses on already existing lines but of making a choice between several projected lines then the employment of the virtual length is still more cumbrous, because in that case we require the actual amounts of the working-expenses which would themselves have to be deduced from the calculated virtual lengths, and would therefore be more simply determined directly.

The conception of virtual length would be entirely unnecessary if its use were confined to simply comparing the working-expenses of several lines with the object of selecting one of them.

But the idea of virtual length is valuable as a basis on which to determine **rates proportional to the working-expenses.**

In connexion with the employment of virtual lengths for the fixing of the rate-distance or **tariff-length**, opinions differ as to the meaning of the term. Many engineers, and Schübler amongst them, would include the construction-cost and maintenance-expenses of the line in the virtual-tariff-length.

Schübler* proposes that the average amount of the interest on the capital for the whole railway system of a country incident per tonne-km. and the maintenance-expenses of the line—which he fixes for Germany at ·525 pf.—should be added to the working-expenses of

* Schübler: Ueber den Begriff der Virtuellen Länge und die praktischen Anwendungen derselben. *Centralblatt der Bauverwaltung.* 1886.

the section (train- and traction-expenses) and from the unit-cost thus obtained to determine the virtual-traffic-length which is then made the basis for the fixing of the rate of freight.* The working-expenses on a flat-land line plus the transport-expenses* would amount per tonne-km. of the gross-load of goods-trains accordingly to

$$\cdot 337 + \cdot 525 = \cdot 862 \text{ pf.}$$

and on a grade of ·025, to

$$\cdot 914 + \cdot 525 = 1 \cdot 439 \text{ pf.}$$

So that the virtual-tariff-length of the latter would be

$$\frac{1439}{\cdot 862} = 1 \cdot 78.$$

But the following calculation will show that the optimum or most advantageous kilometric freight-charge, and therefore the tariff-length which is to serve for fixing of the same is entirely independent of the road-expenses.

If d be the unit freight-charge per km., k the kilometric working-expenses, exclusive of the road-expenses as given in Tables IX and X, $A\ i + U$ the kilometric maintenance-expenses and interest on the capital, and C the volume of traffic, then the net-gain from working, after deducting the interest on capital, is

$$N = C\left[\,d - k - \frac{A\,i + U}{C}\,\right]$$

or

$$N = C\,(d - k) - (A\,i + U).$$

The volume of traffic will increase according to some function C of the freight-rate, say $F\,(d)$, when the latter diminishes.

Putting

$$C = F\,(d)$$

then

$$N = (d - k)\ F\,(d) - (A\,i + U).$$

Whence the condition for the optimum freight-rate for which N is a maximum is

$$(d - k)\,F'(d) + F(d) = 0. \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

Thus the most advantageous freight-rate depends solely on the absolute working-expenses, k, and on the form of the function F (d), and is entirely independent of the road-expenses.

We have seen in Part I.—§ 13—that the optimum freight-rate is a definite multiple of the working-expenses, and therefore varies proportionately as the working-expenses.

Quite apart from the circumstance that in determining the best freight-rate, and the tariff-length for fixing the freight-rate, the volume of traffic, which depends on the rate of freight and hence the average cost of carriage per unit of traffic, cannot be assumed as constant, the tariff-length of Schübler does not succeed any better than any other in giving the same *i.e.*, a uniform net-gain per tonne-km. on any and every section of the line. This it would only do in the particular case in which the freight-rate on the flat-land line were made exactly ·862 pf., and thus higher by the average amount of the cost of carriage,† viz., ·525 than the working-expenses.

For example: were the freight-rate by Schübler's tariff-length on the flat-land line 1 pf., and thus = 1·78 pf. for a section on a raising gradient of ·025, then on the flat-land line a net-gain there would be obtained of

$$1 - \cdot 337 = 663 \text{ pf.},$$

and on a gradient of ·025 one of

$$1 \cdot 78 - \cdot 914 = \cdot 886 \text{ pf.}$$

* In an earlier work: "Virtual Length, Virtual Grade, and Tariff Length of Railways"—*Organ für die Fortschritte des Eisenbahnwesens:* 1879, I also had considered it necessary to take into consideration the road-expenses in determining the tariff-length, but subsequently I became convinced of the erroneousness of this view.

[† *Scil.* interest on capital cost + cost of maintenance.]

The interest on the capital and the maintenance-expenses of the line—independent of the amount of the traffic—which under all circumstances are constant items of outlay, can never influence the optimum freight-rate producing the maximum attainable net-working gain.

The net-gain from working arises from the excess of the freight-rate over the working-expenses per unit of traffic and is a multiple, proportionate to the volume of traffic, of this net-profit per unit.

But since the volume of traffic decreases as the unit freight-rate increases and *vice versâ*, there is evidently a certain definite optimum freight-rate which depends on the amount of the increased working-expenses of the transport of the traffic-unit but which is entirely independent of the constant road-expenses.

Were the most advantageous ratio of freight-rate to operating-expenses to be determined on the basis of the law of the dependency of the traffic volume on the freight-rate, then the freight-rate should be fixed according to this relation not only for the different kinds of merchandise but also for the different traffic-sections.

Thus the amount of the freight-rate is determined not only by the length of the section over which the transportation is made but also by the amount of the operating-expenses on a given section when these expenses increase proportionally with the number of units transported.

The freight-length should therefore be equal to the virtual length as given by the Formulæ 46 and 47.

§ 30.

Virtual Gradient.

If the equivalent grade of a line be s_2 of which the ruling gradient is s and its length l then the operating-expenses for the transportation of a unit of traffic will be—according to § 11— of the form

$$K = (\alpha + \beta s + \gamma s_2)\, l.$$

Now to ascertain the operating value of the line we can determine a **gradient** s_3 — the length of the line remaining constant—which would have **the same working-expenses as the line in question** if this gradient were uniform throughout the whole length of the line and were at the same time the ruling gradient. For such a line then

$$K = (\alpha + \beta s_3 + \gamma s_3)\, l.$$

This gradient s_3—which we may term the **virtual gradient** of the line—is given by the equation

$$(\alpha + \beta s_3 + \gamma s_3)\, l = (\alpha + \beta s + \gamma s_2)\, l$$

and is

$$s_3 = \frac{\beta}{\beta + \gamma} s + \frac{\gamma}{\beta + \gamma} s_2$$

For Goods-traffic—from § 12, $\beta = 10, \quad \gamma = 17$:

so that the virtual gradient for goods-traffic is

$$s_3 = \cdot 37\, s + \cdot 63\, s_2 \quad \ldots \quad \ldots \quad \ldots \quad (49)$$

For Passenger-traffic, in the same § it was found that

$$\beta = 8, \quad \gamma = 22,$$

so that for passenger-traffic the virtual gradient is

$$s_3 = \cdot 27\, s + \cdot 73\, s_2 \quad \ldots \quad \ldots \quad \ldots \quad (50)$$

By the aid of the virtual grade we obtain the operating-expenses for the transportation of a tonne gross-load of goods-trains over the whole line as

$$K = (\cdot 24 + 27\, s_3)\, l$$

or

$$K = \cdot 24\, (1 + 112{\cdot}5\, s_3)\, l \quad \ldots \quad \ldots \quad \ldots \quad (51)$$

And per tonne of passenger-train

$$K = (\cdot 725 + 30\, s_3)\, l$$

or

$$K = \cdot 725\, (1 + 41{\cdot}4\, s_3)\, l \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

Numerical Example. Let the ruling gradient of a line be ·015, its equivalent gradient for goods-traffic, ·00643, for passenger-traffic ·00779, as was found in the Numerical Example of § 11. Then the virtual grade of this line for goods-traffic is

$$s_3 = \cdot 37 \times \cdot 015 + \cdot 63 \times \cdot 00643 = \cdot 0096$$

and for passenger-traffic it is

$$s_3 = \cdot 27 \times \cdot 015 + \cdot 73 \times \cdot 00779 = \cdot 0097.$$

The *equivalent* grade for passenger-traffic is always greater than that for goods-traffic; whereas the *virtual* gradient for both descriptions of traffic is always within narrow limits the same.

Dividing Eqns. 51 and 52 by the costs on an flat-land line, *i.e.* by ·337 and ·875, we obtain the virtual length in terms of the virtual gradient—and this for goods-traffic is

$$\lambda = \cdot 71\, (1 + 112{\cdot}5\, s_3)\, l \quad \ldots \quad \ldots \quad \ldots \quad (53)$$

and for passenger-traffic,

$$= \cdot 83\, (1 + 41{\cdot}4\, s_3)\, l \quad \ldots \quad \ldots \quad \ldots \quad (54)$$

The virtual length is for goods-traffic therefore equal to 2, if the virtual grade is ·0115; but for passenger-traffic only so when the virtual grade rises to ·0282.

It is perhaps hardly necessary to point out that the calculation of the working-expenses and also the virtual length is more simply and briefly made without the previous determination of the virtual grade.

The idea of the virtual grade is of little value in the location of the trace, and is only useful as a criterion of the operating-value of a line.*

[* NOTE.—The term "Virtual Grade" has a totally different signification in American railway work. McHenry, Chief Engineer, Northern Pacific Ry. Co., says:—"A gradient of equivalent resistance to the *force exerted by the engine* is the "virtual grade"—or real resistance taxing the engine cylinders." See Rules for Railway Location and Construction of the Northern Pacific Ry. Co., 1899. New York, Engineering *News* Publishing Co.—TR.]

SECTION III.

THE LOCATION OF THE TRACE.

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§ 31.

General and detailed Tracing.

General location is distinguished from **detailed** location; in other words, **the general** and **the detailed preliminary studies** are distinct and separate from one another.

The object of the general or preliminary project is to determine whether the line is worth constructing or not; and where the line is projected by a private company its object is to frame the requisite design and estimate for the Governmental sanction to the line; whereas that of the detailed location is to provide the engineer with the essential and fundamental data for the construction of the line, viz., a project fully worked-up in its minutest details.

For the **Preliminary project** there is required:

(1). A **General Map** to a scale of from $\frac{1}{200000}$ to $\frac{1}{50000}$ in which the line is divided into kms. and numbered at every 5 km.

In Prussia for this purpose the General Staff map of $\frac{1}{100000}$ must be used, and the line is to be laid down thereon in vermilion.

(2). A **Longitudinal Section** showing the elevation of the ground and of the projected line and its grades, the length of station-sites, tunnels, and of bridges, openings, road-overways, the depths of river channels, and H. F. levels of waters of all kinds, the height of the subsoil-water, the nature of the soil, and the kind of cultivation of the ground, and the details of the curvature of the line in the space set apart for the purpose.

In Prussia the longitudinal scale prescribed is $\frac{1}{10,000}$: this is also the usual scale elsewhere, and for the vertical scale $\frac{1}{500}$ or, more commonly, $\frac{1}{1000}$. As regards all other details of procedure and as to the colouring of the longitudinal section, see the Regulations cited hereafter in treating of the detailed project.

(3). A **Survey Plan** of the immediate neighbourhood of the line; this is as a rule placed under the longitudinal section, and shows ground to a distance of 250^m on either side of the centre-line.

By the Prussian Regulations the Survey Plan is to be drawn to a scale of $\frac{1}{10000}$;

When the shape of the ground is such as to influence the location of the trace, contour lines should be shown at every 5^m or 10^m in vertical height. The North point must be carefully laid down.

For lines in mountainous or in hilly ground it may be necessary to show the form of the ground by a more extensive use of contours on separate sheets.

(4) A **Descriptive and Explanatory Report** of the line, with **a statement** of the major works occurring thereon, and **of the reasons for the location chosen, and of the principles which have guided the location in every detail.** An exposition of the financial value of the projected line will be given either here or in some especial form as an appendix.

(5) A **General Estimate of Cost** under the same heads as given in the detailed estimate of cost.

In the **Detailed Project** there must be given an exact representation of the line in plan and elevation and a statement of the technical grounds for the acquisition of the land, and for the choice of type of earthwork, the design of the permanent-way adopted, the bridges, smaller openings, road over-crossings, retaining walls, tunnels, and for any special or extra-ordinary works, and finally for the stations and their individual sites.

In Prussia the following orders for the preparation of the Detailed Project were issued in October 1871:—

(1) **The longitudinal section** of the line shall show distances to a scale of $\frac{1}{2500}$, and heights to a scale of $\frac{1}{250}$. The line is to be subdivided or "stationed" by marks 100^m apart, and the "stations" are to be numbered, consecutively from the initial point, immediately under the verticals of the section: the kilometric numeration will likewise be made underneath at each "station" and indicated by Roman figures.

The distance of the centres of the stations and the positions of any other points that may possibly be in any way noteworthy lying between stations are to be indicated from the preceding station centre as zero; and similarly the position of the changes of grade, the commencement and ends of curves, tunnels, station-plots, position of bridges, culverts, road over crossings, etc.

All elevations are to be referred to the zero of the Amsterdam-gauge. If in any case this gives too long ordinates for some of the sheets the datum may be raised, but always by multiples of 10^m, measured vertically above the horizontal. The elevations are of course always referred to zero irrespective of any such elevation of datum.

All the heights relating to the existing condition of things previous to the location of the line are to be written in black figures, also the R. L. of the ground at the chainages at the centres of stations and any other intermediate points worthy of notice, the crown-width edges of banks and heights of roadway of roads either crossed by or lying in the neighbourhood of the line, also the crown and springing-heights arched bridges spanning such roads, the undersides of the structures in the case of metallic or wooden bridges, the bottom and edges of sheets of water, the zeros of any gauges met with in the neighbourhood of the line, the heights of weirs and their bottom sills, the flood-marks on neighbouring buildings, etc.

The lowest, mean, and highest flood-levels of all water surfaces, the surface of water in wells, and the figures relating to the borrow-pits are to be indicated in blue ink figures: all other numerical data of the project are to be marked in vermilion.

The results of the examination of the ground as to its character, depth, and stratification; the bottoms of morasses and swamps, and the economic uses to which the surface of the ground is applied—viz., whether arable-land, grazing, forest, garden, etc.: and finally, the boundaries of the parish, county, etc., are to be indicated on the longitudinal section.

The gradients of the line are written-in from one change of grade to another above the grade-line along with the length of the individual grade lengths: the P. C. and P. T. of curves, their radii, lengths, and the lengths of the intermediate straights are written immediately under horizontals in the space set a apart for the purpose.

The height of the cultivated ground-surface, *i.e.* the bottom of the banks and the top of cuttings, is shaded-off downwards in some brown colour. The areas of the cuttings are coloured grey, and those of the banks a pale red. The figures showing the heights and depths of banks and cuttings, respectively, are in red.

The bridges, culverts, road over-crossings, and tunnels are represented in section corresponding to the horizontal and vertical scales with the clear width, or clear length, as the case may be, and the name of the stream, river or road indicated.

The points of commencement and termination of station-grounds are indicated by ordinates prolonged above the formation-level, and adorned with terminal flags. The name of the station or halting-place is written between these flags.

Each sheet of the Horizontal Section is to be 46 cm. long and 64 cm. wide, and must be furnished with a title giving the name of the railway and subscribed with the name of the Engineer by whom the survey was made; *it must also be dated.*

Three or four sheets may be fastened together by means of linen guards.

Benchmarks may be advantageously established at distances of, say, 1000^m and should be placed *outside* the area of the operations of construction.

(2) The **Plan** shows ground for a width of at least 250^m on each side of the line.

The Plan according to the Prussian Regulations must be drawn to a scale of $\frac{1}{2500}$. The ground may be continued or repeated so far as it may appear necessary to determine the plan and elevation of the centre-line on each sheet for distances of 1^m to 5^m. *The North-point must be shown on each sheet.*

The centre line, divided into "stations" and kms., is dotted-in in vermilion, and the figuring similarly in vermilion. The earthwork of banks and cuttings with its side-pits and slopes and all the subsidiary works, such as road-crossings and diversions, service roads, protection-strips, bridges path-diversions, etc., completely represented, and all and everything actually existing is to be represented in black, and all the projected works are to be drawn and described in vermilion.

At the beginning and end of each sheet both in the plan and in the longitudinal section a piece of the preceding and following sheets representing at least 100^m., but as a plain line only, is to be shown.

(3) **Cross-sections** of the line are to be drawn horizontally and vertically to a scale of $\frac{1}{250}$. The point of the longitudinal section at which the cross-section is taken is to be given below the datum of the cross-section and the distance of all points thereon is to be referred to the centre-line of the railway. For the rest all the Regulations prescribed for the longitudinal section apply equally to the cross-section.

The cross-sections are as a rule drawn in an album of a certain specified size. The number of cross-sections depends on the configuration of the ground: in flat land two or three may suffice per km., whereas in hilly or mountainous often 50 to 100 or more may be necessary.

(4) The **Drawings** of Retaining Walls, Bridges, Tunnels, and Buildings are, as a rule, to be drawn to a scale of $\frac{1}{100}$. All the ground-plans, cross-sections, and elevations necessary for the complete understanding of the project must be given; and in the case of any extra-ordinary or important works separate detailed drawings on a larger scale are to be given.

The nature of the ground, and the highest and lowest water-levels are to be shown in the above: and the principal dimensions are to be in figures.

(5) The general arrangement of the **track** structure* and that of the road-crossings at rail-level are to be shown to a scale of $\frac{1}{50}$: the cross-section of the rails and fish-plates, the bearing-plates, fish-bolts, spikes, etc., are to be drawn full-size, and their individual weights are to be specified on the drawings.

(6) The **plans of station yards** and stopping-places are to be prepared to a scale of $\frac{1}{1000}$: and not only all the buildings but also their immediate surroundings, together with the approach-roads are to be shown. Also all the tracks and sidings, points and crossings, loading-ramps or platforms, turn-tables, traversers, water-cranes, goods-platforms, ashpits, washing-out pits, lanterns, signals, drains, underground-passages, etc., and the curvature and grade of the line are to be fully represented thereon. *Also the North-point.*

All **drawings** are to be on sheets of the same size as those of the longitudinal-section (46 × 64 cm.); and where necessary are to be put up so as to fold. For works of continual recurrence such as water-ways, culverts, minor bridges, road-crossings, over- and under-crossings, many kinds of station-buildings, switches, traversers, etc., it is advantageous to adopt **standard types.**

(7) The **Explanatory and Descriptive** Report is to contain full particulars regarding the district or country passed through, and an estimate of the probable traffic; also the fundamental reasons for the direction of the line chosen and its position in elevation, and a statement of the conditions governing the location, together with description of the line and of its works of art; the method or scheme of construction is to be described and accounted for, also some data are to be given as to the time within which the line and its accessories will be fully completed.

* [The so-called "permanent-way".—Tr.].

(8) **The Estimate of the Cost.** This is to be distributed under the following heads :—

I. Acquisition of land, and Compensation for damages.

II. Earthwork, grading, retaining walls—inclusive of these items for road over-crossings.

III. Maintenance of the sub-grade structures, and of that of the track during the period of construction and for one year after the opening of the line.

IV. Fencing.

V. Road crossings, including over- and under-passages, with all accessories.

VI. Culverts and Minor Bridges up to 10^m clear span.

VII. Major Bridges.

VIII. Tunnels.

IX. Special provisions and equipment for the working of inclines.

X. Permanent-way, together with all sidings and switches.

XI. Signals, with the requisite huts and dwellings of the staff.

XII. Stations, permanent and temporary, with all accessories such as buildings, turntables, water-cranes, etc.

XIII. Other and Special works, as stream-diversions, river training works, protection works, etc.

XIV. Rolling-Stock,

XV. Administration.

XVI. General.

XVII. Interest on Capital during the period of construction.

§ 32.

Location in Flat Land.

Flat-land lines are those lines in plains or undulating ground of which the grades are so moderate that **brakes are never required when running down-hill,** and the curve-radii are so large that there is **no noticeable increase in resistance due to curvature.**

On such lines, accordingly, the grades never exceed the value of the coefficient of resistance $w =$ **·0036,** and the curve-radii are always greater than **1000^{m}.**

To adapt the Eqns. 10 and 11 of § 13, which give the working-expenses per tonne-km. of paying-load, and per passenger-km., to flat-land lines the curve-resistance c must be made $= 0$, and s_1 must be put $= w$. Consequently, for goods-traffic, putting $s_1 = \cdot 0036$ and for the passenger-traffic $s_1 = \cdot 0055$, the operating-expenses for a line of length l km. on which there is annually moved T tonnes paying-load and P passengers—in pfennigs—are

$$K = T(\cdot 703 + 23\tfrac{1}{3}s)\,l + P(1\cdot 134 + 10\tfrac{2}{3}s)\,l \quad \ldots \quad \ldots \quad (55)$$

The working-expenses thus depend—apart from the volume of traffic and the total length of the line—only on the size of the ruling gradient.

The first task of technical locating is this: To determine **provisionally** the position in elevation and grading of the series of lines previously fixed by the principles of Commercial Tracing. This is to be done according to the principles laid down in § 22. The minimum distance apart of the gradients is to be proportional the speed of travel on the line: and while following as closely as possibly the configuration of the ground, and equalizing as far as practicable the cuts and fills, attention must be paid to the height of the subsoil or ground-water.

In the location of stations, and of the crossings of roads and streams, suitable elevations are to be sought for: in the case of stations, such that will give a yard of the requisite length, either level or at least one of which the gradient lies somewhere between 0 and ·0036.

While this preliminary working-up of the longitudinal section is in hand the angular points of the commercial trace may be replaced by curves of radii of not less than 1000^{m}, and sufficiently long straights inserted for the sites of the stations.

After this provisional work on the section the next step is to determine the most favourable ruling gradient. If in Eqn. 55 the actual numerical values of T, P, and l are inserted we obtain a simple form for the operating-expenses in the form

$$K = D + Es.$$

Numerical Example: putting $T = 400{,}000$, $P = 250{,}000$, $l = 120$, then the working-expenses in M. are

$$677{,}640 + 14{,}400{,}000\, s.$$

Supposing in the first fixing of the grades the maximum gradient occurring be ·0036 on a length of 4 km.: then by flattening the line on this length to a grade of ·003 there would be a saving in working-expenses there on of

$$14{,}400{,}000 \times \cdot 0006 = 8{,}640 \text{ M.}$$

Reducing the ruling gradient would be pecuniarily desirable if the interest on the consequent increase of the cost of construction were less than this amount—or at a rate of 4%, if the increase to to capital were less than 216,000 M.

Supposing this flattening of the grade proves to be desirable, then it would have to be next examined whether a further flattening of this maximum gradient of ·003—perhaps now on a total length of 7 km.—to one of ·0027 would be desirable.

The consequent increase in the capital-cost of the line should not entail the payment of a greater sum of interest than the reduction obtained in the working-expenses, viz., of

$$14{,}400{,}000 \times \cdot0003 = 4320.$$

if the flattening of the grade is to confer any pecuniary advantage.

By the aid of such trial calculations the best value of the ruling gradient is quickly obtained.

However, the above method of determining the best ruling gradient is not suitable for lines of small traffic. And for this reason: the flattening of the ruling gradient reduces the working-expenses only by enabling heavier trains to be hauled. So that if there be no necessity at all for hauling heavier trains, as on lines of small traffic, then flattening the ruling gradient will not result in a reduction of operating- expenses.

For example: if a Local line has an annual goods-traffic of 90,000 paying-tonnes=210,000 tonnes gross-load, and a passenger-traffic of 120,000 persons=150,000 tonnes train-load, or an annual total-traffic of 360,000 tonnes-train-weight, then the daily load hauled would be 990 tonnes, or 3 trains daily in each direction of 165 tonnes each. If locomotives weighing 30 tonnes and a tractive-force coefficient of ·08—thus giving a tractive-force of 2·4 tonnes - were employed then the grade on which a train of the above weight could be hauled up-hill is given by the expression

$$(165 + 30)\ (\cdot0036 + s) = 2\cdot4.$$

or

$$s = \cdot0087.$$

If it do not appear desirable to reduce the number of trains and so increase the train-weight, then on a flat-land line no reduction of working-expenses is possible if the ruling gradient is less than ·0036.

In such cases it will often be a question well worthy of study whether a saving in cost of construction cannot be obtained by altering the character of the line, and using a ruling gradient greater than ·0036.

This question would then be treated in the manner laid down in § 33.

Having determined the ruling gradient, making it less than or equal to ·0036, thus maintaining the character of a flat-land line, we may then pass on to the third stage in the technical locating.

This consists in examining whether for some one reason or another it might be advantageous to alter the plan of the line in parts of the trace. In weighing all the pros and cons it must be remembered that the working-expenses of a flat-land line, once the ruling gradient has been fixed, depends only on the length of the line.

According to Eqn. 55 the operating-expenses per km. of a flat-land railway of which the ruling gradient is, say, $s = \cdot003$, and on which there is a traffic of 400,000 tonnes paying-load and 250,000 passengers, is about 6,000 M.

If the kilometric-expenses for road-maintenance and line-supervision be 3,000 M. per annum, then the working- and maintenance-expenses per running m. are 9 M., which at 4°/₀ interest corresponds to a capital sum of 225 M.

An alteration in the plan of the trace is therefore advantageous if when the line is lengthened x metres the capital-cost is lessened by more than 225 x M., or if when the line be shortened by x m. the extra capital outlay is less than 225 x M.

In this way any and every question that can arise as to changes in the trace may be very simply solved.

Considerations affecting the acquirement of land, the character of the ground, or the existence of road and river-crossings may render necessary alterations in the trace.

Fig. 16

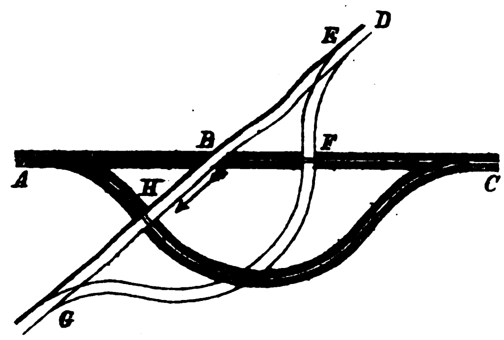


Fig. 17.

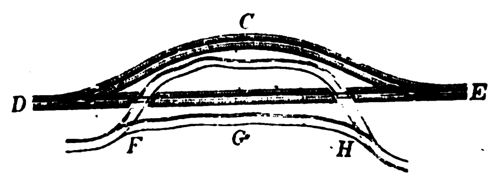
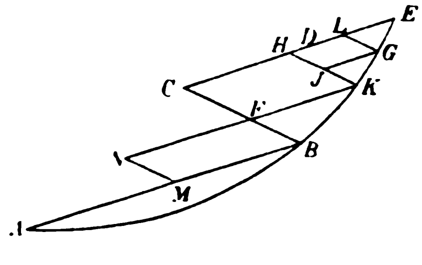


Fig. 18.



As regards land-acquisition, built-over ground or land valuable from other considerations should be gone round; running through private property is to be avoided; churches and cemeteries will usually not be interfered with.

The configuration of the ground will rarely occasion a diversion of the trace; but the character of the ground may easily do so: thus bogs and marshes and all ground unsuited to embankments are to be avoided.

Road-crossings will seldom afford sufficient ground for altering the trace in plan, and probably only in the case of Local lines; this is so because a sharply-angular crossing is usually more cheaply avoided by a diversion of the road crossed than by diverting the railway.

Changes in plan of the initial location of the trace are most commonly caused by the conditions under which streams and rivers affect the line. In such cases the line should be placed as far outside flood-areas as possible, and running-through standing water is to be avoided. Running water should be bridged at the place where the stream is uniform and controlled, and where it offers a suitable ground for foundations and a rectangular crossing.

To avoid a diagonal crossing either a diversion of the stream or an alteration in the plan of the line may be adopted.

If it be desirable to divert the course of the *stream* from—**Fig. 16**—the line *E B G* into the line *E F G*, then the saving due to the rectangular bridge at *F* as compared with the oblique bridge-crossing required at *B* ought to exceed the cost of diverting the watercourse. But the deviation of the *line* or trace from *A B C* into *A H C* is only justifiable if the saving in the rectangular bridge at *H* is greater, as compared with the diagonally-crossing bridge at *B*, than the additional expense of building the line *A H C* as compared with *A B C*, plus the additional 225 x M. for the maintenance and working of the x^{m} of extra length of line.

Repeated crossings may be likewise avoided by a diversion of either the stream or of the trace—**Fig 17.**

In both cases the cost of two bridges at *A* and *B* is saved; but in the one case the stream must be diverted into the line *F G H*, in the other, the line must be deviated into the line *D C E*. This involves an increased capital outlay, and in addition there is 225 x M. be added for operating- and maintenance-expenses for the increased length of x metres.

§ 33.

Location in Hilly Country.

The trace in hilly country will have gradients in several of its sections which exceed the limits of "non-injurious" gradients. The plan of the trace is influenced by the configuration of the ground, which latter will in the Technical Trace usually occasion very considerable deviations from the Commercial Trace.

The simplest case is when both terminal points of the trace lie in one and the same valley, so that the line follows the direction of the thalweg or water-channel. But if the terminal points lie in different valleys unconnected with each other, then the trace has to be carried over one or more watersheds or "divides." In that case the best crossing-point of the watershed has to be determined both in plan and elevation; the whole length of the trace is thus divided by these watersheds into sections connecting points in one and the same valley.

To surmount the watershed, the lowest points therein, viz., the saddles, or passes, are, as a rule, to be chosen, and amongst them there is usually one which is better than the other adjacent ones. Only in exceptional cases will it be advantageous to cross a watershed at some higher point, where it may be that the formation of the ground is decidedly more advantageous for tunnelling work, or where a considerable shortening of the line can be obtained as compared with the line over the pass.

For each point of the watershed that can possibly to come up for consideration as a possible site for the crossing, the best position elevation of the trace must be determined before a comparison with rival points of passage can be made.

Technical tracing in hilly land when there are two or more watersheds to be crossed begins with the determination of the best height at which to pierce them. With this object that watershed is first studied in the approaches to which the steepest gradient presumably occurs, which then becomes the ruling gradient for the whole line.

If the ruling gradient s is carried up in a length l_1 to the watershed then the working-expenses on it per tonne net-load are from Eqn. 10—p. 31,

$$k_1 = (\cdot 56 + 23\tfrac{1}{3}s + 39\tfrac{2}{3}s)\, l_1$$

If on the other side of the ridge the down line has an equivalent grade s_1 for a length l_2 then on this the working-expenses per tonne of paying-load are

$$k_2 = (\cdot 56 + 23\tfrac{1}{3}s + 39\tfrac{2}{3}s_1)\, l_2$$

If, finally, the length of the remaining length of the trace is l_3 and its equivalent gradient is s_2 then the working-expenses on it per tonne net-load are

$$k_3 = (\cdot 56 + 23\tfrac{1}{3}s + 39\tfrac{2}{3}s_2)\, l_3$$

The sum of the working-expenses for the whole length of the line is $k = k_1 + k_2 + k_3$; and putting $l = l_1 + l_2 + l_3$
then

$$k = (\cdot 56 + 23\tfrac{1}{3}s)\, l + 39\tfrac{2}{3}\, (l_1\, s + l_2\, s_1 + l_3\, s_2).$$

Again, putting the height to be surmounted on either side of the watershed $s\, l_1 = h_1$ and $s_1\, l_2 = h_2$
then

$$k = \left(\cdot 56 + 23\tfrac{1}{3}\,\frac{h_1}{l_1}\right) l + 39\tfrac{2}{3}\,(h_1 + h_2 + l_3\, s_2)$$

If the proposed height at which the ridge is to be pierced is reduced by x km., h_1 and h_2 are therefore smaller by x, and the saving in working-expenses per tonne is

$$e = \left(23\tfrac{1}{3}\frac{l}{l_1} + 79\tfrac{1}{3}\right)x$$

Similarly, the saving in the working-expenses incurred in hauling a passenger over the whole length of the line is

$$e_1 = \left(10\tfrac{2}{3}\frac{l}{l_1} + 58\tfrac{2}{3}\right)x$$

If the annual traffic over the line is T tonnes paying-load and P passengers then the saving in M. due to lowering the piercing of the range by x metres is

$$E = \left[T\left(70\frac{l}{l_1} + 238\right) + P\left(32\frac{l}{l_1} + 176\right)\right]\frac{x}{300{,}000} \quad \cdots \quad \cdots \quad (56)$$

For example: if $T = 400{,}000$: $P = 250{,}000$: $l = 120$: $l_1 = 8$, then
$E = 2{,}264\,x$ M.

The apex of the trace at the watershed must therefore be lowered until any further lowering of it by one metre would increase the interest on the capital-cost by 2,264 M. If the tunnel per running-metre cost 3,000 M, involving payment of interest of 120 M. then the floor of the tunnel must be lowered until a further lowering of one metre would entail a lengthening of the tunnel of $\frac{2264}{120} = 18{\cdot}9$m.

In deducing Eqn. 56 for the saving in working-expenses produced by a lowering of the apex-height of the trace by x metres, it has been assumed that the ruling gradient is diminished in a corresponding ratio with the lowering of the height to be surmounted. If, however, the ruling gradient be unaltered by the lowering of the apex then when this latter is lowered by x metres the resulting saving is only

$$E = \left[238\,T + 176\,P\right]\frac{x}{300{,}000} \quad \cdots \quad \cdots \quad \cdots \quad (57)$$

For example: if $T = 400{,}000$, $P = 250{,}000$, the saving E is only $= 464\,x$. M. so that for the assumed cost of the tunnel an elevation would have to be found for which a further lowering by one metre would produce a lengthening of the tunnel by more than $\frac{464}{120} = 3{\cdot}9$m.

When several places suitable for tunnelling the watershed are available for choice it is often possible to select one by the aid of a similar calculation; but in other cases the work of location must be carried out further to enable the several lines to be compared together.

When the trace apex-height in the watershed has been determined the whole length of the line is *ipso facto* divided into sections each of which for its entire length lies in one and the same valley.

If the line crosses a valley, descending from a watershed to the bottom of a valley and from this again rising to the next watershed, then the best elevation of the lowest point in the valley is fixed by the same method of calculation as in the determination of the best height at which to tunnel the dividing ridge. Of course in that case we cannot go lower than the height or point at which it is necessary and proper to bridge the stream.

When the highest and lowest bends of the trace have been in this way fixed in plan and elevation, we have obtained the valley-sections of the trace each of which rises from a lower initial point to a higher terminal point in one and the same valley. In many cases in which the line in its general course follows the length of the valley there is a free choice for the location of the line as between one-side or the other of the stream. The best is that side of the valley which offers the cheapest land, the greatest security against landslips, where the sidelong slope is least, which is faced by the sun, and on which the smallest and least important side-valleys debouch. All these requisites are seldom found together, and therefore the choice of one side of the valley or the other often demands the most careful consideration of all the

above conditions; and not seldom it will be found advisable and advantageous at certain selected points to pass the trace from one side of the stream to the other.

The further working-up of the trace is begun in those valley-sections which contain the ruling gradient, by examining whether the previously fixed grade cannot be still further improved. This examination may lead to modifications of the location of all the remaining sections, and may even have a retrospective effect on the fixing of the tunnel-elevation in the watershed.

Since the working-expenses of the whole line will be less the smaller the ruling gradient is, it must first be determined whether the ruling gradient can be carried out in one uniform ascent from the lowest to the highest point only broken, if at all, by stations.

A line with one *uniform gradient* is to be sought for on a contoured plan according to the method given in § 38, hugging the natural features of the ground as closely as possible. Most usually such a line will follow the valley, curving in and out with the bends and turns of its slopes. After having determined this initial location, which may be regarded as a "*natural line*" of ascent, we have next to consider whether the trace cannot be better located within a narrower band of lines of shorter lengths and having correspondingly steeper grades, by reducing, or even entirely eliminating, the curves of the "natural line" of ascent, thus giving deeper cuttings and higher banks, or by replacing these by tunnels and viaducts.

In greatly winding and moderately rising valleys it has to be examined whether it would not be better to entirely abandon a line lying in the valley-bed, and instead to ascend by one or more crossings of the valley and tunnelling of the ridges separating the bends of the stream, to reach the watershed from the lower initial point in the valley by a much shorter and steeper line.

The choice between these two different traces **is to be made in the following manner.** Let the probable future annual traffic of the line be T tonnes paying-load and P passengers. Let the portion of the trace to be carried out on the ruling gradient s have a length l_1 and the remaining part of the line a length l_2 on the equivalent grade s_2 : then the working-expenses are

$$K_1 = T\,(\cdot 56 + 63\,s)\,l_1 + P\,(\cdot 967 + 40\,s)\,l_1 + T\,(\cdot 56 + 23\tfrac{1}{3}\,s + 39\tfrac{2}{3}\,s_2)\,l_2 + P\,(\cdot 967 + 10\tfrac{2}{3}\,s + 29\tfrac{1}{3}\,s_2)\,l_2$$

and the maintenance-expenses are

$$K_2 = U\,(l_1 + l_2).$$

If the height to be surmounted on the ruling gradient is h km., and if there be curves of $\alpha°$ total central-angle on this grade, then from § 23, we have

$$h_1 = h + \cdot 0000175\,\alpha\,; \text{ and } \;l_1 = \frac{h_1}{s}.$$

Substituting these values, and since

$$K = K_1 + K_2$$

we obtain

$$K = \Big[T\,(\cdot 56 + 39\tfrac{2}{3}\,s_2) + P\,(\cdot 967 + 29\tfrac{1}{3}\,s_2) + U\Big]\,l_2 + (63\,T + 40\,P)\,h_1 + (\cdot 56\,T + \cdot 967\,P + U)\,\frac{h_1}{s} + (23\tfrac{1}{3}\,T + 10\tfrac{2}{3}\,P)\,l_2\,s. - \text{in pf.} \quad \ldots \quad (58)$$

For example: let $T = 400{,}000$; $P = 250{,}000$; $U = 300{,}000$: $l_2 = 30$: and further, let the first term of the above equation which is independent of s and h_1 be put $= M$: them we have in Marks.

$$K = M + 352{,}000\,h_1 + 7658\,\frac{h_1}{s} + 3{,}600{,}000\,s$$

If the line be carried along the valley following in the closest possible manner the configuration of the ground whereby possibly $s = \cdot 005$ and $h_1 = \cdot 215$, then the working- and maintenance-expenses would be

$$K = M + 422{,}974 \text{ M.}$$

On the other hand, if the series of lines forming the trace were laid out so as to occupy a less width laterally, thus shortening the length of the trace, whereby $h_1 = \cdot 210$, and $s = \cdot 0052$, then the working-and maintenance-expenses would be diminished to

$$K = M + 401{,}905 \text{ M.},$$

or 21,069 M. less than in the previous case where the line more closely followed the ground. This more direct line should thus have the preference if the extra cost of construction were less than $\frac{21069}{\cdot 04}$ $= 526{,}725$ M.

Supposing a third line were possible on which, by cutting out the valley-windings the watershed could be reached on a shorter line; then if here $h_1 = \cdot 204$, and $s = \cdot 001$, the working- and maintenance-expenses would amount to

$$K = M + 264{,}231 \text{ M.}$$

Thus compared with the first line examined, the working- and maintenance-expenses would be smaller by 158,743 M.; so that this line should have the preference if the cost of construction does not exceed that of the first line by more than $\frac{158743}{\cdot 04} = 3{,}968{,}575$ M.

In this way choice between several possible locations, and consequently also, that the best ruling gradient is very easily made.

In the above calculation a uniform through-grade to the watershed has been assumed thus reducing the working- and maintenance-expenses to a minimum. Apart from the fact that a break is rendered necessary by the intermediate stations, the carrying out of such a through uniform ascent would involve a very considerable outlay in construction, although of course less in hilly ground than in mountainous country. If the through-grade should cause the trace in some parts of the line to rise high above the bottom of the valley, or side valleys to be crossed on high banks or on viaducts, or the line to lie along on steep side-slopes, or to be moved away from the intermediate localities it is desired to serve then it will have to be considered whether **by abandoning the uniform through-grade and by improving the sections at the bottom of the valley** more cannot be saved than the resulting increase in working- and maintenance-expenses amounts to.

The calculations requisite to determine this point are made in the following manner.

The working- and maintenance- expenses of a line of l_1 km. in length, of which the ruling gradient of which the equivalent grade for goods-traffic is s_1 and for passenger-traffic is s_2 are given by

$$K = T\,(\cdot 56 + 23\tfrac{1}{3}\,s + 39\tfrac{2}{3}\,s_1)\,l_1 + P\,(\cdot 967 + 10\tfrac{2}{3}\,s + 29\tfrac{1}{3}\,s_2)\,l_1 + U\,l_1 \qquad \ldots \quad (59)$$

Were the line carried out on a uniform grade of $s = \cdot 0052$ and thus s_1 and $s_2 = \cdot 0052$, and if $l_1 = 40$, $T = 400{,}000$, $P = 250{,}000$, $U = 300{,}000$ pf.; then from the above Eqn. the working- and maintenance-expenses would be

$$K = \frac{400{,}000}{100}\,(\cdot 56 + 63 \times \cdot 0052)\,40 + \frac{250{,}000}{100}\,(\cdot 967 + 40 \times \cdot 0052)\,40 + 3{,}000 \times 40 = 379{,}516 \text{ M.}$$

On the other hand, were the line carried out on a broken-up ascent of a total length of 42·8 km., and fitted more closely to the bottom of the valley, of which 15 km. were on the ruling gradient $s = \cdot 008$, 8 km. on a grade of ·006, 2 km. on a grade of ·005, and 17·8 km. on grades under ·0036; and if there occurred in these last sections curves of 360° total central-angle, and on the "injurious" grades curves of 400° central-angle, then—§ 11—the equivalent grade for goods-traffic would be,

$$s_1 = \frac{1}{42\cdot 8}\Big[\cdot 0036 \times 17\cdot 8 + \cdot 178 + \cdot 000018\,(2 \times 360 + 400)\Big] = \cdot 00613$$

and for passenger-traffic,—

$$s_2 = \frac{1}{42\cdot 8}\Big[\cdot 0055 \times 19\cdot 8 + \cdot 168 + \cdot 000018\,(2 \times 360 + 400)\Big] = \cdot 00694$$

in which the altitudes surmounted on "injurious" ascents are inserted, viz. ·178 and ·168 km, respectively.

The working- and maintenance-expenses are therefore,

$$K = \frac{400{,}000}{100}(\cdot 56 + 23\tfrac{1}{3} \times \cdot 008 + 39\tfrac{2}{3} \times \cdot 00613)\,42\cdot 8 + \frac{250{,}000}{100}(\cdot 967 + 10\tfrac{2}{3} \times \cdot 008 + 29\tfrac{1}{3} \times \cdot 00694)\,42\cdot 8$$

$$+\, 3{,}000 \times 42\cdot 8 = 432{,}280 \text{ M.}$$

The working- and maintenance-expenses are thus on the line with the broken ascent greater by $(432,280 - 379,516) = 52,764$ M. than on the line having the uniform through-grade. If in common with this section an additional length of 30 km. is worked, then on this latter, as a consequence of the increase of the ruling gradient from ·0052 to ·008, the working-expenses increase by

$$K_1 = \frac{1}{100} (400,000 \times 39 \cdot \tfrac{2}{3} + 250,000 \times 29 \cdot \tfrac{1}{3}) (\cdot 008 - \cdot 0052)\ 30. = 19,488 \text{ M.}$$

Thus the total increased outlay for working- and maintenance-expenses is

$$52,764 + 19,488 = 71,252 \text{ M.}$$

which must therefore be counterbalanced by a saving in cost of construction of more than

$$\frac{71,252}{\cdot 04} = 1,781,300 \text{ M.}$$

before the proposed change from the through uniform-grade can be commercially justifiable.

There is a much larger scope for choice in the arrangement or disposal of grades in those sections in which the ruling gradient is already fixed by the location existing in other sections. Omitting the stations on the level or on a grade of 1 in 400, the grades for the whole length of the line—from the principles of § 27—are to be made throughout either as "injurious," or "non-injurious." **When the elevations to be surmounted and the length of the line permit the exclusive employment of "non-injurious" grades.** (which for goods-traffic must be less than ·0036,) **then ascents followed by descents are permissible. In the reverse case when "injurious" grades have to be exclusively employed**—which are to be kept within the ruling gradient and the limiting value of the "non-injurious" grades of ·0055 for passenger-traffic—**all the grades must ascend in the same direction.**

In accordance with this **rule** the grades will be so chosen that the cost of construction shall be the least possible, remembering at the same time to obtain sufficient elevation to keep the line clear of water and to cross roads, etc., and further that a too frequent change of gradient should be avoided.

But occasionally a deviation from the above rule will cause a saving in the cost of construction which will exceed the simultaneous increase in working-expenses.

For example: if in a trace, in which only "non-injurious" gradients are to be used, there were introduced a gradient of ·0045 for a length of 2 km.: then for passenger-traffic there would be no additional increase in the working-expenses: for the goods-traffic, however, there would be one of

$$39\tfrac{2}{3} (\cdot 0049 - \cdot 0036)\ 2 = \cdot 0714 \text{ pf. per tonne;}$$

and consequently, for a traffic of 400,000 tonnes paying-load an increase in working-expenses of 286 M. The adoption of the grade of ·0045 would then be justified if at the same time thereby, as compared with a grade of ·0036, there were $\frac{286}{\cdot 04}$ saved $= 7,150$ M. in cost of construction. If the grade were further increased to ·006 then for passenger-traffic, also, there would be an increase in working-expenses and for a traffic of 250,000 persons the increase would amount to

$$\frac{1}{100}\ 400,000 \times 39\tfrac{2}{3} (\cdot 006 - \cdot 0036)\ 2 + \frac{1}{100}\ 250,000 \times 29\tfrac{1}{3} (\cdot 006 - \cdot 0055)\ 2 = 835 \text{ M.}$$

to justify which it would be necessary to show a saving in cost of construction of 20,875 M.

If, to take another instance, the trace had to be carried out throughout on "injurious" grades, and there were a length of 1 km. on a grade of ·004; then for goods-traffic there would be no additional expense; whereas per passenger the journey would become dearer by

$$29\tfrac{1}{3} (\cdot 0055 - \cdot 004) = \cdot 044 \text{ pf.}$$

If a grade of ·003 were used the increase in expense per passenger would rise to

$$29\tfrac{1}{3} (\cdot 0055 - \cdot 003) = \cdot 073 \text{ pf.}$$

And for the paying-tonne also there would be an increase in working-expenses of

$$39\tfrac{2}{3} (\cdot 0036 - \cdot 003) = \cdot 0238 \text{ pf.}$$

In the subsequent more detailed working up of the trace it may happen that alterations become necessary in its plan and consequently in the curves and in length of the line, arising from the considerations mentioned in discussing location in flat country—the acquisition of land, character of the ground, crossing of roads, and water-courses.

In all these cases, in order to **intelligently** make choice between the various sections of the trace under consideration, the sum of the working-expenses, maintenance-expenses and the interest on the capital sunk has to be ascertained; as is illustrated in what follows.

Suppose that for a line of which the volume of traffic amounts to 400,000 tonnes paying-load and 250,000 passengers, and of which the maintenance-expenses are 3,000 M. per km., and which has a ruling gradient of $s = \cdot 01$, there are two possible traces to choose from, *A* and *B*, connecting two terminal points of which the line *A*'s estimated cost of construction is 600,000 M. and that of *B* is 540,000 M. The length of the line *A* is 2·5 km. of which 2 km. is straight on a grade of ·009, and consequently attains an elevation of ·018 km.; and the remaining ·5 km. is level and on a curve having 30° of central-angle.

The equivalent grade of this line for goods-traffic is

$$s_2 = \frac{1}{2\cdot 5}\left[\cdot 5 \times \cdot 0036 + \cdot 018 + \cdot 000018 \times 2 \times 30.\right] = \cdot 00835,$$

and for passenger-traffic, is

$$s_2 = \frac{1}{2\cdot 5}\left(\cdot 5 \times \cdot 0055 + \cdot 018 + \cdot 000018 \times 2 \times 30.\right) = \cdot 00873.$$

Consequently, the working-expenses are

$$K_1 = \frac{400,000}{100}\left(\cdot 56 + 23\tfrac{1}{4} \times \cdot 01 + 39\tfrac{1}{2} \times \cdot 00835\right) 2\cdot 5 + \frac{250,000}{100}\left(\cdot 967 + 10\tfrac{1}{2} \times \cdot 01 + 29\tfrac{1}{4} \times \cdot 00873\right) 2\cdot 5$$
$$= 19,558 \text{ M.}$$

If to this is added the maintenance-expenses, 7,500 M., and the interest on the capital cost, 24,000 M, then we obtain the total annual expenses as 51,058 M.

The line *B* is 3 km. in length on a grade of ·006, and has 100° of central-angle

The equivalent grade both for goods- and passenger-traffic, is then

$$s_2 = \tfrac{1}{3}\left(\cdot 018 + \cdot 000018 \times 100\right) = \cdot 0066.$$

Accordingly, the working-expenses are

$$K = \frac{400,000}{100}\left(\cdot 56 + 23\tfrac{1}{4} \times \cdot 01 + 39\tfrac{1}{2} \times \cdot 0066\right) 3 + \frac{250,000}{100}\left(\cdot 967 + 10\tfrac{1}{2} \times \cdot 01 + 29\tfrac{1}{4} \times \cdot 0066\right) 3$$
$$= 22,166 \text{ M.}$$

Adding to the above 9,000 M. for maintenance-expenses and 21,600 M. for interest on the capital-cost we obtain as the total annual charge 52,766 M. Consequently, the line *A* is the financially better line by the annual amount of 1,612 M.

§ 34.

An Example of the choice between two possible Locations in Hilly Country.

As a complement of the discussion in the foregoing § 33 we will take an actual example for investigation.

For **the branch line Sarnau-Frankenberg**, which will be opened to traffic in the year 1888, there were two possible lines for consideration.

This normal-gauge Local line is to be worked by mixed-trains only, and the working-expenses of goods-trains on Main lines may be taken as fairly approximating thereto. The traffic will probably at the outset not amount annually to more than 300,000 tonnes gross-load. But having regard to the further possible increase of traffic we may assume as an estimate—according to § 12 in Part I of this Work—a normal volume of business of 400,000 tonnes.

One of the possible lines has a maximum grade of $\frac{1}{60}$ with curves of 300^{m} radii; consequently, a Ruling gradient of

$$s = \cdot0166 + \cdot0033 = \cdot02$$

The whole length of line is 26·14 km. of which 10·9 km. is on "non-injurious" gradients: the remaining length is on "injurious" grades and ascends to a height of ·1879 km. In the sections on "non injurious" grades there are curves having a total central-angle of 722°: on the "injurious" grades curvature amounts to 918° central-angle.

The Equivalent grade of the line is therefore

$$s_2 = \frac{1}{26\cdot14}\Big[\cdot0036 \times 10\cdot9 + \cdot1879 + \cdot000018\,(2 \times 722 + 918)\Big] = \cdot0103:$$

and the Working-expenses are

$$K = \frac{26\cdot14 \times 400{,}000}{100}\Big(\cdot24 + 10 \times \cdot02 + 17 \times \cdot0103\Big) = 64{,}315 \text{ M.}$$

On the second line the maximum grade is $\frac{1}{53}$ so that with curves of 300^{m} radius the ruling gradient is

$$s = \cdot01887 + \cdot00333 = \cdot0222$$

Of the whole length of 26 km. there is 12 km. on "non-injurious" grades; and the remainder, on "injurious" grades, ascends to a height of ·1931 km. On the "non-injurious" gradients there is 699° of central-angle, and on the "injurious" grades the curvature is 956°.

The equivalent grade of this line is therefore

$$s_2 = \frac{1}{26}\Big[\cdot0036 \times 12 + \cdot1931 + \cdot000018\,(2 \times 699 + 956)\Big] = \cdot0107$$

and the working-expenses,

$$K = \frac{26 \times 400{,}000}{100}\Big(\cdot24 + 10 \times \cdot0222 + 17 \times \cdot0107\Big) = 66{,}966 \text{ M.}$$

The cost of working this steeper second line is thus greater than that of the first by,

$$66{,}966 - 64{,}315 = 2{,}651 \text{ M.}$$

but since it is ·14 km. shorter, the maintenance-expenses are somewhat smaller; so that, allowing for this, the extra operating cost can be put down at 2,400 M. But since this line with the steeper ascent is cheaper by 110,000 M. in construction, or by an annual

interest-charge of 4,400 M, than the other line with grades of $\frac{1}{60}$, it is better than the other by an annual amount of $4,400 - 2,400 = 2,000$ M., and for this reason is to be preferred.

The choice might turn out otherwise if we took into consideration the fact that a future prolongation of the line is in prospect, in which the ruling gradient will be the maximum gradient of the section here considered.

Supposing that after 6 years an extension of 25 km. is probable, causing the traffic on the whole line to increase to 500,000 tonnes gross-load, viz. an increase of 100,000 tonnes; then the lesser working-expenses of the 1 : 60 line would increase by $\frac{2651}{4}$ M. or 663 M. On the 25 km. extension the working-expenses per tonne-km. of gross-load, if the ruling gradient were ·02 instead of ·0222, would decrease by

$$10\,(\cdot 0222 - \cdot 02) = 022 \text{ pf.}$$

and so, as a whole, by

$$25 \times 500,000 \times \cdot 022 \frac{1}{100} = 2,750 \text{ M.}$$

If the line with grades of 1 : 60 had been chosen, then after building the extension of the line there would be a total saving in working-expenses of

$$2,400 + 663 + 2,750 = 5,813 \text{ M}:$$

and therefore, after deducting the interest of the cost of the extension, there would be an annual saving of

$$5,813 - 4,400 = 1,413 \text{ M.}$$

By selecting the line with 1 : 60 grades there would be on the completion of the extension an annual loss compared with the other line on a gradient of 1 : 53 of 2,000 M. which at compound-interest would in 6 years amount to

$$2,000\left(\frac{\overline{1 \cdot 04}\,^{6} - 1}{\cdot 04}\right) = 13,420 \text{ M.}$$

of which the interest, 537 M., is to be deducted from the gain to be obtained later on; so that for the 1 : 60 line there remains an advantage of

$$1,413 - 537 = 877 \text{ M.}$$

But if the extension of the line were not carried out within a period of 14 years, the annual loss on the 1 : 60 line would amount to about 36,000 M., of which the interest is greater than the advantage which would then be derived; so that the selection of the 1 : 53 line would have been the better one.

The choice of the trace, quite apart from the uncertainty of the estimate of the probable traffic, as the above example shows, often depends upon circumstances the influence of which can only be intelligently guessed at.

§ 35.

Location in Mountainous Regions.

The conditions of location in hilly districts, where the lengths of the valleys occasion circuitous deviations often involving increased cost of construction to shorten them, differ from those in mountainous regions. In the latter the valleys are often so short that to obtain the best gradients a line has to be sought for which exceeds the valley in length. Also, in hilly country, the climatic and hydraulic conditions have only exceptionally any influence on the position of the trace, whereas in mountainous regions they are often of very great importance.

When it is a question of reaching a place lying amongst mountains which can be attained without crossing a watershed then, since the elevation to be surmounted is known, the first and most important problem is the determination of the best gradient.

In most instances, however, a range or "divide" has to be crossed. In that case not only the best positions and gradients of the **approach-lines** on either side of it have to be determined but also the best point in plan and elevation for the **crossing of the water-shed** or range.

If there be more than one possible point of crossing of the watershed for selection then that pass is to be looked-for which offers at the same time **the lowest elevation, shortest length, most convenient and suitable ground, the best and most favourable climatic conditions, and of which the approach-valleys offer the most favourable conditions for the location of the approach-lines leading up to the crossing, and which at the same time in their lower sections approximate most closely to the Commercial Trace.**

Regarding the choice of the valleys by which the approach is made, the following points have to be considered :—

1. The **Shape**
   - (*a*) The longitudinal fall of the valley; this should differ as little as possible from the ruling gradient subsequently fixed on.
   - (*b*) The cross-section of the valley, of which the enclosing sourrounding adjacent heights should be neither too high nor too steep.
   - (*c*) Its ground-plan—this should not have too acute bends or windings.
   - (*d*) The existence of lateral subsidiary valleys offering facilities for the development of the line.
2. The **Nature of the Ground.**
   - (*a*) The ground should be as easily excavatable as possible, and yet should not require too flat slopes for the cuttings and enbankments.
   - (*b*) The danger of slips and slides should be as small as possible. In this connexion it is particularly necessary to note whether there are any traces of former landslips.
   - (*c*) There should be as little chance as possible of stone-avalanches.
   - (*d*) Building materials, such as sand, ballast gravel, quarry stone, etc., which can often be had at small cost and of good quality.
3. **Hydraulic Conditions.**
   - (*a*) The size of the flood-areas, and the amount of annual rainfall. This will enable conclusions to be come to regarding the flow per second at low water, at the ordinary level, and in floods.

(b) The existence of water-power either already in use or utilizable.

(c) The presence of torrents which form alluvial cones or moraines.

(d) The presence of torrents which simply fall in spray.

4. **Climatic and Weather** Conditions.

(a) The aspect of the valley with reference to the sun.

(b) The length of time the snow lies on the ground, and its depth.

(c) The occurrence of avalanches.

(d) The average temperature, particularly during the winter.

(5) The **Economic** Conditions.

(a) The economic employment of the soil and its degree of productivity. This is of moment as regards the price of the land and the volume of the local traffic.

(b) The residential conditions of the population; these are of importance as affecting the amount of the probable local traffic, for the locating of the station-sites, and for the housing of the work-people during the period of construction.

(c) The nature of the industrial occupations of the inhabitants.

(d) The location and condition of the existing roads.

The just balancing of all above conditions which is requisite in weighing the claims of the different valleys is of course a very difficult problem, only solvable when considerable intelligence is coupled with a wide experience.

In the majority of those cases—usually rare—where a choice between the merits of different valleys has to be made, some of the abovementioned conditions may be of such predominant importance that all the others can be left out of consideration.

Only quite rarely will it be necessary to completely work up rival projects in order to come to a final decision. However, for the exact fixing of individual portions of the trace all the abovementioned conditions for valleys are of importance.

Having selected the valleys on both sides of the range for making the ascent the most suitable elevation at which to cross it is then to be determined, bearing always in mind the mutual connexion of the height to be surmounted with the gradient of the approaches on both sides.

The lower the watershed-crossing is the less is the height of the ascent and the smaller will the most suitable gradients (as seen from Eqn. 31.) of the approach inclines be; so that in general each change in the apex-height of the trace necessitates an alteration in the length and the gradients of the approach lines.

For the fixing of the height of the crossing the determining circumstance is: **that the open portion of the line must cease at that point at which the working of the line first becomes impeded by the presence of snow and ice.**

The greatest altitude of uncovered or open line above sea-level is for Alpine valleys:—

On the Gotthard Line	—North side	1109^m.
" " "	—South side	1145^m.
" Pusterthal "	... ...	1213^m.
" Mont Cenis "	... ...	1297^m.
" Brenner "	... ...	1373^m.
" Arlberg "	—East side	1302^m.
" " "	—West side	1215^m.

Very much greater elevations are crossed in other parts of the world.

In America the Denver and Rio Grande Railway crosses the La Vata Pass at a height of 2950^m, and the Marshall Pass at an elevation of 3430^m. The Peru-Bolivia Railway from Arequipa to the Titicaca crosses the Portes del Cruzera at 4470^m. and the Lima-Oroya Ry. at 4778^m—the greatest elevation above the sea-level hitherto (1888) reached by a railway.

The difficulties experienced in working lines at high elevations arise from the reduced adhesion of the driving-wheels of the locomotive on the rails, and from the presence of snow.

According to the experiments of **Stöckert*** on the Kaiser- Ferdinands Nordbahn, extreme cold decreases the adhesion. At 0°C. this decrease amounts to 5%; at—5°C. to 10%; at—10°C. to 15%; and at—15°C. to 20%. Thus, for example, if the mean temperature during 6 winter months were—6° the weight of trains would have to be 11% smaller than in summer.

In addition to this disadvantage there is the increased cost of track-maintenance due to the expense of clearing away snow, and the inconvenience of frequent interruptions of traffic by severe snow-falls and snowdrifts.

At certain elevations, which vary enormously with the geographical position, the severity of the weather and the difficulties, interruptions, and increased expenses arising therefrom in the working of the traffic and in the maintenance of the road reach such a degree that any higher rise of the trace is impracticable.

But even, if the climatic conditions should permit of an **open line**, it will only be exceptionally possible to cross a mountain by a line in open cutting, as is done, for example, at the Brenner. The general rule will be that in order to cross at the best elevation the watershed must be pierced by a **tunnel.**

In examining the best elevation for this apex-tunnel it will most usually be found that the only possible choice lies between single positions of the tunnel at definite heights apart, and that no other intermediate elevation lying between these points is practicable: since the ends of the tunnel must be so situated as to allow of a sufficient space for the requisite workshops, etc.; and generally, also, for a station site; and must, in addition, be suitably situated for the junction of the lines of approach from the plain below.

Suppose that for one of the possible positions of the appex-tunnel the entrances A and B, and consequently, the length of the tunnel $A\ B$, are fixed provisionally, then the positions of the lower ends C and D, of the approach-inclines $A\ C$ and $D\ B$ have to be determined. Consequently, it has to be examined whether the stations at which the working of the mountain sections is to commence would not be more advantageously located beyond C further down the valley towards E; and similarly beyond D downwards to the valley F. The sections $E\ C$ and $D\ F$, and the tunnel $A\ B$, are further to be run over by the unbroken-up trains jointly with the approach-inclines $A\ C$ and $D\ B$ on the ruling gradient.

Let the sum of the lengths of the appex-tunnel and of $E\ C$ and $D\ F$ be $= l$, and the equivalent grade of the same be s_2. To the heights h_1, h_2, which the inclines $A\ C$ and $D\ B$ surmount on the ruling gradient there is an addition to be made for the reduction of the grades in curves which will amount, for $\alpha°$ of central-angle, to $\cdot 000018\ \alpha$. To the height to be surmounted there is to be added the height lost in the reduced grade in long tunnels. In the case of the Gotthard Ry. this reduction of the ascent was taken at 3% for all tunnels of greater length than 500^m. Adhering to this rule, for a total length λ of all tunnels of length greater than 500^m, this would amount to $h = \cdot 003\ \lambda$ metres.

The height to be surmounted is consequently for both ramps

$$h = h_1 + h_2 + \cdot 000018\ \alpha + \cdot 013\ \lambda$$

* Die Alternativ-Trassen der Albergbahn—von Franz Ritter von Stöckert. Wien. 1880.

In this way having determined the ratio $\frac{l}{h} = m_{,}$ the amount of the kilometric cost of construction A, the kilometric road maintenance-expenses U, the number of the units T of traffic, viz. the sum of the tonnes of paying-load and of passengers despatched annually, then the proper gradient for the ascent is calculated from Eqn. 31.

The tractive-force coefficient is in this calculation to be taken at ·1, which may be assumed as correct for mountain railways in general.

If, as frequently happens, this calculation yields such a steep ascent that the lengths of the approaches turn out shorter than the length of the valley forming the approach to the apex, then the trace loses one of the distinctive features of a mountain line, namely, the necessity for **a development of length** exceeding the length of the valley, although in other respects it may have the character of a mountain line.

But it must be emphasised that the calculation of the best gradient by Eqn. 31 is not entirely determinative of the choice of the ascent, since this equation cannot take account of many defects which a steep ascent possesses. Danger in the working of the line increases with the gradient, there is an increased wear of rails and tires due to the extra and excessive use of brakes, and further there is a reduction of the speed of travel which is undesirable in the case of passenger-traffic; all of which circumstances increase the outlay on train-staff and the interest on the capital outlay in locomotives and tenders—which in the formula is assumed as invariable for all gradients. And finally, in steep ascents, owing to the comparatively small weight of the individual trains, a larger number becomes necessary, and increasing traffic renders a double track necessary earlier than would otherwise be the case.

Now since a flattening of grade only but slowly increase the transport expenses involved in Eqn. 31 (as Table XIV shows) and in order to allow for the abovementioned circumstances not taken account of in the calculation which make a steep gradient disadvantageous, a gradient considerably flatter than that given by Eqn. 31 must be chosen.

But even allowing fully for all the circumstances making for a flat grade, it is impossible to escape the conviction that **in many mountain railways** characterised by a great development of length **the grades have been made too flat.** Thus, for example, the trace of the **Schwarzwald** Railway from Offenburg to Villingen—so cleverly located by its engineer **Gerwig**—must in view of the selected grade of 1 : 55 be regarded almost as an extravagance. Also the trace of the **Gotthard** Railway, located in such a masterly manner by **Hellwag**, would from the economic point of view have been certainly more advantageously built on a steeper grade.* Apparently, the location was influenced entirely by the mandate not to go beyond a maximum grade of $\frac{1}{40}$, and thus the grades of ·026 and ·027 have not been exceeded.

For the **Northern approach-incline of the Gotthard Tunnel,** which between Erstfeld and Goeschenen forms a distinct working section, **the best gradient** would have been obtained according to Eqn. 31 as follows.

The height of ascent between the Erstfeld Station and the Goeschenen entrance to the tunnel is ·6 km. If it is assumed that there are in this section curves of 2000° of central-angle then, owing to the reduction of grade and consequent loss of height in the curves, this height must be increased, for the purpose of the calculation, by

$$2000 \times {\cdot}000018 = {\cdot}036 \text{ km.}$$

Further, there are three tunnels each above 500^{m} in length, so that applying the requisite reduction in height in each, a further addition of ·003 × 3 = ·009 km. is to be made to the length of ascent, which accordingly becomes

$${\cdot}06 + {\cdot}0036 + {\cdot}009 = {\cdot}0645 \text{ km.}$$

[* Conf. on this interesting point Wellington: Economic Theory of Location: par. 915-4: and especially Wellington: Correspondence on Economical Railways: Mins. Procs. I. C. E. Vol. LXXXV: Part III: p. 184.—Tr.]

Since, it may be anticipated that the result of calculation would show that the ruling gradient need not begin at the Erstfeld Station, but on the contrary the trace from Erstfeld onward can follow the bottom of the valley on low grades for the first few kilometres, and since further the stations are on the level, it may be assumed that in addition to the ramps on the ruling gradient, there will be, approximately, say, $l = 6$ km. of bank on low grades, of which the equivalent gradient will be, say, $s_2 = \cdot 0014$. The value therefore of the ratio m is $= \frac{l}{h} = \frac{6}{\cdot 645} = 9 \cdot 3$.

The construction-cost per km. may be taken as 700,000 M., the kilometric maintenance-expenses at 4,000 M, and the volume of the traffic (the sum of the tonnes of paying-load) and passengers, allowing for a gradual increase, at 800,000.

Further, let the load-coefficient $b = 2\frac{1}{3}$, the rate of interest $i = \cdot 04$: $w = \cdot 0036$; $s = \cdot 1$; $L = 60$; $B_0 = 32$; $a = 25$: so that

$$B = 32 + \frac{25 \times 60}{2} = 107.$$

Also, let $e = 2$ and $f = \cdot 15$.

The virtual number of trains is, consequently—§ 23,

$$n = \frac{2\frac{1}{3} \times 800000 \times \cdot 0036}{(\cdot 1 - \cdot 0036)\, 60} = 1162.$$

and the best gradient— from Eqn. 31— is

$$s = \frac{\cdot 1 - \cdot 0036}{1 + \sqrt{\frac{\cdot 1}{\cdot 0036} \times \frac{107 + 9 \cdot 3 \times \cdot 0964 \left(32 + \frac{1}{2} \times 25 \times 60 \times \cdot 008 + \frac{2 \times 60}{\cdot 1} \cdot 0964)^{2}\right)}{107 + \frac{1}{1162}\left(70{,}000{,}000 \times \cdot 04 + 400{,}000 + \cdot 05 \times 2\frac{1}{3} \times 800000\right)}}}$$

that is, $s = \cdot 045 = 4\frac{1}{2}\ \%$.

With such a gradient the surmounting the height of ·645 km. would require a length of 14·33 km. Since the length of the valley from Erstfeld to Goeschenen is 20 km. there would remain 5·67 km. over for the station-yards and for the initial section in proximity to the bottom of the valley. Thus the result of the calculation, made on the assumption of a lengthening of the ascent, shows that no such development was necessary—since the length of the valley is greater than the length of the incline requisite when using the optimum gradient.

Now since it is impossible to shorten the valley the working-expenses would be reduced to a minimum if, in accordance with § 33, the trace from its starting point at Erstfeld up to the tunnel-entrance at Goeschenen were carried on a uniform grade—exclusive of the reductions of ascent which must be made in curves and in the longer tunnels.

Allowing a level of 500^{m} for each of the three intermediate stations there would be a straight open length of 20 km. on a grade of ·035. The line as actually executed, on the contrary, is 28·9 km. long, and has 24·3 km. on a grade of ·026, and curves of 300^{m} radius, and consequently, a ruling gradient of

$$\cdot 026 + \frac{1}{300} = \cdot 0293.$$

The working-expenses as also the cost of construction of the shorter and steeper line would unquestionably have been less than they are on the line actually carried out.

On the present existing line, in which curves of some 2800° central-angle occur, there is ·6 km. on "injurious" grades and 4·6 km. on "non-injurious" grades. The equivalent grade is therefore, for goods-traffic

$$s_2 = \frac{1}{28 \cdot 9}\left(4 \cdot 6 \times \cdot 0036 + \cdot 6 + \cdot 000018 \times 2800\right) = \cdot 0231.$$

whereas for a line with a ruling gradient of ·035, it would be

$$s_2 = \frac{1}{20}\left(1{\cdot}67 \times {\cdot}0036 + {\cdot}6 + {\cdot}000018 \times 2000\right) = {\cdot}0321.$$

The working-expenses per tonne gross-load for the **actual** line are, therefore—Eqn. 7,

$$k = 28{\cdot}9\left[{\cdot}15 + 2 \times {\cdot}0293 + \frac{32\,({\cdot}0036 + {\cdot}0293) + \frac{1}{2} \times 25 \times {\cdot}1 \times 60\,({\cdot}0036 + {\cdot}0231)}{60\,({\cdot}1 - {\cdot}0036 - {\cdot}0293)}\right]$$
$$= 28{\cdot}6 \text{ pf.}$$

and on the line **without developement** they would be

$$k = 20\left[{\cdot}15 + 2 \times {\cdot}035 + \frac{32\,({\cdot}0036 + {\cdot}035) + \frac{1}{2} \times 25 \times {\cdot}1 \times 60\,({\cdot}0036 + {\cdot}0321)}{60\,({\cdot}1 - {\cdot}0036 - {\cdot}035)}\right]$$
$$= 25{\cdot}6 \text{ pf.}$$

Accordingly, for 800,000 tonnes gross-load of goods-trains, such as now pass over the Gotthard, there would be saving on the steeper and shorter line of

$$800{,}000\,(28{\cdot}5 - 25{\cdot}6)\,\frac{1}{100} = 23{,}200 \text{ M.}$$

Allowing for the slower speed of travel and the consequent increase in the expenses of train-staff and interest on the cost of rolling-stock, this saving might probably be reduced to 20,000 M. The saving in the working-expenses of the passenger-service—counterbalancing the disadvantages of slower running—are thus left entirely out of consideration. In addition to the saving in working-expenses here put at 20,000 M. there is also a reduction in cost of line-maintenance of, say, 4,000 M. per km.; and therefore for 8·9 km. this amounts to 35,000 M., so that for the shorter and steeper line the total saving under these heads, working and maintenance, would amount to 55,000 M.

The cost of the construction of the steeper line—following the valley without development, and of some 7 km. in length—would be somewhat less in the higher sections where it would lie closer to the valley-bottom than the existing line does; and the lower 13 km. even of this line, although it would have been located in very difficult and dangerous places on the valley-side, would have probably been cheaper than the 21·9 km. of the actual line.

With the same total length of tunnel of 7·3 km. as on the present line all the difficult and dangerous places in the shorter line could certainly have been overcome. But even if the cost of its construction had been greater by an annual interest-charge of 550,000 M. the shorter line would have been the better.

A perfectly indisputable opinion, as to the cost of construction could, of course only be arrived at by an actual detailed construction-estimate. Still so far as it is possible to judge in the absence of such an estimate, a steeper gradient on the Northern approach to the Gotthard Tunnel would have been preferable, and such a gradient would no doubt have been carried out had not hampering stipulations been laid down as to the grade to be employed. Accordingly it has not been our intention to give here an expert opinion covering exhaustively all the circumstances of the Gotthard location; we have simply desired to illustrate by a striking example the kind of calculations and considerations which are necessary to determine the optimum *i.e.* the financially most advantageous, gradient of a mountain railway.

It may further be pointed out that the adoption of a steeper ascent for the Gotthard Railway would have rendered imperative at an earlier date the construction of a second **track**; and this, notwithstanding the low grades, is even now quite unavoidable.*

The best value of the ruling gradient having been satisfactorily determined, we have then to investigate whether the provisionally-assumed elevation of the apex tunnel has not been altered thereby, thus necessitating its re- location.

[* And has since been carried out.—Tr.]

If the apex-height of the trace can be attained on the previously determined ruling gradient s only by increasing the length of the line, then by lowering or raising the summit by h km. the length of the ramps of approach will become smaller or larger by the amount $\frac{h}{s}$. Thus, if the interest of the cost of construction, the maintenance- and the working-expenses per km. of the approach-ramps is K, the expense accruing from the alteration in height of the point of passage through the watershed is $2\,K\,\frac{h}{s}$.

A good example in illustration of such a case is afforded by the **Alberg Railway.** As is well known, there were two rival schemes, an upper and a lower; regarding the relative merits of which at the time there raged a lively discussion.

On the Arlberg the traffic preponderates heavily in the direction east to west, and in this direction the ruling gradient was fixed at $\cdot 0264$; whereas in the other direction, west to east, of lesser traffic, the gradient was made $\cdot 0314$.

Assuming the paying-load in the latter direction to be only half of that in the main direction; and that the load-coefficient b in the principal direction of the traffic $= 2\frac{1}{2}$; that the tractive-power coefficient $z = \cdot 1$, the coefficient of resistance $w = \cdot 0036$; then with a ruling gradient in the principal direction $s = \cdot 0264$, the gradient in the other direction may be, according to Eqn. 12, $s_1 = \cdot 0317$. From Eqn. 16 for the working-expenses, for an unequal traffic in both directions of the amount stated, we obtain a load-coefficient $= \frac{2b}{1+r} = \frac{2 \times 2\frac{1}{3}}{1+\frac{1}{2}} = 3\frac{1}{9}$ as our basis.

The working-expenses per tonne-km. of paying-load on the ruling radient are therefore—from Eqn. 4,

$$k = 3\frac{1}{9}\left[\cdot 15 + 2 \times \cdot 0264 + \frac{32\,(\cdot 0036 + \cdot 0264) + \frac{1}{2} \times \cdot 25 \times \cdot 1 \times 60\,(\cdot 0036 + \cdot 026{:})}{60\,\,(\cdot 1 - \cdot 0036 - \cdot 0264)}\right]$$
$$= 3\cdot 157.\ \text{pf.}$$

For the passenger-traffic, which may be assumed as equal in both directions, the stiffer gradient in the subsidiary direction, *i.e.*, $\cdot 0314$, becomes the ruling gradient; so that the working-expenses per passenger-km. on the ruling gradient are

$$k = 1\frac{1}{3}\left[\cdot 405 + \frac{27\,(\cdot 0055 + \cdot 0314) + \frac{1}{2} \times \cdot 25 \times \cdot 1 \times 54\,(\cdot 0055 + \cdot 0314)}{54\,(\cdot 1 - \cdot 0036 - \cdot 0314)}\right]$$
$$= 1\cdot 9\ \text{pf.}$$

Assuming a traffic of 500,000 tonnes paying-load and 300,000 passengers, the working-expenses per km. on the ruling gradient are

$$K = \frac{1}{100}\,(500{,}000 \times 3\cdot 157 + 300{,}000 \times 1\cdot 9) = 21{,}485\ \text{M.}$$

By the addition of the cost of construction varying from 5,000 to 6,000 M. per km. this amount becomes 27,000 M.

The lower trace has an apex-tunnel of 10270^{m} in length and its entrances are 1215^{m} and 1302^{m} above sea-level: whereas the higher trace has an apex-tunnel 7000^{m} long, and entrances at 1382^{m} and 1378 above sea-level. The higher trace is longer than the lower by 5·4 km., and this is on the ruling gradient; so that as regards working-expenses and maintenance, it is,

$$5\cdot 4 \times 27{,}000 = 145{,}800\ \text{M.}$$

more costly than the lower.

The higher trace has, in addition, 3 stations more than the lower, and is 7 km. longer in open length; so that taking into account the stations and the expense for snow-clearing the extra amount of the working-expenses and maintenance is to be put down at about 160,000 M., which at 4% corresponds to a capital sum of 4 million M.

v. Stöckert (*loc. cit.*) reckons the extra expense of constructing the line on the lower trace, taking into account the loss occasioned by payment of interest during the period of construction—which latter he fixes at 3 years for the approach-ramps, 4½ years for the upper tunnel, and 5 years for the lower one—at 2¾ million gulden, or about 4¾ million M.

Accordingly, the upper trace is better than the lower by a capital sum of 500,000 M. or an annual sum of 20,000 M. Nevertheless, as is well known, the lower trace has had the preference, although its supporters estimated its extra cost—on the basis of a smaller traffic than has been here assumed—at more than double the amount just calculated, because it was feared that its exposed position and its great height above sea-level would give rise to serious difficulties in, and continual obstructions to, its working.

These apprehensions have been completely justified by the experience of the winter of 1888 when even on the lower trace the traffic suffered very considerable obstruction from snow.*

As to the difficulties in construction and working and the greater cost which are to be expected on lines situated at great elevations above sea-level, an opinion of any value can of course only be arrived at after long experience fortified by a thorough and searching study of the local conditions. But not alone the elevation above sea-level but frequently also the elevation relatively to the bottom of the valley enhances the cost of construction and the expense of working to such a degree that a radical change of principle on which the location has been carried out may have to be considered—as the following § will show.

[* Conf. Alpine Engineering: L. F. Vernon-Harcourt. Mins. Procs, I. C. E., Vol. XCV., Part I., for descriptions of above Railways.—Tr.]

§ 36.

Continuous and Broken Grádes.

Since valleys, in general, gradually diminish in degree of fall from above downwards, a line uniformly falling at the mean rate of the valley from the watershed downwards will gradually depart from the valley-bed and, at a certain point in its length will attain its greatest vertical height above it and finally will run into the bottom of the valley.

Owing to irregularities in the longitudinal fall of the valley—see § 1—it may happen that a line on a uniform grade would in certain parts of its length fall below the valley, and have to be carried by a tunnel under the bed of the stream.

Now although the construction of such a tunnel would ordinarily present no extraordinary difficulties from an engineering point of view yet the presence of water, might make it impossible. However, it is mainly on the score of the expense that the location of the trace below the bed of the stream is to be avoided. That point of the valley which would have the greatest vertical height above such a trace on a uniform through grade becomes a technical "**fixed-point**" in the elevation of the trace. Thus the upper-exit of the **Dazio George** in the valley of the Ticino, in the southern approach-ramp of the Gotthard, forms such a point, see Fig.

Let *A*—**Fig. 18**—be the lower starting point of a line ascending a valley; *E*, the upper terminal, being either the apex-point, or some other technically fixed-point in the trace. If the grade fixed upon as suitable is flatter than the actual average grade of the valley between *A* and *E*, then a development of the trace will have to be carried out in one of the ways, described in § 37. In determining the requisite length of the trace, the reduction of the grade in stations, in curves, and in tunnels must of course be taken into account.

If the uniform ascending grade starting from *A* cuts the valley-bottom at *B*, and if *E D C* be a uniform grade descending from *E*, then from the vertical distance *D B* between the two the requisite length of development to connect them can easily be found.

This development can be made at a single place by means of the development-line *B C D*, or at several suitably chosen points along the lines *B F K H E*, or *B F K J G L E*, or *A M N K H E*, etc. In general it is best to make the development at those points where the ascending line intersects the thalweg, and not to give too much of it at any single place, in order that the trace may not be anywhere too high above the bottom of the valley.

A trace high above the valley is undesirable for several reasons, although the acquisition of land there is usually less expensive than in the neighbourhood of the bottom of the valley. Amongst the **disadvantages of a trace at a considerable height above a valley** may be mentioned:

1. Its distance from the population, usually found in or near the bottom of the valley—thus rendering difficult its access to the stations.
2. Its distance from the roads, which as a rule lie in the neighbourhood of the bottom of the valley—thus rendering difficult the bringing up of building materials.
3. The steeper sidelong inclination of the ground—entailing increased earthwork.

4. The usually more irregular shape of the longitudinal section of the ground surface, entailing more frequent curves and formidable earthworks.

5. The greater danger to the working—arising from falling stones, avalanches, and torrents.

6. The difficulty in finding a location—arising from the presence of alluvial cones, or of torrents.

7. The greater difficulty in working the traffic—arising from the lessened adhesion due to moisture, fogs and clouds, and ice on the rails.

When the trace does not lie on the bottom of the valley at a safe distance from the foot of the valley-sides so as to be outside the region of the cones of deposit of torrents, the range of the spray of cascades, the path of avalanches, and falls of stones, but is located on the sides of the valley, then in the higher mountainous regions there is hardly any means of avoiding avalanches, falls of stones, and cascades other than that of placing the line in the face of the mountain and tunnelling under the dangerous places. The cones of deposit, if not cut into too far below their apexes, can be crossed by a bridge of a single span. If intersected more deeply such comes are preferably traversed by a tunnel, which as a rule will have to be located in the firmer ground of the side of the valley, and only exceptionally[1] can be built in an open cut or excavation through the body of the cone which is afterwards covered-in.

Under certain conditions by adhering to **uniform ascent** with its consequent considerable elevation of the trace above the bottom of the valley the difficulties and cost increase so enormously that it is better to altogether abandon it. Instead of seeking—as we do when adhering to the uniform ascent—for the line most closely conforming to the assumed most favourable gradient, **the Guiding Principle should be to seek a favourable location in the neighbourhood of the valley-bottom, and to keep the line there so long as this can possibly be done without exceeding the maximum permissible grade and minimum curvature, and to begin surmounting the difference in height by development only where the natural formation of the ground is no longer practicable for the continuance under favourable conditions of the line in the neighbourhood of the valley-bottom.**[2]

In this way the trace is kept for a great part of its length so close to the bottom of the valley that no extraordinary constructional difficulties arising either from the conformation of the ground nor from natural phenomena can occur; thus providing a sufficient degree of safety to the line and its working. Only when a stiffer ascent of the valley offers obstacles to the further progress of the line along the valley should any lengthening of the line be undertaken.

These principles were laid down by **Hellwag** for the location of the Gotthard Railway in his remarkably concise and luminous "Report on the Location of the Centre-line and Longitudinal Section of the Gotthard Ry.",[3] and were carried out in its construction.

It is clear that a trace with broken ascents in which the general direction of the valley is followed in closest proximity to the valley-bottom on grades changing to suit the shape of the ground and where the slope of the valley-bottom never exceeds the ruling gradient cannot attain the elevation which would have been got by the continuous ruling gradient. Since the height so lost has to be recovered in the development on the ruling gradient, the line must of necessity be longer than that on the continuous grade ascent.

To warrant a deviation from the continuous grade, the increase in working-expenses due to the lengthening of the line must be covered by the saving in cost of construction.

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1 Conf. Kowatsch: Beiträge zu Trassestudien über Eisenbahnanlagen im Bereiche von Schuttkegeln. Wien 1881. Verlag von R. von Waldheim.

[2 Conf. Wellington: Chap. XVI, pars. 745--753. Also Wellington: Correspondence on Economical Railways, Mins. Procs. I. C. E., Vol. LXXXV, p. 189, for the American form of this maxim.—Tr.]

3 Zürich: Verlag von Zürcher und Furter. 1876.

As an example, may be cited **the Southern Approach incline to the Gotthard Tunnel.**

The valley of the Ticino from the Biasca station, situate at an altitude of 296^m, up to the entrance to the apex-tunnel at Airolo, 1145^m above sea-level, has a length of 36 km.; so that taking into account the level-lengths forming the stations and the reduced ascents in curves and tunnels, a through ruling gradient of ·0026 to ·0027 could be obtained without resorting to development. But this uniform ascent would have lain for a considerable distance in and above the Dazio gorge, under the fissured rock-bed of the Ticino; so that the upper-end of the Dazio Gorge at 940^m elevation had to be taken as a technical fixed-point in the elevation of the trace. From here upwards the trace follows the valley and rises without development, firstly on a very flat grade and subsequently on a grade of ·0026 up to the entrance to the tunnel. In the lower sections the trace follows the valley from Biasca downwards for some 12 km. on varying grades, the maximum being ·0027: then on a development in 2 **spirals** it ascends the upper-end of the rapids of the Ticino above Giornico, then again follows the valley, in proximity to the valley-bottom, for some 7 km., and finally ascends in 2 spirals to the upper-end of the Dazio Gorge. See Fig.

In the whole of the south approach-incline there are developments of length only at the two falls in the valley at the Dazio, and above Giornico, by which a height of ·27 km. is obtained; the trace following the course of the valley for a distance of 34 km. Since in the development there occur curves of a total centre-angle of some 1500° and some 6 km. of tunnels in which the gradient is diminished by ·003, the effective height to be surmounted by developing is

$$h = \cdot 27 + \cdot 000018 \times 1500 + 6 \times \cdot 003 = \cdot 315.$$

The ratio of the length of line without any development—viz. $l = 34$ the equivalent gradient of which is ·018—to this height is therefore

$$\frac{34}{\cdot 315} = 108.$$

Noting that the construction-cost per km. of the development is about 1,000,000 M., and inserting this value in pf. (100 million) for A in Eqn. 31, and putting $m = 108$, $s_2 = \cdot 018$, and keeping the other quantities the same as determined in § 35 for the Northern approach-incline then from Eqn. 31 the best gradient for the development is

$$s = \cdot 03.$$

In curves of 300^m radius this gradient must be decreased by ·0033, and where these curves occur in tunnels, by ·003, and is consequently to be fixed at ·0237. In the spiral-tunnels there is, actually, a grade of ·023: so that the correct figure has been hit-off almost exactly.

The working-expenses per tonne gross-load for the line as carried out between Biasca and Airolo, a distance of 48 km. of which the equivalent grade $s_2 = \cdot 02$, and of which the ruling gradient is $s = \cdot 03$ (*i.e.* a grade of ·027 with curve of 300^m radius) are

$$K = 45\left(\cdot 15 + 2 \times \cdot 03 + \frac{32\,(\cdot 0036 + \cdot 03)}{60\,(\cdot 1 - \cdot 0036 - \cdot 03)} + \frac{1}{2}\,\frac{25 \times \cdot 1 \times 60\,(\cdot 0036 + \cdot 02)}{60\,(\cdot 1 - \cdot 0036 - \cdot 03)}\right)$$
$$= 41 \cdot 6 \text{ pf.}$$

If instead of the discontinuous ascent closely conforming to the valley-bottom from the Dazio Gorge to the upper-end of the valley, the ascent had been uniform then, while retaining the ruling gradient of ·03, the length of the valley could have been utilized, thus reducing the length of the trace to 38 km., viz. a shortening, as compared with the actual line, of 7 km. In that case the working expenses would have been 35·1 pf. per tonne of gross-load—or 6·5 pf. less. On the assumed normal volume of traffic of $2\frac{1}{2} \times 800{,}000$ tonnes gross-load this gives an annual saving of about 120,000 M. in working-expenses.

If we omit from consideration the maintenance-expenses and assume that the shorter line, owing to its higher position on the valley-sides, would have required the same maintenance-expenses as the existing 7 km.-longer line, then the construction-cost of the 38 km.-trace with uniform grade might have been $\frac{120000}{\cdot 04} = 3{,}000{,}000$ M. greater than the existing 45 km.-line on discontinuous grades.

Now although in the present instance—where, according to Hellwag, the construction of the line on a uniform grade offered difficulties of a practically insurmountable order—the location on discontinuous grades was justifiable, yet as a general rule it will only in rare instances be advantageous to deviate from a continuous through grade.

§ 37.

Development of the Trace.

Lengthing the trace to obtain flatter gradients may be effected in several ways depending on the configuration and nature of the ground—viz. by running up into side-valleys, by lacets or loops, spirals, and zigzags.

(1) **By running up into lateral valleys—Fig. 19.** The trace leaves the main valley at *A* and ascends on the line *A B C D* into the lateral valley, turns-round at *D* into the line *D E F G* and continues the ascent from *G* along the main-valley. The gain in height corresponds to the difference in length of the curved line *A B C D E F G* and the chord *A G*. The entrance into the side-valley at *A*, the turn at *D*, and the exit or return at *F* into the main-valley is, as a rule, only possible with the minimum permissible curve-radius coupled with the appropriate reduced grade. The length gained is further reduced if the entry into the valley, the turn, and return to the main-line require longish tunnels in which the gradient has again to be reduced. If therefore the parts *B C* and *E F* are of no great length the gain in elevation is but small.

In order that the part of the development entering the side-valley shall not run too rapidly into the valley-bed—which latter usually becomes rapidly very steep towards the head—it is necessary that the trace when turning off at *A*, shall be already at a considerable height above the bed of the side-valley.

It is generally the turn-round at the head of the valley—particularly in narrow valleys—where a sufficient distance between the branches *E F* and *B C* at *E* and *C* at the end of the side-valley can only be obtained by deep cuttings or by tunnelling in sidelong ground that is the expensive part of the work. It is often advantageous to utilize a bifurcation of the valley to make the turn, as is shown in Fig. 19.

Of the two parts of the line running round the side-valley the entering one is usually on the lower, the other branch on the higher or upper slope of the lateral valley; in this way the crossing of the side-valley at considerable height at, or in the neighbourhood of, its opening into the main-valley may be avoided. This circumstance, when side-valleys are suitably formed, may even make the running up into side-valleys **desirable** solely **on the ground of economy**—even though a development of the traces be not necessary, or when increased length is in itself undesirable.

In exceptional cases *both* limbs, *A D, E D*, of the side-valley development may lie on the higher or upper-side of the side-valley, the trace crossing the entrance to the side-valley before entering it. But as such-side valley-crossings must always occur at a considerable elevation—an expensive feature—such a location can only come up for consideration when and if the lower slope of the valley offers great physical difficulty in carrying the entering limb (*A C*) of the development along it.

A change from one side to the other is not unusual when running up a lateral valley for development—particularly in the case of the lower-lying branch of the trace.

In narrow side-valleys running almost parallel with each other it may happen that the entering line lies in the lower valley and the returning branch in the upper and parallel lateral valley—the connecting turn piercing the range separating the parallel valleys.

Fine examples of the utilization of lateral valleys for development are offered by the **Semmering** and the **Brenner** Railways

(2) **Lacets or Loops**—Here the trace follows the main-valley—**Fig. 20**—until it almost intersects the gradually increasingly steep valley-bed and then turns a semicircle;

Fig. 19.

Fig. 20.

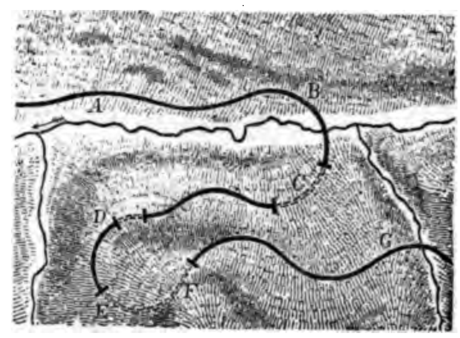


Fig. 21.

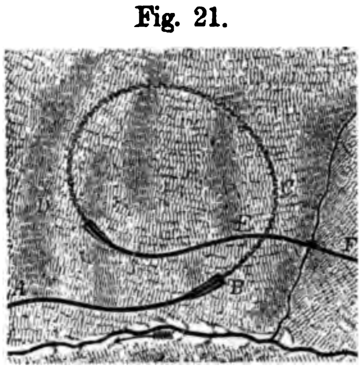
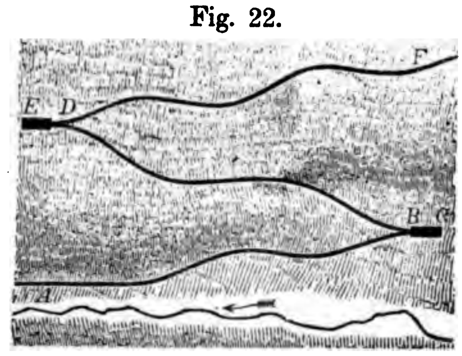


Fig. 22.



rising from the valley-bed it advances ascending in the opposite direction, *i.e.* down the valley-bed and finally returning on itself, in a semicircle, it ascends in the original direction up the valley.

Three portions of the length of this loop-development lie opposite to each other in a cross-section of the valley: these are the lower part *A B*, the reverse *C D E*, and the returning part *E F G*. The gain in length is greater than obtained by running up a side-valley, since the whole length of the loop *B C D E F G* forms the increase in length. But the sharp curves and the usually indispensable tunnels for the turn and return, by the reduction of the grade therein, diminish the length gained.

A further advantage as compared with the employment of lateral valleys lies in the circumstance that the trace in the main-valley may, by means of loops, be carried upwards therein until it almost intersects the valley-bed; whereas when carried-up into a side-valley the development must commence when the trace is already at a considerable height above the valley bottom.

If the shape of the ground allows of so doing it will be advantageous both for the turn and the return to utilize the entrance side-valleys in the manner shown in Fig. 20. In this way it becomes possible to put the turn and the return at least partially in open cut on the slopes of the side-valley.

At the commencement of the loop the trace, as a rule, crosses the main-valley at an elevation just sufficiently great for the construction of a bridge.

It is seldom that this crossing of the valley at the beginning of the loop can be avoided and the return be made towards the hill-side instead of towards the river or valley—bottom, so that the three lengths of the loop opposite one another shall all lie on the same side of the stream.

When there are several loops one above the other in a finite length of the valley—as is usually the case with the ordinary cart-roads but only rarely occurs with a railway line—this form of trace is described as serpentine, each of the bends being about 180°.

Beautiful examples of loop-development are to be found on the **Black Forest** Railway.

(3) **Spirals** are a form of development in which the curvature of the return continues in the same direction as that of the turn, thus forming a complete revolution.

At the end of the spiral the return limb *D E*—**Fig. 21**—must be carried over the turn *A B C*. At the point *E* where the trace crosses itself the whole length of the development furnishes elevation. Both for the turn and for the return of the spiral the presence of side-valleys may be advantageously utilized on the slopes of which the trace may be carried in cut and, under some circumstances, the tunnel work may be confined to the ridge separating the two valleys. In that case we may regard the spiral as a special case of utilization of parallel valleys in which the entering portion of the trace lies in the upper, and the leaving limb in the lower side-valley.

Spirals may, however, be carried out quite independently of side-valleys by penetrating the hill-side by a spiral tunnel and continuing the spiral turning until the direction of the main-valley is reached, emerging in open—cut on to the valley-slope. In narrow valleys without available side-valleys and with steep side-slopes spirals may sometimes be the only possible method of development, however costly such a course may be.

Tunnel-spirals have been employed hitherto only on the **Gotthard** Railway, and on this account the trace of that line is unique and highly interesting.

(4) **Zigzags**. Here the direction of running of the trains is changed at the station at the head of each zigzag.—**Fig. 22.**—Zigzags are as a rule the cheapest form

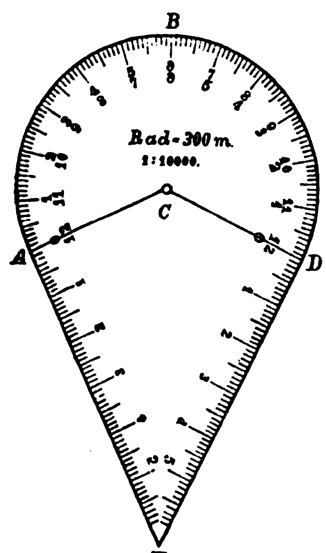
of length-development—only to be used, however, when necessity compels, owing to the inconvenience of the movement and to the ensuing loss of time due to the change of pirection.

An example of a zigzag is that at **Elm** in the ascent of the watershed between **Main** and Fulda on the Frankfurt-Bebra Line. Much more remarkable are the zigzags on the **Lima-Oroya** Railway.

By means of zigzags at *B* and *D*.— Fig. 22.—development *B D F* is obtained in the direction of the valley without sharp curves and generally without long tunnels so that by it height is gained directly and without any reductions.

This advantage is purchased at the expense of the additional lengths of the head-stations, *B C* and *E D* which add unpropitably to the actual length of the line, unless they are also required for or by the traffic, and they increase the outlay on construction and working.

Fig. 23.



§ 38.

Locating the Trace on a Contoured Plan.

The location of the trace by means of a contoured plan cannot be begun until conclusions—provisional and susceptible of alteration—have been arrived at regarding the following points :—

(1) The plan and elevation on the watershed of the upper terminal of the ascent.

(2) The position of the lower terminal in the valley.

(3) The value of the ruling gradient.

(4) The minimum permissible curve—radius.

(5) The minimum length permissible of intermediate straights between reverse curves.

(6) The reduction to be made in the ruling gradient in curves of which the lengths exceed a given quantity and of which the radii are less than a given length.

(7) The reduction in the ruling gradient to be made in tunnels exceeding a certain length.

(8) The position, length, and the maximum permissible gradient of stations.

The location of the trace is usually best commenced at the upper terminal—because its position is more clearly defined than that of the lower one.

If the length of the valley is not sufficient in which to attain the requisite elevation on the assumed grade then we have to examine at what point the development should commence and the particular form it should take—§ 37—to gain the elevation desired. The side of the valley is then to be fixed upon in accordance with the principles of § 33. According to the side of the valley chosen, the point of crossing from one side to the other, the point at which to develop the trace and the mode of development chosen, there will be a number of possible lines for examination and comparison.

To determine the trace, a length $\frac{h}{s}$ — where h is the vertical distance apart of the contours, and s the grade—is taken up in pair of compasses and stepped constantly downwards from each of its intersections with a contour. When the contours curve sharply we measure in the same way the chord of the subsequently required arc making allowance approximately from the outset for the reduction of grade required in the curve.

If when crossing a valley or a gulch the next lower-lying contour of the opposite slope cannot be reached with the length taken in the compasses then we go downwards two or more contours with two or more compass-lengths.

The provisional trace thus obtained forms a series of straight zigzaging lines the ends of which lie on the contours. To this is now to be fitted as closely as possible a series of lines formed of curves and tangents, beginning at the upper-end, and working on a length at a time of one or two kilometres. The arcs are best drawn by means of a template formed of thin card, or preferably of transparent horn, shaped as shown in **Fig. 23**. A number of such templates are prepared for the curves most commonly in use. On the tangent of the template is the linear scale of the contoured plan, and at the beginning of the curved portion the edges are subdivided to the same scale. On the back of the template there is an identical scale for curves in the reverse direction. At the centre of the template there is a small hole enabling the centre of the curve to be marked on the plan.

The template is used as follows: several templates of different radii are moved about on the paper until the one is found, which best suits the zigzags of the line of ascent and the proper connecting length of curve is then templated with a lead-pencil. In this way the arc from A to B is drawn, and template rotated until the end D of the template-arc comes to B, and the tangent to the curve at the end of the latter in the line $D\ E$ is then drawn.

When the trace thus formed of a succession of arcs and tangents has been laid down on the contoured plan it is divided-up into steps of elevation of 1^m or 2^m each.

For example, if the upper terminal of the line to be traced on a ruling gradentof ·0125, $(=\frac{1}{80})$ lies at a height of 500^m then a length of 80^m is to be taken in the compasses on the scale of the plan and, going downwards on the trace, the heights 499, 498, etc., are marked off. If when thus proceeding the point of commencement of a curve of 300^m radius is met in which the grade must be reduced to $\cdot 0125 - \cdot 0033 = \cdot 0092$ then a length of $\frac{1}{\cdot 0092} = 108{\cdot}7^m$ is to be taken in the compasses and with this length the subdivision in the curve continued. Supposing the change of grade occurs, say, at the height of $496{\cdot}4^m$ then from the height 497 to the height 496 the length to be marked-off is

$$\cdot 6 \times 80 + \cdot 4 \times 108{\cdot}7 = 91{\cdot}5^m.$$

Having marked-off on the plan the elevations of the trace in vertical steps of one metre then at each point of division is to be determined a point in the ground, perpendicular to the trace of equal altitude. The line connecting the points so determined running from the top to the bottom of the trace is termed the **zero-line.*** In those parts of the trace which are in bank it lies on the hill-side; and in cuttings, on the valley-side.

If the area contained between the trace and this zero-line is lightly shaded with a a soft pencil, then after a little practice an idea can be easily formed as to whether the banks and cuttings balance one another, or whether to procure this balance the trace should be moved. When the trace crosses deep valleys or lies in tunnels the zero-line is useless.

The trace having been shifted if necessary so as to equalize cuts and fills it is subdivided into lengths or "stations" of 100^m. A small shifting of the points of change of grade is desirable when by so doing the grade-length is made an even multiple of 10^m.

It is now easy from the lengths, altitudes and the contours on the plan to plot a longitudinal section of the line, and for this work millimetre section-paper is most useful. A rough estimate of the banks and cuttings based on the section so obtained will probably indicate places where the trace should be moved inwards or outwards to obtain a better balance of cuts and fills. This completes the general preliminary work of location.

In the detailed working-up of the project cross-sections on the ground taken, at sufficiently small distances apart at an average distance of, say, 10^m are indispensable, for obtaining the best plan and elevation of the trace.

In many cases it is useful in order to avoid a needlessly—wide contoured plan to fix the initial zigzag series of lines on the actual ground by the aid of a clinometer, and to this zigzag line to fit to a second series of polygonal and *longer* lines, and to confine the contoured plan, to the immediate neighbourhood of this line.

[* Vide Wellington: par. 1246, p. 890. This is the line or "trace" along which the amount of earthwork would be an absolute minimum.

"Sometimes when the line runs on a side slope it is considered necessary to lay down the so-called zero-line. "This is the trace on the surface of the ground which would be made by a horizontal line moving continuously "perpendicularly to the centre line. However, it is rare that the local conditions are so simple that any trustworthy "conclusion can be drawn from the relative positions of the zero-line and the centre line as to the equalization of cuts "and fills or the improvement of the location, and for this reason the writer never made any use of it in his practice."

F. Kreuter: Linienführung der Eisenbahnen, p. 132—Tr.].

The remaining stages of the general location, viz. **the joining-up of the straights by curves to form the final trace, its graduation into altitudes, the determination of the zero line, the more exact fitting of the trace to the zero line, the subdivision of the length of the trace, and the plotting of the longitudinal section,** cannot be carried out properly until a contoured plan has been prepared, so that the use of a clinometer does not in any way render the preparation of a contoured plan unnecessary; it simply prevents a needless width of survey being made. And when as usually happens in intricate ground more than one line presents itself for consideration a considerable area of contoured plan is unavoidable and the use of a clinometer then affords no special advantage.

However, the consideration of these questions and indeed the whole of the preceding description of the location of the trace is really an excursion into the province of the practical part of location and is foreign to our immediate object.

FINIS.

APPENDIX.

APPENDIX A.

THE ECONOMICS OF RAILWAY LOCATION.

On the theory and practice of railway location, much has been written, but in most cases the subject has been treated as a general proposition, and analyzed in a more or less mathematical style. A broad discussion of the underlying economic principles which should govern location because of its important relation to construction and operation, has been seldom undertaken. For this reason we have been particularly interested in the new book of rules and instructions for the engineering and construction departments of the **Northern Pacific Ry.**, a copy of which we have recently received from Mr. E. H. McHenry, M. Am. Soc. C. E., Chief Engineer of that company, under whose direction it has been prepared. It covers location, construction and maintenance, and some portions are reprinted elsewhere in this issue by special permission.

Perhaps the most important feature of this book, and certainly one of the most novel, is that which relates to the economics of location, or the relations between location and operation. It is, we believe, the first attempt to publish condensed instructions on this important subject, and to reduce the principles involved to practical rules for the guidance of engineers in the field. These rules are given, practically in full, in another column, and it will be seen that the subject is treated in a thoroughly scientific manner, and with a clear perception of its bearing upon the problem of economical operation.

The rules and instructions will be of exceptional interest to our readers, and might form the subject of more than one editorial, but we will here refer only to the subject of grades. We find no less than four classes of grades treated of: (1) ruling grades, which limit the maximum weight of trains; (2) maximum grades, which may be operated by heavier engines or assistant engines; (3) virtual grades, whose rate represents the real resistance (in excess of rolling friction) taxing the engine cylinder; (4) momentum or velocity grades, introduced to avoid increasing ruling grades by giving a run at the hill. The discussion of the relations of these grades to operating conditions and train service is of special interest at the present time, when so much attention is being given to the reduction of operating expenses by increasing the train loads until they approximate pretty closely to the maximum power of the engines.

Some of the matter contained within the instructions has been drawn from the late Mr. A. M. Wellington's well-known book on "The Economic Theory of Railway Location," supplemented by original matter derived from experience on the Northern Pacific Ry. The theory of virtual grades given in that work has been considerably extended and reduced to practical rules. Mr. McHenry states, in sending us these instructions, that in the discussion of this problem Mr. Wellington erroneously assumed equal tractive power of the engine at all speeds. The engine power may be regarded as a constant, being the product of the speed and tractive factors. These factors vary reciprocally, as shown by a diagram of engine horse powers, which is reproduced in another column, being taken from the instruction book above mentioned.

As an instance of the practical influence of location upon construction and operating expenses, it may be noted that within recent years the Northern Pacific Ry. has expended considerably more than $1,000,000 in grade reductions. The practical application of the theory of "virtual grades" has saved the railway company not less than $300,000 in the first cost of the improvements. In addition to this, it has made it practicable to reduce the ruling rate of grades 0·4 per cent., at far less cost than was originally estimated for reductions to 0·5 per cent. only.

Nearly all our trunk lines have carried out extensive works of a similar kind, and based more or less upon the economic principles and traffic conditions herein referred to, but very little has been published regarding the calculations upon which the works have been planned and carried out.

Going back to the question of original location, it may be well to remind our readers that in the earlier days of our railway history, and up to within less than twenty years ago, the location of railway lines was considered almost a branch of engineering in itself; in fact its relation to the operation of railways was hardly comprehended. There were, of course, certain limitations as to maximum grades and curves, but these were usually of a very arbitrary character, sometimes adopted before even the reconnoissance surveys were made. The effect of the distribution and arrangement of grades, curves, and other features upon train service and operating expenses did not enter into consideration, as a general thing. The engineer, and especially the locating engineer, thought it the end of his business to lay out a favorable and suitable line, and then, perhaps, to superintend its construction. Beyond that point he considered that his field did not extend.

It was in those days that the opinion obtained that locating engineers, like poets, were "born, not made," and much was heard of the wonderful powers of a man with "an eye for country." Location was regarded as something mysterious, and the idea scarcely existed that it could be analyzed on a scientific basis, and its principles made comprehensible to any man. We by no means intend to say that there is nothing in an "eye for country," or that one engineer is as good as another for location purposes. Some men have greater intuitive perception of topographical conditions than others, and many of the old-time locating engineers unquestionably did good work in their day. The "born" locating engineer, however, was apt to consider himself superior to considerations of mere scientific and economic principles. Had he but understood them, the quality of his work might have been of vastly higher grade. He was also apt to be ignorant of, or regardless of, the operating conditions which might be affected by his work. On the other hand, a good bridge engineer or municipal engineer might make a poor hand at location, but if he had carefully studied the latest and best information on the subject, he would probably have a better basis upon which to work than if compelled to rely upon his "eye for country" and his general knowledge of surveying and engineering.

The work of the "practical" locating engineer, as opposed to the scientific engineer is evident on many of our railways, and has in many cases compelled railway companies to incur large expenditures for improvements in their lines in order to facilitate the handling of traffic. Not all the changes and improvements in our railways are due to such a cause. Many of them are due to the fact that the roads were located and built at a time when railway engineering was in its infancy, many of the underlying principles then remaining undiscovered and when it was impossible to foresee the future conditions of traffic. Much of the important work of the locating engineers of recent years has been the designing of improvements in existing lines, **based upon traffic conditions** the problems **including factors never dreamed of in the days when the main study of the locating engineer was to put a line through without exceeding a certain grade. Even in quite recent years traffic conditions have been generally looked upon as entirely foreign to questions of location.**

Difficulties in operation are pretty sure to arise upon a line whose locating engineer failed to look beyond the construction, and where the questions of keeping within the prescribed limits of grade and curve, and of keeping the cost of construction as low as possible, have been regarded as of prime importance. As a matter of fact, the most important feature of railway location is **its influence upon the operation of the railway when built.** The effects of a location which is disadvantageous in this respect will be felt severely in the future, either by the continual expenses and difficulties of operating, or by the expenditures of large sums to improve the original conditions. Two important problems which frequently enter into consideration in regard to operating and economic conditions, are as follows: (1) The comparative advantages of a long developed surface line (permanent or temporary) and a shorter tunnel line involving heavy work: (2) the comparative merits of a line with cheap structures which must subsequently be renewed, or one built with substantial structures at the beginning.

In the book of engineering rules of the Northern Pacific Ry., it is pointed out that in location the aim should not be to secure a line of uniform low cost, but one of least total cost.

It is a common error to reject routes with short sections of heavy and expensive construction, in favor of more uniform (although inferior) routes of greater total cost. We cannot refrain from repeating the following rule, which indicates that when location is apparently easy it is apt to be carelessly done, with the result that it may give a very expensive line:

The possibility of obtaining a very good line should not preclude the search for a better one. The greatest and most costly location errors occur most frequently in prairie regions.

In regard to the tendency in practice to introduce timber trestles (not required for waterways), in order to save time or to avoid difficulties in building embankments, it is explained that the maintenance cost of such structures is far in excess of that of embankments of equal first cost. The rule is therefore given, that such a structure should not be built unless the cost of an embankment exceeds both the first cost of the bridge and the subsequent cost of filling it in. The Northern Pacific Ry., like most of our great western roads, has had a large amount of temporary work, and has spent much time and money in filling trestles and effecting other improvements. Whether these structures might, to some extent, have been avoided in first construction, under the economic and financial conditions then existing, we are not prepared to say. Rapidity of construction was then a prime consideration, and it is probable that in many cases problems were solved in the quickest and easiest way, rather than studied out with a view to future conditions, with which the engineer perhaps conceived that he had little to do. At any rate, it is evident that the Chief Engineer of the Northern Pacific Ry. intends that in the location and construction of future extensions such problems must be worked out upon an economic basis.

In this connection we may refer our readers to the profiles of transcontinental roads which were given in our issue of June 10, 1897, with some discussion upon their general features and the arrangement of their grades. In that article it was stated that the great Northern Ry. gets through the mountains with an easier profile than that of the Canadian Pacific Ry. This is true as far as the mere question of grades is concerned, but from an operating point of view the Canadian Pacific Ry. has far the better location. The heavy grade sections of the former road cover a distance of 650 miles, while the heavy grades of the latter are bunched in one operating section of 125 miles, and westbound trains have but one lift, and eastbound trains two lifts. By reason of this concentration of heavy grades, it is claimed that for a corresponding distance from the Pacific coast this road can move a given amount of tonnage with a smaller expenditure of power than can be done on any other of the transcontinental lines. Our attention has also been called to the fact that while the two roads have total ascents of 15,305 ft. and 23,051 ft., respectively, these figures are for distances of 1,780 and 2,906, miles. Comparing the routes from St. Paul to the Pacific by the Great Northern Ry., and the Soo Line and Canadian Pacific Ry., the latter has nearly 3,000 ft. less elevation than the former. Similar results are obtained from a comparison of the main line of the Great Northern Ry, with that of the Canadian Pacific Ry. for equal distances from the Pacific Coast.

In closing, we may refer to one more point, that of short direct routes as compared with longer but easier routes between the same points. In several cases, undue importance has been attached to directness of route, without consideration of operating conditions. In the article on "Notes on Colorado Railways," in our issue of April 6, reference was made to a particular case in which a short direct route is far inferior to the roundabout route. The late Mr. Louis de Busschere, Engineer-in-Chief of the Belgain State Railways, published only a short time before his death, a pamphlet on "**Virtual Lengths of Railways,**" discussing the difference between the actual length in miles, and the virtual length as affected by conditions of grade and curvature, with their influence upon the speed. His discussion was more particularly in regard to high-speed lines, and showed that a line of shorter actual length might be unable to compete economically with one of shorter virtual length, owing to the unfavorable conditions under which the former had to be operated.

Engineering News:—20 *April* 1899.

APPENDIX B.

RAILWAY GRADIENTS.

BY H. HAUPT, C.E.

The paper now published was written in 1870, as part of a report on the reconnaissance for the Shenandoah Valley Railroad; reference was made to it in the published portion of the report, and several requests made by members of the engineering profession for copies, which could not be conveniently complied with.

As the article is not precisely in proper shape for a scientific magazine, and as the author cannot spare the time required to rewrite it, this explanation may possibly be accepted as an apology for defects.

GRADIENTS.

Two important objects should be sought in the arrangement of the gradient on any important line of road, and these are economy in construction and in operation.

To understand the propriety of the proposed recommendations it is necessary to observe, that an ordinary run of a locomotive is about 100 miles per day, but it is preferable to make the working distances on a long line as nearly as possible 50 miles, so that each engine will travel this distance and return.

By this arrangement the repair shops will be located at intervals of about 100 miles. Each engine will receive its repairs at the same shop, so that accounts can be properly kept and responsibility fixed, and at the end of each run of 50 miles time is afforded for a thorough examination and adjustment preparatory to the return trip.

The load of an engine on any working division of 50 miles is determined by the maximum resistance offered by grade and curvature, and except where assistant engines are employed no greater load can be carried over the division than can be carried over the point of greatest resistance. While, therefore, it may be expedient to make large expenditures to reduce the maximum resistances on any division, yet when these have been established it is not proper to sacrifice capital by reducing points of less resistance, as not a single car could thereby be added to through trains, or any appreciable economy of operation secured.

The angle of friction is that on which a train would just move or continue in motion by gravity alone.

This angle has been practically determined by the actual load of engines on different gradients of the Pennsylvania Railroad, where the eastward trains are loaded to the capacity of the engine; the angle of friction was thus found to be 24 feet to the mile, and the gross load on a level 1,200 tons with the standard engine there used.

On a grade of 24 feet, therefore, the power required to move 1,300 tons would be doubled, or an engine would carry but 600 tons; on a grade of 30 feet the load would be 541 tons; on 84 feet, 270 tons, or one-half the load on 30 feet.

This being understood, an attempt will be made to prove, contrary to the generally received opinion, that **undulating gradients below the limits of maximum resistance are not objectionable** and that while admitting of great economy of construction they do not materially increase the cost of operation, as compared with uniform and low gradients between the same points; also that the use of **higher gradients for part of a given distance will often result in greater economy of operation than a lower and a uniform gradient for the whole distance.**

It was formerly the practice of engineers to compare different lines of railway by conceding a given amount of rise and fall as equivalent to a mile of distance. This is not correct practice; the profile of the line and the direction and amount of tonnage, or, in other

words, **the maximum resistances and their distribution over the line, are the elements to determine questions of relative economy of operation**; the rise and fall affect the question very slightly.

If the maximum resistances can be concentrated at one point and overcome at once with the aid of assistant engines while lighter gradients in favor of the direction of the tonnage prevail on all the rest of the route, the line will be operated **cheaply**; but if the maximum resistances are scattered over the whole line at intervals more or less remote, the operation will be **expensive**.

In the solution of the problems in railway economics it is not safe to apply general rules or principles too freely.

Each line presents a problem in itself, the solution of which should depend on the particular data which the case presents; millions of dollars have been sacrificed by conforming to general rules and theories when circumstances required variations.

As a case in illustration, 30 miles of a railroad in Massachusetts were located on a uniform gradient below the maximum resistance of the division, and if constructed on that line the cost would have exceeded a million and a half of dollars. A million dollars were saved by a change of location and of gradients which did not reduce the load of an engine over the division a single pound, the gradients used being still much below the limits of maximum resistance of the other portions of the division.

One or two practical illustrations will be given, having reference to the system of gradients recommended, which will make this subject sufficiently clear to the practical reader. Suppose the gradient which determines the load of an engine over any division is of 30 feet to the mile, and that it would be practicable by an expenditure not excessive to locate a portion of the division with a uniform ascending gradient of 6 feet to the mile, but that by ascending at the rate of 36 feet to the mile for five miles and descending 24 feet to the mile for the remaining distance a large saving could be effected in construction. There are few perhaps who would not make a sacrifice to secure the lower gradient, and yet it can be shown that it has no advantages even in economy of operation over the undulating and much higher gradient.

A 6-feet gradient requires one and a quarter the power required on a level, and in 10 miles the power may be expressed by $1\frac{1}{4} \times 10 = 12\frac{1}{2}$. On a 36-feet gradient the power required is $2\frac{1}{2}$ times that on a level, and for five miles would be represented by $2\frac{1}{2} \times 5 = 12\frac{1}{2}$, but in descending the next five miles at the rate of 24 feet to the mile no power will be required, as gravity will move the train; therefore the whole expenditure of power in passing over the ten miles will be the same in the second case as in the first.

Again, assume as before that the ruling gradient which determines the load of an engine is 30 feet to the mile, and that an elevation of 1,090 feet in 23 miles may be overcome either by using a uniform gradient of $47\frac{1}{2}$ feet for the whole distance, or a 30-feet gradient for 15 miles, and an 80-feet gradient for 8 miles.

At the same cost of construction a majority of engineers would adopt the uniform and lower gradient; but it can be readily shown that the higher gradient for part of the distance will be much more economical. As the engine is supposed to be fully loaded for a 30-feet gradient, any increase above this limit involves the use of additional power, or, what is generally done, the splitting up and reconstructing of trains. If assistant power is used, economy requires that the power should be worked to its capacity; there would be great waste in sending an engine only half-loaded. Now, the gradient which doubles the power required on 30 feet is 84 feet, and it requires no argument to prove that it would be cheaper to use the double power on 8 miles rather than on 23, and therefore to extend the 30-feet gradient for 15 miles and use 84 feet for the remaining 8 miles is far better than $47\frac{1}{2}$ feet for the whole distance of 23 miles.

These figures and this illustration have been used because they apply exactly to a case in point in overcoming the summit between New River and James River, at the head of Potts' Creek; but there is yet another and a very important consideration in favor of the adaptation of the higher gradient, which arises from the fact that streams near the summit almost invariably fall much more rapidly than the average for the whole distance, and along a valley bounded by mountains the uniform grade would throw the line high above the stream, encountering spurs and deep gorges and greatly increasing the cost of construction as compared with a line conforming more nearly to the fall of the stream.

Another consideration is that the higher gradient allows advantage to be taken of favorable ground; and to avoid difficulties, flats on the sides of mountains may often be reached and spurs avoided by variation of grade within the maximum limit where the uniform gradient would admit of no variation.

To those who have not given the subject particular attention, the cost of operating a high gradient by assistant power when concentrated at a single point will be surprisingly low.

Taking the report of the Virginia & Tennessee Railroad as furnishing data nearest the locality of the Shenandoah Valley Extension, the cost of an engine per mile run, for fuel, oil, waste, repairs, engineer, fireman, etc., averages 35 cents. For a distance of eight miles it would be \$2·80, and although the engine would return empty this allowance will be doubled for the round trip, making it \$5·60, which should be in excess of actual cost.

The gross load in 30-feet gradients being 540 tons, the net load would not be less than 240 tons, and the cost of assistant power per ton would therefore be 23 mills.

If one such gradient occurred in 100 miles, the cost of assistant power would be less than a fourth of a mill per ton per mile, and if only once in 200 miles the cost would be less than one-eighth of a mill per ton per mile.

The cost per mile run of freight trains on the Virginia & Tennessee Railroad was last year, for transportation, machinery and road expenses, 111 cents; as the whole cost of assistant power on a round trip of 16 miles was \$5·60, it is obvious that the increased expense of overcoming the elevation assumed for illustration by an 84-feet gradient would not be more than the expense of operating 5 additional miles of 30-feet gradient, or 2½ miles when the return trains are taken into account.

Simplicity rather than rigid accuracy has been sought in these illustrations.

The object has been to remove unreasonable prejudices against high gradients when properly employed, and to satisfy the board of directors that the system of high gradients recommended for the Shenandoah Valley Line admits of an economy of operation that cannot be approached in any of the great leading lines of the country crossing elevated summits or in mountainous regions. The nearest approach is the Pennsylvania Railroad, and the credit of this is due to J. Edgar Thomson, by whom the gradients were arranged before the writer became connected with the road. It is probable that upon the Shenandoah Valley Line there will be five tons carried east for one in the opposite direction; it is necessary to determine, therefore, the maximum gradient, ascending westward, that will be equivalent to 30 feet ascending with full trains eastward.

The gross load of an engine on a 30-feet gradient was given at 540 tons, the net load 240; one-fifth of this would be 48 tons, which added to the dead weight of engine and cars would give the average gross weight of westward trains 380 tons, which would be the load on a grade of 53 feet nearly. If assistant power should be used, the grade ascending westward equivalent to 84 feet eastward would be about 130 feet (see, for computation of effect of gradients in paper read before the American Philosophical Society and published in *Van Nostrand's Magazine*).

If, then, it should be found practicable without excessive expenditure to descend from the summits eastward with gradients not exceeding 53 feet, it should be done; if the expense should be too heavy and higher gradients employed so as to require the use of assistant engines ascending westward, then gradients as high as 130 feet in that direction might be employed to advantage so as to limit the use of such engines to the smallest number of miles.

My conclusion in regard to gradients is, that a line can be located between the point of intersection with the East Tennessee, Virginia & Georgia Railroad, near Russellville and Covington, on the Chesapeake & Ohio Railroad, with no ascending gradient eastward exceeding 30 feet, and no ascending gradient westward exceeding its equivalent for single engines in that direction of 53 feet, excepting at one and possibly at the two main summits dividing the waters east and west of New River, where the equivalent gradients for assistant engines of 84 feet ascending east, and 130 feet ascending west should be employed. These high gradients are recommended.—*Van Nostrand's Engineering Magazine.*

Reprinted in The Railroad Gazette:—5 *July*, 1873.

APPENDIX C.

THE THEORY OF COMPENSATING GRADIENTS.*

At one time the doctrine of compensating gradients was held as an article of faith by large numbers of engineers. Enormous use was made of it in committee rooms. It is trotted out even in the present day not unfrequently, and made to do service in Parliamentary contests by astute lawyers. It is a very simple doctrine. Those holding it believe that *the cost of working the traffic on a line with many steep inclines is no greater than that of working a dead level.* The trains require, it is true, a great deal of power to pull them up hill; but then they run down hill of themselves without any pulling whatever. Consequently the cost of working is independent of the gradients, the running down hill compensating for the up-hill work. The original Bill for the London and South-Western Railway was stoutly opposed by the Great Western, one of the grounds of opposition being that the inclines on the South-Western road would be found too steep to admit of the line being worked with commercial success. The theory of compensating gradients was, however, urged with so much force *by Dr. Lardner and others,* that the preamble of the South-Western Bill was taken as proved, and the Bill itself was passed, the ruling gradient of the South-Western being taken as 1 in 250. The point had been very keenly disputed, and the Parliamentary decision on the measure ended the matter. Finally, Dr. Lardner induced the British Association to appoint a committee and carry out experiments, the results of which were as is well known entirely unexpected. In the present day the matter is better understood; but the doctrine has cropped up again in connection with light railways, and even in connection with much more important undertakings it will probably be heard of again during the next Session of Parliament. It is by no means unnecessary, therefore, to say a few words here on the subject, and to clear away misconceptions into which it is very easy to fall.

In laying out a line of railway the engineer has to deal very largely in compromise. It is certain that we cannot have all that we would like to have, and from first to last we must balance considerations and conditions against each other. *Theoretically, the best line is one dead level and perfectly straight, raised just high enough above the surrounding country to provide for good drainage.* The purpose for which the railway is constructed must be always borne in mind, which means that the civil engineer must never forget that locomotives will haul trains on it. Sir Benjamin Baker said last week—and said truly—that whatever the world wanted it got. No matter how steep the inclines or how sharp the curves, it is possible to make locomotives to haul trains up and round them, but only at a price. Mechanically, inclines may in a sense be made compensatory; pecuniarily, never. The steeper the inclines the greater will be the cost of motive power. *The whole theory of compensation is founded on error.* It is enough to state a very simple case to prove this. On a certain line, 20 miles long, there is about midway a chain of hills to be got over, or got through, or got round. To get over these we have, say, two inclines, each five miles long, rising to meet each other at the top like the rafters of a roof. The gradient is 1 in 75. The remainder of the line is *fairly* level. Now, the leading condition here is the power of a locomotive, which can haul a coal train, say, of 450 tons gross, to the top of the incline. It matters nothing that when the train has been got to the top, it will run down hill for five miles of itself. It will be necessary to apply brakes all the way down, and the wear and tear will be great, while the saving in coal will be very small. The locomotive must be of necessity very large and powerful. How large and how powerful our readers can judge for themselves, if they will turn to page 462 in our issue for November 8th. The cost of such engines is very great; the permanent way must be of the heaviest type; the ballast, the sleepers—every detail, in fact, must be very good in order to stand the great hauling effort put forth by the locomotive. The engine will be very much too powerful for 15 miles of line out of 20. A favourite way out of this difficulty is to employ a banking engine to help a train engine up the hill. But this represents *probably* much more expenditure than the use of a heavy train engine. It means the wages of a second

*[The Translator is responsable for the italics.]

driver and fireman, a largely increased fuel expenditure, more shop work, *&c.*, &c. The use of banking engines is always to be discouraged, and besides, the moment it is conceded that a banking engine is wanted, the whole theory of compensating gradients melts away. We have only to frame a mental comparison between the working of the steep gradient line with one on a dead level, or nearly so—because a *tunnel has been driven through the hills*—to perceive how *overwhelming is the advantage possessed by the level line.* If this be true, however, of a line which is not crowded with traffic, it becomes much more evident when a line has as much and more than it can accommodate. *The capacity of a line for work is profoundly influenced by the gradients.* It would be found, for example, very difficult to work trains of more than 450 tons on inclines of 1 in 75. Such trains nearly represent the limit of locomotive power on the normal gauge. Banking heavier trains than this would be possible, but extremely objectionable. In fact, the working of heavy trains on very steep hills is an operation by no means free from risk, very costly, and a *constant source of anxiety to all concerned.* But there need be no difficulty in hauling 900 or even 1000 tons on a level or nearly level road with very moderate engine power.

But, it may be urged that a pair of inclines of 1 in 75 is seldom met with—at least in Great Britain. This may be true; but inclines of 1 in 100 or 1 in 120 are not. Indeed, we happen to know of projects which will probably soon be brought before Parliament in which such inclinesare contemplated: and the advocates of the light-railway-along-the highway system think nothing of hills of 1 in 30, or even less. If our readers have followed us thus far, they will have no difficulty in seeing that while the 1 in 75 inclines represent an extreme case on the one hand, the road on a dead level represents an equally extreme case on the other hand. Between the two comes the road with inclines less steep than 1 in 75. The civil engineer has, indeed, a *somewhat complex* problem to solve. The more nearly the profile of the railway conforms to the general lie of the country traversed by it, the less will be the capital outlay on its construction. Generally speaking, a level line is one with heavy cuttings and lofty embankments. The civil engineer must consider then what return he can get for his outlay in levelling the line by augmenting his earthwork, *and act accordingly.* Let us suppose, for example, that the cost of a given line will be £9,000 per mile with a ruling gradient of 1 in 200, and £13,000 per mile with a ruling gradient of 1 in 350. Is it worth while to flatten the line? Will the difference between the *working expenses* in favour of the flatter line pay reasonable interest on the extra outlay? It will not do to say that 5 per cent. can be earned on the £9,000, and to rest content. If 5 per cent. can also be earned on the £13,000, then there ought to be no hesitation in adopting the flatter road. *The arguments in its favour are overwhelming.* The steeper road never, under any circumstances, can possess a working capacity equal to that of the more level road, because, no matter how powerful the locomotives may be that are placed on the hilly line, the same engines could pull much heavier trains on a level track. The capacity question is very frequently overlooked. It seems to be tacitly taken for granted that two lines of rail are always enough, but recent railway history proves that this assumption is wrong. Roads are being doubled daily, at very heavy cost, simply because the carrying capacity, which is measured by the number of trains that can be run on them, has been raised. When the lines are fairly level, the difficulty can be got over, as far as minerals and heavy goods are concerned, by running much longer trains. *It is for the simple reason* that the capacity of a level road is much greater than that of a hilly road can be, *that it is worth while to incur a large outlay in flattening a line.* Every case, however, must be considered on its merits; no hard-and-fast rule can be laid down.

Where lines are hilly they are usually crooked as well. Here, again, we have matter for careful consideration. To straighten out a curve may be a very expensive operation. Can it be made to pay? Again, we must bear in mind what are the working conditions. It will be seen that in New South Wales Mr. Eddy and his fellow commissioners have thought it well worth while to expend large sums in altering curves and cutting down hills. It may be said with truth that the New South Wales roads are peculiar and exceptionally severe. But it *does not follow at all that they teach us nothing in this country.* No one can, we think, read

the admirable report, the publication of which we concluded in our last issue, without seeing that much of it applies to all railway systems. It teaches us that the cost of working steep inclines *must be very heavy*. Mr. Eddy has nothing to say about compensating inclines. In setting out a road in this or any country, the engineer will have to consider what is to be *urged in favour* of a sharp curve as well as what can be said against it. *But he must never forget* that curves represent great loss of hauling power, great increase of resistance, wear and tear, and risk. They may, however, also represent a very considerable reduction in first cost.

Up to this point we have had principally in view heavy coal or mineral traffic. The theory of compensation certainly breaks down with it, *if not wholly*, then so far that the most that can be said is that *less coal is burned running down hill than would be burned on a level. When*, however, *we come to deal with fast trains the compensating theory fails completely—disappears in thin air*. Up even *moderately* heavy inclines high speeds are impossible of attainment, and an attempt to run down grades as compensation soon reaches a limit beyond which it is impossible to go. Let us suppose that a line is laid on the compensating principle, and that the termini are on the same level. If the inclines are so steep that a velocity of not more than forty miles an hour can be attained up them, then the down grades must be traversed at eighty miles an hour if an average speed of sixty miles an hour is to be reached. When we come to examine the theory of compensation we find that it breaks down with heavy slow traffic, because nothing that the descending grade can do will help to reduce the work of getting up hill—the only exception is when the adverse inclines are so very short that they can be "rushed," like some on the North London Railway, for example—and that as a consequence as much locomotive power must needs be provided as though there were no inclines to descend. With fast traffic, on the other hand, the compensating action of the down grade cannot be fully utilised, because its utilisation would mean speeds which are excessive. It follows from all the considerations that we have here set before our readers, that the civil engineer should make the road as level as he can under the governing conditions, *the probability* being that the extra cost entailed will give a better return than money invested in any other way in connection with the railway. If steep inclines are, however, unavoidable, then they should as much as possible be concentrated, in order that banking engines may be used to the best advantage either for fast passenger or heavy goods trains.

The Engineer:—22 *November*, 1895.

APPENDIX D.

THE INFLUENCE OF GRADIENTS ON SPEEDS AND LOADS.

On all our colonial railways the time comes sooner or later when the question comes to be considered whether the heavy gradients which made construction easy and inexpensive does not involve a heavier charge on working expenses than is permissible in the interests of economy. In **New South Wales** this was recognised, the gradients were flattened at considerable expense, but the result was much heavier loads, and therefore a greater net revenue per train-mileage. In **South Africa** this question has also been engaging attention in recent years; the necessity for reducing the high maximum gradient of 1 in 40 has been clearly established in view of the heavy inward load to be hauled up these gradients, and a re-survey of the whole of the Midland and Eastern systems from this point of view was undertaken and completed in 1894. About 80 per cent. of the latter system was on the maximum grade. A sum of £25,011 was spent in reducing the maximum gradient to 1 in 80 near Zwatkops and Caerney on the Midland; but in dealing with these improvements the reasonable principle has been laid down, that **unless the cost of banking engines and other locomotive traffic and maintenance expenditure, equals or exceeds the interest on the extra capital to be sunk in the change of gradient and curves, it should not, as a rule, be undertaken**; and that relief should be sought rather in increasing the weight of the locomotives up to the maximum which the 3 ft. 6 in. will admit, than in costly alterations in the location of the line. If this principle is adhered to, no doubt much disappointment will be avoided. The saving actually to be effected by costly changes in the alignment of lines which were laid out with extreme care to suit the natural features of the country traversed, is comparatively small.

The **running of special test trains** between Port Elizabeth on the Midland, East London on the Eastern, and Springfontein on the Orange Free State Northern was determined upon with the object of ascertaining the actual and relative cost of haulage between those points as well as the cost of return empties. Two test trains were run from each port each way along the nearest route by the respective systems, to the common terminal point on the Northern system, and an exhaustive statement of the results was published. This statement, and the returns accompanying it, are most instructive, giving as they do the actual and contingent cost of running goods trains of known weight on lines with varying ruling gradients, the time employed in running them under working conditions, and the absolute fuel consumption. The engines were of an unusually heavy type, with eight wheels.

TABLE I.
Gradients, Distances, Speeds.

Items.	Midland.			Eastern.				Percentage in favour of	
	Port Elizabeth to Cradock.	Cradock to Springfonte in.	Port Elizabeth to Springfontein.	East London to Cyphergat.	Cyphergat to Burgersdorp.	Burgersdorp to Springfontein.	East London to Springfontein.	Midland.	Eastern.
Ruling gradient...	1 in 40 2·5 p.c.	1 in 80 1·25 p.c.	...	1 in 40 2·5 p.c.	1 in 80 with load 2·5 p.c.	1 in 80 1·25 p.c.	...	on 1 in 40 25·44 on 1 in 80 61·03	
	m. ch.	m. ch.	m. ch.	m. ch.	m. ch.	m. ch.	m. ch.		
Distance (open mileage)... ...	181 38	180 40	361 78	204 38	38 75	70 26	313 59		13·32
Train mileage of total load ...	327½	180½	507¾	430½	71½	70¼	572½	11·25	
Excess of train mileage over open mileage per cent. due to difference in ruling gradient ...	80·32	0	40 24	110·78	82·92	0	82·54	105·11	
Up Loaded.									
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.		
Running time exclusive stoppages for through trains up	13 0½	14 32½	27 53	16 37½	2 55½	6 17	25 50	...	7·39
	m.	m.	m.	m.	m.	m.	m.		
Running time, average speed ...	13·41½	13·04½	13·33	12·30	13·53	11·10½	12·35	7·12	
Down Empty.									
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.		
Running time down	14 37	11 19	25 57	14 26½	2 55½	35	22 36	...	14·82
	m.	m.	m.	m.	m.	m.	m.		
Running time, average speed ...	12·42	15·95	13·91	14·43	13·36	13·78	14·32	...	3·00

Remarks.—The maximum speed on the up journey was 15·12 miles on the Midland and 13·75 miles on the Eastern, and on the down journey 18·48 miles and 16·26 miles respectively.

TABLE III.

Trains, Weights, Loads.

Items.	Midland.			Eastern.				Percentage in favour of	
	Port Eliza-beth to Cradock.	Cra-dock to Spring-fontein.	Port Eliza-beth to Spring-fontein.	East London to Cypher-gat.	Cypher-gat to Burgers-dorp.	Bur-gers-dorp to Spring-fontein.	East London to Spring-fontein.	Midland.	Eastern.
Class of engine	Eight wheels coupled, with tender in bogie			Eight wheels coupled, with tender and bogie					
(Up Loaded).									
Gross weight of train in pounds (exclusive of engine and tender).	496,482 396,840	893,322	893,322	438,620 485,373	506,720 416,275	923,995	923,995	3·31	
Net weight of paying load.	297,115 247,656	544,671	544,671	264,225 305,350	307,900 261,675	569,575	569,575	...	4·28
								On 1 in 40	On 1 in 80
Net load per cent. of gross load	59·84	60·98	60·98	60·24	60·76	60·54	60·54	0·66	0·54
Non-paying load per cent. of paying load...	67·10	64·08	64·08	66·00	64·54	65·73	65·73	1·95	1·83
Additional weight hauled on 1 in 80 per cent. of that on 1 in 40	...	79·63	...	...	...	85·05	...	...	5·12
(Down Empty).									
Gross weight of train in pounds (exclusive engine and tender)	249,957	366,765	308,000	210,255	247,200	395,216	255,462	...	20·56
Up.									
Composition of train, 9 ft. and 10 ft. wheel-base.	9 6 trucks 1 van	15 trucks 1 van	15 trucks 1 van	9 6 trucks 1 van	7 8 trucks 1 van	15 trucks 1 van	15 trucks 1 van		
Down.									
Composition of train, 9 ft. and 10 ft. wheell-base.	12 trucks 1 van	16 trucks 1 van	16 trucks 1 van	9 trucks 1 van	12 trucks 1 van	18 trucks 1 van	14 trucks 1 van		

The trial trains were made up chiefly of the following types of stock: Long bogie flats, weighing 9 to 11 tons, and carrying 15 to 18 tons; sheep trucks, weighing 4¼ to 5½ tons, and carrying 7¾ to 8¼ tons; shorts flats, weighing 8¾ tons, and carrying 6 tons. The loads were made up of: 1. Rails; 2. Timber; and 3. General merchandise. The vans weighed 5¾ tons on an average.

TABLE IV.

Expenditure and Receipts.

Items.	Midland.			Eastern.				Percentage in favour of	
	Port Eliza-beth to Cradock.	Cradock to Spring-fontein.	Port Eliza-beth to Spring-fontein.	East London to Cypher-gat.	Cypher-gat to Burgers-dorp.	Burgers-dorp to Spring-fontein.	East London to Spring-fontein.	Midland.	Eastern.
Up (Composite).	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	per cent.	per cent.
Actual cost of haulage (including water, oil, tallow, waste fuel, wages, &c.)	14 16 5	10 6 4	25 2 9	22 2 2½	2 10 7	4 4 6½	28 17 4	12·92	
	d.	d.	d.	d.	d.	d.	d.		
Actual cost of 1 ton (2,000 lb.) of goods between given points ...	13·07	9·10	22·17	18·63	2·13	3·56	24·32	8·84	
Actual cost per train-mile ...	10·87	13·72	11·88	12·32	8·69	14·44	12·11	1·90	
" " ton per open mile	0·0720	0·0503	0·0612	0·0911	0·0547	0·0506	0·0775	26·63	
" " " train-mile	0·0399	0·0503	0·0436	0·0432	0·0248	0·0506	0·0425	...	2·58
Total cost of haulage, including actual and all other locomotive charges (except office, general superintendence, and maintenance of carriages) and guards' wages	£ s. d. 28 0 4	£ s. d. 17 17 9	£ s. d. 45 18 1	£ s. d. 44 7 8	£ s. d. 6 14 10½	£ s. d. 7 13 5½	£ s. d. 58 16 0	21·98	
	d.	d.	d.	d.	d.	d.	d.		
Total cost of 1 ton of goods between given points	24·71	15·77	40·48	37·40	5·68	6·47	44·55	18·30	
Total cost per train-mile... ...	20·55	23·77	21·20	24·74	22·64	26·21	24·34	12·90	
" " ton per open mile..	0·1311	0·0873	0·1118	0·1829	0·1460	0·0920	0·1579	41·23	
" " " train-mile.	0·0754	0·0873	0·0798	0·0868	0·0795	0·0920	0·0865	8·39	

TABLE IV—*continued.*

Items.	Midland.			Eastern.				Percentage in favour of	
	Port Eliza-beth to Cradock.	Cradock to Spring-fontein.	Port Eliza-beth to Spring-fontein.	East Lonton to Cypher-gat.	Cypher-gat to Spring-fontein.	Burgers-dorp to Spring-fontein.	East London to Spring-fontein.	Midland.	Eastern.
Down (Single).									
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.		
Actual cost of haulage	5 5 7	5 4 10	10 10 5	5 13 10½	1 10 4½	2 1 3	9 5 6	...	12·89
" " per train-mile and open mile	d. 6·98	d. 6·97	d. 6·975	d. 6·68	d. 9·34	d. 7·04	d. 7·10	3·22	
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.		
Total cost (as above)	12 0 4	11 18 8	23 19 0	15 4 10	3 6 9	5 14 0½	24 5 7½	1·38	
" per train-mile and open mile	d. 15·88	d. 15·87	d. 15·875	d. 17·8	d. 20·55	d. 18·97	d. 18·56	16·53	
Freight rate per ton of 2,000 lb....	2	2	2	2	2	2	2		
" " " 2,240 " ...	2¼	2¼	2¼	2¼	2¼	2¼	2¼		
	£	£	£	£	£	£	£		
Receipts from net load	412·0	409·5	821·5	485·5	92·5	167·0	745·0	10·27	
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.		
" per open mile	2 5 4½	2 5 4½	2 5 4½	2 7 6	2 7 6	2 7 6	2 7 6	...	4·69
" " train-mile	1 5 2½	2 5 4½	1 12 5	1 2 7	1 5 10½	2 7 6	1 6 0½	2·70	
Actual expenses per cent. of receipts	3·60	2·52	3·06	4·55	2·73	2·53	3·87	26·11	
Total expenses per cent. of receipts	6·80	4·37	5·59	9·14	7·29	4·59	7·89	4.·14	

Note.—The prices upon which the charges were estimated and calculated out were the following: 600 lb. extra fuel was in each case allowed for raising steam.

TABLE V.

Item of Charge.	Midland.		Eastern.		Average.
	Rates.	Cost per Train-Mile.	Rate.	Cost per Train-Mile.	
Coal (uniform rate) per ton ...	13s. 9¼d.	Loaded 7·60d. Empty 3.15d.	13s. 9¾d.	Loaded 7·04d. Empty 3·304d.	Loaded 7·32d. Emty 3·225d.
Store charges	4 per cent.	Loaded 0·00d. Empty 3·125d.	...	Loaded 0·29d. Empty 0·133d.	Loaded 0·295d. Empty 0·129d.
Wages per hour of running time:					
Driver	1s. 2d.	1.30d.	1s. 2d.	1·345d.	1·32d.
Fireman	0·8d.	0·75d.	0·8d.	0·77d.	0·76d.
" incidental, shed time, washing-out days, relief men, lodging money, &c.	40 per cent. of driver's & fireman's wages.	0·825d.	40 per cent. of driver's fireman's wages.	0·845d.	0·835d.
Shedmen, coalmen, and labourers, per ton of coal consumed.	12s.	Loaded, 1·253d. Empty, 0·51d.	2s.	Loaded, 1·15d. Empty, 0·53d.	Loaded, 1·19d. Empty, 0·52d.
Cleaners, at per trip... ...	6s.	0·40d.	6s.	0·70d.	0·55d.
Sick fund, average current ...	0·26d.	0·26d.	0·38d.	0·38d.	0·32d.
Guard's wages per hour running time.	9d.	0·825d.	9d.	0·615d.	0·72d.
Oil, tallow, waste, &c., current average.	0·53d.	0·53d.	0·91d.	0·91d.	0 72d.
Water, current average ...	0·70d.	0·70d.	0·58d.	0·58d.	0·64d.
Engine repairs, current average	2·55d.	2·55d.	4·46d.	4·46d.	3·50d.
" replacement on life of 600,000 miles.	1d.	1·00d.	1d.	1·00d.	1·05d.
Repair and maintenance of wagons and goods vans.	3d.	3·00d.	3½d.	3·50d.	3·05d.
Banking engine	2s.	Loaded, 0·625d. Empty, —	2s.	Loaded, 0·52d. Empty, —	Loaded, 0·57d. Empty, —
Average charges ...	...	Loaded, 21·90d. Empty, 15·92d.	... ...	Loaded, 24·10d. Empty, 19·50d.	Loaded, 23·00d. Empty, 17·50d.

Remarks.—The total locomotive department charges per train mile barely exceeded 6½ per cent. of the receipts, and as these are direct charges always due whenever a train is run, this shows how exceedingly heavy indirect and incidental charges are on railways, as well as the losses incurred by running trains with light instead of full loads, in comparison to the direct and really unavoidable charges, these being on these two systems from one-tenth of all the charges, and amounting in 1894 to from 49·7 per cent. to 64·7 per cent. of the total receipts.

TABLE VI.

Fuel Consumption per Ton of Load Hauled (Weight of Engine alone Excepted) per Mile and per Train-Mile.

Section.	Full Loads. Colonial Coal = Welsh Steam. Per Ton of Load Up. per lb. 2000	per lb. 2040	per lb. 2000	per lb. 2240	Per Train-Mile Up. Colonial Coal = Welsh Steam.		Empty. Per Ton of Load Down. Colonial Coal = Welsh Steam. per lb. 2000	per lb. 2240	per lb. 2000	per lb. 2240	Per Train-Mile Down. Colonial Cost = Welsh Steam.		Ordinary Working Conditions. Average per Section. per Ton of Load (2240 lb.)	Per Train Mile.	Remarks.
													Welsh Steam Equivalent.		
Village Drift Coal.	lb.	lb.	lb.	lb.	lb.	lb.	lb.	lb.	l .	lb.	lb.	lb.	lb.	lb.	General average per ton of load per mile, 22¼ lb. Ditto per train mile, 42 lb.
Port Elizabeth to Cradock...	37¼	·40¾	·24¾	·27¾	92·56	61·71	·21¾	·24⅜	·14½	·16¼	39·77	26·51	·22	44·11	
Cradock to Springfuntein ...	23⅛	·26	·15½	·18¼	106·37	70·91	·30¾	·34½	·20½	·23	38·63	25·76	·20⅝	48·33	
Average	30 3/61	·33⅜	·20⅛	·23	99·45	66·30	·26¼	·26¼	·17½	·19⅜	39·21	26·14	21 5/6	46·22	
Cypherghat and Fair View.					Welsh coal = Colonial coal × 1·5.										
East London to Cypherghat..	44¾	·50	·25¼	·28¼	100 64	57·00	·39	·43¾	·22⅛	·24¾	37·04	20·00	·26½	38·50	
Cypergbat to Burgersdorp ...	18½	·20¾	·10½	·11⅓	47·06	26·67	·51⅜	·57⅝	·29⅛	·32¼	63·51	35·88	·22¼	31·27	
Burgersdorp to Springfontein	20⅛	·22½	·11⅜	·12¾	92·79	53·57	·27⅛	·24¾	·12½	·14½	43·87	24·85	·13½	31·21	
Average ..	35¾	·39¾	·20¼	·22⅜	92·33	52·26	·36¼	·41½	·20⅛	·23¾	41·82	22·70	23	37·72	
					Welsh ooal Colonial coal × 1·875.										

REMARKS.—The average are all higher on the Midland than on the Eastern trials, owing probably to less care in the firing in the former than in the latter, and partly to the Tilney grate in use on the latter system being superior for colonial coal to the finger grate used on the Midland, but this item of coal consumption was insufficient by itself to turn the scale in favour of the Eastern system, as the coal was more costly and less efficient in the latter than on the rival system. (*Vide* Tables III. and IV. *ante.*)

TABLE VII.

Fuel Consumption in Relation to Gradients in Welsh Steam Coal Equivalents.

For eight-wheeled coupled goods engine on 3 ft. 6 in. gauge. Speed 12 to 16 miles per hour.

Description.	Consumption per Train-Mile. Minimum.	Consumption per Train-Mile. Maximum.	Consumption per Ton Hauled per Mile. Minimum.	Consumption per Ton Hauled per Mile. Maximum.	Remarks.
	lb.	lb.	lb.	lb.	
1 in 40 against haul	90	120	0·45	0·60	Uniform / Full load } =200 tons = maximum load on 1 in 40.
1 in 40 with haul	10	13⅓	0·05	0·066	Do. represents amount required to keep up steam, train propelled by its own weight.
1 in 80 against haul	57	77	0·28½	0·38½	Do. do.
1 in 80 with haul	12·66	17·11	0·06½	0·08½	Do. partially.
Level	24	32	0·12	0·16	Full load = 200 tons.

REMARKS.—The consumption per train-mile to keep up steam is about one-ninth of the maximum consumption on 1 in 10, and haulage on the level takes about 2½ times the minimum consumption to keep up steam and 5½ times the the minimum working up 1 in 80 and 1½ times down 1 in 80. The consumption on chopping gradients and for variations of load are within the limits above given ever changing even at constant speeds.

Table I. deals with the gradients, distances, and speeds. The most noticeable feature seems to be the excessive train-mileage run on the Eastern system in comparison to the open mileage—this excess being 86½ per cent. on the Eastern as against only 40¼ per cent. on the Midland, resulting in an advantage for the latter of 64½ miles in actual running, of 11·25 per cent. in train-mileage, and of 105·11 per cent. on the relative percentages; in despite of the Eastern route being actually 48 miles 39 chains or 13·32 per cent. shorter in open mileage than the Midland; this advantage in train-mileage for the Midland is due to the comparative

lengths in the two systems where the ruling gradients are 1 in 40 and 1 in 80 respectively; these lengths being as 7¼ on the Midland to 9¾ on the Eastern, giving the former 35·44 per cent. less 1 in 40 and 61·03 per cent. more 1 in 80 than the latter.

This shows unequivocally that **the comparison between two routes to be of practical value must be based upon the two factors of length and gradient, and not on length alone**; and that it is mistaken economy to locate a line so as to make the length a minimum with the object of reducing first cost if the percentage of maximum gradient is largely increased thereby, more especially where the line is likely to be a competing line, as was the case with the Eastern system in a major degree and the Midland system in a minor degree. It is as well to remember that at a constant speed the change introduced by gradients is equivalent to an increase of length of line inversely as the rates of inclination of the respective gradients, as the following Table shows:

TABLE II.

Gradient Equivalents.

Gradient.	Rate of Inclination.	Equivalent Length on the 1 in 40 Standard.	Gradient.	Rate of Inclination.	Equivalent Length.
	per cent.	miles.		per cent.	
1 in 40	2·5	1	1 in 90	1·111	2·25
1 „ 45	2·22	1·12	1 „ 95	1·052	2·31
1 „ 50	2·00	1·25	1 „ 100	1·000	2·50
1 „ 55	1·82	1·31	1 „ 120	0·833	3·00
1 „ 60	1·63	1·50	1 „ 150	0·666	4·00
1 „ 65	1·53	1·63	1 „ 200	0·500	5·00
1 „ 70	1·43	1·74	1 „ 300	0·333	7·50
1 „ 75	1·33	1·88	1 „ 400	0·250	10·00
1 „ 80	1·25	2·00	1 „ 500	0·200	12·50
1 „ 85	1·176	2·12	1 „ 1000	0·100	25·00

Thus if the two systems be compared as equalised on the basis of the ruling gradient of 1 in 80, the eastern route, instead of being shorter by 48 miles 39 chains, or 18·22 per cent., actually turns out to be longer by 13 miles 29 chains, or 3·60 per cent. The comparative trials were made under service time-table conditions, and not as specials, and the Midland maintained on the up journey a remarkably uniform rate of running, equal to 13½ miles average; but on the return journey the running was quicker on the 1 in 80 (16 miles), and slower on the 1 in 40, though the average was still under 14 miles. On the up journey the running on the Eastern was slower—one mile per hour less on an average, least of 1 in 80—and less uniform, while on the down journey the reverse was the case.

The percentage ratios of net load to gross load and non-paying load to paying load are intrinsically unfavourable, *i.e.*, much more deadweight has been hauled and is generally hauled even with full loads than should be the case. And a design of truck even more economical in respect to deadweight than the standard American long bogie for the standard gauge, is not an impossibility which should reduce these percentages to 44 and 50 respectively, and improvement in this respect is even more urgently called for on the 3 ft. 6 in. gauge, than on the 4 ft. 8½ in.

In neither system did the loads hauled on the 1 in 80 gradient exceed those on the 1 in 40 by the possible 100 per cent., but the 85 per cent. realised on the Eastern was a fair approximation under ordinary working conditions.

Engineering:—13 *Aug.* 1897.]

APPENDIX E.

INVESTIGATION AS TO VIRTUAL GRADES; CANADIAN PACIFIC RAILWAY.

By A. C. Dennis.*

The subject of **virtual grades** is of considerable importance to the railway engineer, and one which deserves more attention than it has generally received. The accompanying diagrams were used in determining the value of **momentum** in connection with the proposed reduction of grades on the Ontario & Quebec Division of the Canadian Pacific Ry. The present line between Montreal and Toronto has maximum grades of 1% in both directions, which, being uncompensated for curvature, are equivalent to about 1·12%. The gross westbound tonnage, including locomotives, is about 62% of the eastbound; and it is proposed to balance the grades to conform to the tonnage by reducing the eastbound to 0·6% compensated, without any reduction in the westbound grades. With such grades, the locomotive rating behind the tender will be 1,600 tons east, as shown on the diagram, and 900 tons west. The latter will usually be somewhat reduced, because the westbound trains are composed largely of empty cars, which **offer greater resistance per ton than loaded cars.**

The assistance in surmounting heavy grades which may be derived from the momentum of a train at high speeds is almost universally understood, yet engineers usually hesitate to adopt a virtual profile, notwithstanding that one may be practicable which would utilize the energy that has been stored in the train. Mr. A. M. Wellington, in discussing the subject, assumed that the tractive power of the locomotive and the train resistance remain practically constant at different velocities. This, as is well known, is far from correct; and the error which is introduced by this assumption increases as the grade increases over that for which the train is rated.

The diagram in Fig. 1 is designed to represent the performance of an engine loaded for an ascending grade of 0·6%, at 7 miles per hour, on grades from 0 to ± 1·1%, with velocities ranging from 0 to 40 miles an hour. The three curves ar as follows: (1) The tractive power (in pounds per gross ton at different speeds) of the 145% freight locomotive in use on the Montreal and Toronto section of the Canadian Ry.; (2) the train resistance curve taken from Wellington's experiments with loaded cars; (3) the curve representing the algebraic difference of the first two curves or the force available for accelerating or retarding the velocity of the train. The latter, when reduced to its equivalent gravity resistance in rate per cent. and added (algebraically) to the actual garde, is the rate per cent. of acceleration or retardation. This rate per cent., divided into the difference of velocity heads gives the distance in which that change of velocity is accomplished. With these distances the diagram shown in Fig. 2 is plotted.

The accompanying Table shows how the computation was made from the curves in diagram 1. The table as printed covers only the speeds from 1 to 10 miles per hour, and grades from −0·5% to +0·5%, and is inserted only to show the method of work. The manner of extending the table to include the same range of speeds and grades as the diagram will be obvious.

As an example of the use of the diagram, in Fig. 2, suppose we have a descending grade of 0·2%, one mile long, beyond which is an ascending grade of 1·0%, and we desire to know how far up this 1·0%, grade an engine can haul a train loaded for an ascending grade of 0·6% grade and arriving at the top of the 0·2% grade with a velocity of 10 miles per hour. Following the vertical line in the diagram, representing 10 miles per hour, it intersects the —0·2% grade at the horizontal line representing 200 ft. The point where the 5,280 + 200 ft. line intersects this same grade corresponds to 25·3 miles per hour, which is the velocity the

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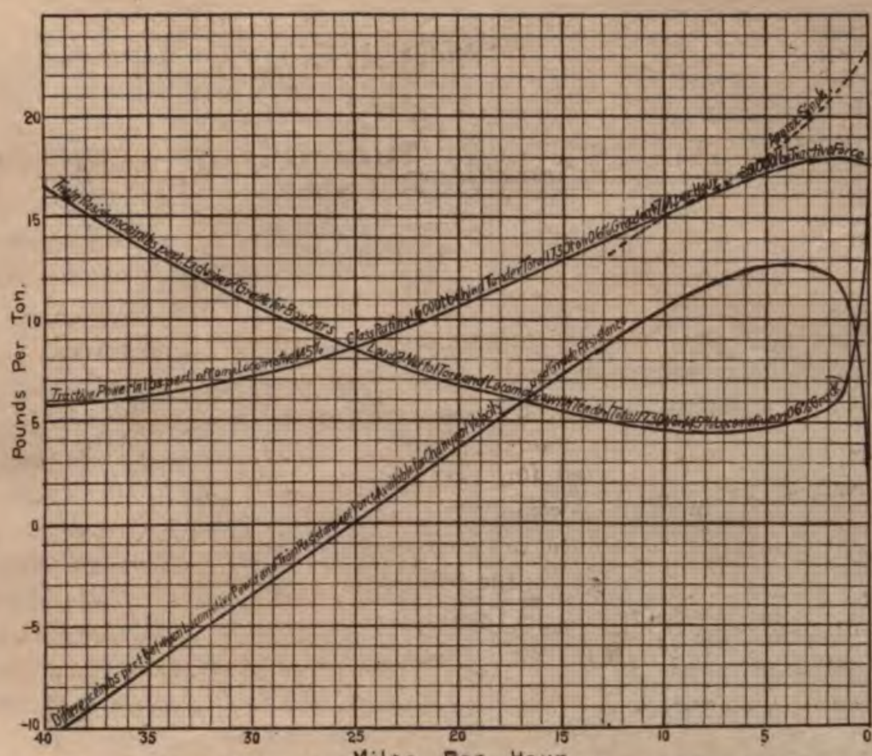

FIG. DIAGRAM OF ENGINE PERFORMANCE ON GRADES; CANADIAN PACIFIC RY.

Put diagram in its place
Level !

Will be attended to when printing

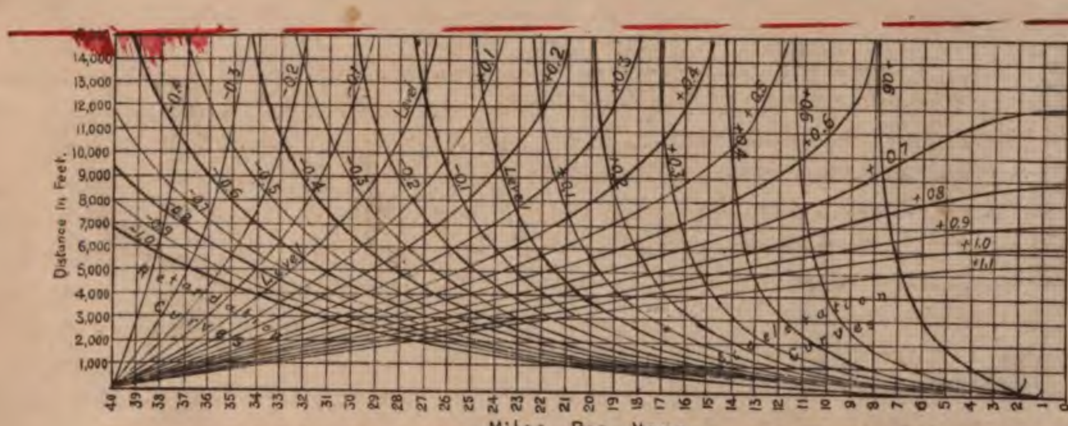

FIG. DIAGRAM FOR DETERMINING VIRTUAL GRADES; CANADIAN PACIFIC RY.

Distances in Stations for Various Changes of Velocity on Grades Shown for 140°/₀ Locomotive Working Compound and Haulting Full Rating for 0·6°/₀ Grade.

Speed miles per hour.	Total velocity head.	Diff. in velocity h'ds.	Diff. locomtive work and train resistance†	Grade equivalent to same rate °/₀.	—0·5 D.	—0·5 T.	—0·4 D.	—0·4 T.	—0·3 D.	—0·3 T.	—0·2 D.	—0·2 T.	—1·0 D.	—1·0 T.	Level. D.	Level. T.	+0·1 D.	+0·1 T.	+0·2 D.	+0·2 T.	+0·3 D.	+0·3 T.	+0·4 D.	+0·4 T.	+0·5 D.	+0·5 T.
					Grade per cent. — Distances in stations.																					
1	0·04	0·04	8·0	—0·40	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	·14	·10	11·8	—0·59	0·1	0·1	0·1	0·1	0·1	0·1	0·1	0·1	0·1	0·1	0·2	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·5	0·5	1·1	1·1
3	·33	·19	12·6	—0·63	0·2	0·3	0·2	0·3	0·2	0·3	0·2	0·3	0·3	0·4	0·3	0·5	0·4	0·6	0·4	0·6	0·6	0·9	0·8	1·3	1·4	2·5
4	·56	·23	12·8	—0·64	0·2	0·5	0·2	0·5	0·2	0·5	0·3	0·6	0·3	0·7	0·4	0·9	0·4	1·0	0·5	1·1	0·7	1·6	1·0	2 3	1·6	4·1
5	·88	·32	12·8	—0·64	0·3	0·8	0·3	0·8	0·3	0·8	0·4	1·0	0·4	1·1	0·5	1·4	0·6	1·6	0·7	1·8	1·0	2·6	1·3	3 6	2·3	6·4
6	1·26	·38	12·6	—0·63	0·3	1·1	0·4	1·2	0·4	1·2	0·4	1·4	0·5	1·6	0·6	2·0	0·7	2·3	0·9	2·7	1·2	3·8	1·6	5·2	3·0	9·4
7	1·71	·45	12·4	—0·62	0·4	1·5	0·4	1·6	0·5	1·7	0·6	2·0	0·7	2·3	0·7	2·7	0·9	3·2	1·1	3·8	1·4	5·2	2·0	7·2	3·7	13·1
8	2·24	·53	11·6	—0·59	0·5	2·0	0·5	2·1	0·6	2·3	0·7	2·7	0·8	3·1	0·9	3·6	1·0	4·2	1·3	5·1	1·8	7·0	2·8	10·0	5·9	19·0
9	2·83	·59	11·4	—0·57	0·5	2·5	0·6	2·7	0·7	3·0	0·8	3·5	0·9	4·0	1·0	4·6	1·2	5·4	1·6	6·7	2·2	9·2	3·5	13·5	8·5	27·5
10	3·55	·72	10·8	—0·54	0·6	3·2	0·7	3·4	0·9	3·9	1·0	4·5	1·1	5·1	1·3	5·9	1·6	7·0	2·0	8·7	3·0	12·2	5·1	18·6	18·0	45·5

D. stands for difference, and T. for total distances, stations. † Difference of locomotive work and train resistance, or force in pounds per ton available for changing velocity or overcoming grade resistance.

train will have acquired at the foot of the – 1% grade. The speed of 25·3 miles per hour on a + 1% grade corresponds to a distance of 2,800 ft.; 0 miles per hour on the + 1% grade corresponds to 6,200 ft. The train, would, therefore, stall at (6,200–2,800) = 3,400 ft. from the foot of the grade, or would run (5,830 – 2,800) = 3,000 ft., before its speed would be reduced to 7 miles per hour. This distance (3,000 ft.) can, therefore, be operated as a virtual + 0·6% grade, beyond which point the grade must be reduced to an actual + 0·6% grade. Experiments made within rather narrow limits of velocity and grade checked very closely with calculations made on the above basis.

It is obvious that a locomotive with a tractive power curve which decreases less rapidly as the speed increases can be operated more favorably as regards momentum grades, so it is important that such grades be calculated for the most unfavorable class of engine likely to run on that section.

It considering the question of reducing the grades on the Montreal and Toronto section it is found that by the assistance of momentum grades only about 30% of these exceeding the new ruling grade of 0·6% will require to be reduced, or that it will be necessary to rebuild only about 10% of the line whereas, to reduce these to actual 0·6% grades as much as 30% of the line would have to be rebuilt. From this it is evident that much of the work usually done in reducing grades to a lower common rate per cent., **without utilising the momentum** which has been acquired by the train, is practically **useless expenditure.** The subject is certainly one which offers a profitable and interesting field for experiment and study by the man who draws the red line on the profile.

Engineering News:—22 *November* 1900.

APPENDIX F.

MOMENTUM GRADES.

By C. Frank Allen.*

The letter on Velocity Grades in the *Railroad Gazette* of Dec. 8, 1899† prompts me to write something on this subject. The principle involved in the use of velocity, or momentum grades is, of course, this: Of the pull or tractive force exerted by the locomotive when in motion, part is used in overcoming the ordinary level tangent resistances (including journal, rolling and atmospheric), part on curve resistance, part on grade resistance, and if these do not consume the entire tractive force, what is left acts to produce an increase in the speed of the train. That is, this unbalanced part of the pull acting through a definite distance accomplishes work which finds its equivalent in the increased energy possessed by the train, and this is expressed in terms of velocity. In a similar way, when the pull of the locomotive is insufficient to cover the resistances on tangent, curve and grade, this deficiency exists through a definite distance acting substantially as a retarding force. Now to meet this, work must be done, and this can be accomplished only by drawing on the energy of the train due to its velocity, and in yielding energy the train suffers loss of speed. Thus, a train starting upon a grade, at a speed of 20 miles an hour and ending at 5 miles an hour, can climb a hill steeper than would be possible at constant velocity.

A proper understanding of the suject may be had by using the method of "virtual heights" and "virtual grades," following Wellington's general method. It is well understood that any body to which velocity has been imparted, has acquired energy sufficient to cause it to rise (barring resistances) to a height given by the well-known formula $h = \frac{v^2}{2g}$. This becomes for velocities in miles per hour $h = 0{\cdot}033445 V^2$, and to this should be added an allowance for the energy due to the rotation of the wheels about their own axes. Wellington finds this to add 6·14 per cent. to the result, making the formula $H = {\cdot}0355 V^2$. The height thus found is the "velocity head" and Wellington, p. 335, gives a table (No. 118) for finding heights ("velocity heads") corresponding to velocities.

Knowing the elevations of the various points on the track, we have the actual profile. Having the actual profile, if we add to the actual height at any point the "velocity head" due to the speed at that point, we thus find what we may call the "virtual height" at that point, and a series of "virtual heights" connected give a "virtual profile," and from this we can find the "virtual grades." These "virtual grades" measure the resistances to be overcome by the locomotive *in addition* to the level tangent and curve resistances. It is evident that the effect of the actual grade is taken into account since the actual grade is used in determining the virtual grade. The matter can better be understood by examples.

(*a*.) Let the full line in Fig. 1 represent an actual profile. A train is assumed to pass Station 0 at a speed of 15 miles an hour, to proceed at the same speed to Station 10, and continue at the same speed to Station 15. The velocity head for 15 miles = 7·99. The "virtual height" at Station R then = 17·99. At 10 the speed is the same, and the virtual height therefore the same. The "virtual grade" is level, and the locomotive has to overcome only the level tangent and curve resistances. At 15 the actual elevation is 17·50, and for the same speed of 15 miles, the "virtual height" is 25·49; the "virtual grade" is parallel (*i.e.*, equal) to the actual grade of + 1·50 per 100, so that in passing from 10 to 15, the locomotive has to overcome a grade of + 1·50 per 100 in addition to level tangent and curve resistances. The pull of the locomotive evidently must be greater here from 10 to 15 than it is from 0 to 10.

The road may be operated, however, so that the locomotive will exert a uniform pull from 0 to 15, having a speed of 15 miles an hour, however, both at 0 and 15. The virtual heights will be at 0, 17·99; and at 15, 25·49; a rise of 7·50 in 15 stations, or + 0·50 per station,

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† See Appendix G.

so that the "virtual grade" under this method of operation is + 0·50 per 100. At 10, the "virtual grade" line is at elevation 17·99 + 5·00 = 22·99, or 12·99 above the actual profile at 10. But 12·99 is the "velocity head" for a speed of $19\frac{1}{10}$ miles per hour. The locomotive which leaves Station 0 exerting on a level track the pull necessary for a + 0·50 grade, pulls between 0 and 10 more than the amount of the level tangent and curve resistances, and the surplus pull serves to raise the speed from 15 miles to $19\frac{1}{10}$ miles. From 10 to 15 the pull due to a + 0·50 grade, is insufficient for the actual + 1·50 grade, but here the train parts with some of its surplus energy, loses velocity and yet reaches Station 15 with a speed of 15 miles per hour.

(*b*.) Similarly in Fig. 2, assume as conditions that the train passes Station 0 at 25 miles per hour, and reduces speed to 10 miles per hour at Station 60. The locomotive is to exert a uniform pull.

The virtual elevation at 0 = 20·00 + 22·20 = 42·20.

The virtual elevation at 60 = 60·00 + 3·55 = 63·55.

The virtual grade will be $\frac{63{\cdot}55 - 42{\cdot}20}{60} = + 0{\cdot}356$ per 100.

The velocity head at 20 = 42·20 + 7·12 − 10·00 = 39·32.

Examples of the Application of Momentum Grades.

Fig. 1.

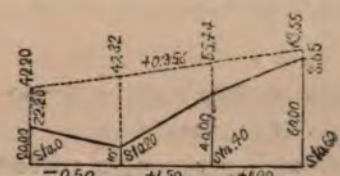

Fig. 2.

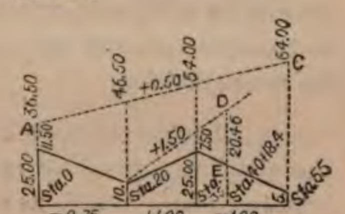

Fig. 3.

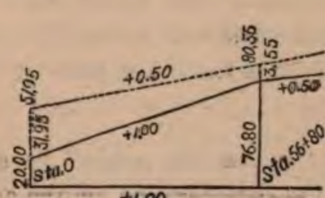

Fig. 4.

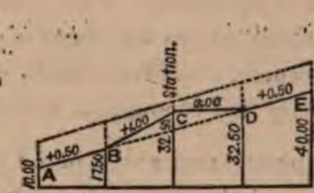

Fig. 5.

The velocity head at 40 = 42·20 + 14·24 − 40·00 = 16·44.

The speed at 20 = 33·3 miles, and at 40 = 21·5 miles per hour.

The principle of "momentum grades" has been made use of mainly in connection with steep grades. The train is operated so as to pass the foot of the grade at high speed and reach the summit at very low speed; in this way the rate of "virtual grade" becomes much less than that of the actual grade, but it is the "virtual grade" which is the effective "maximum grade" and which measures the resistance that the locomotive must overcome. That the principle involved is an important one, and should be made use of in many cases, goes without saying. That "momentum grades" should be used indiscriminately in original location admits of serious doubt. To the writer it appears that there are serious objections to placing complete dependence on the use of momentum on grades for several reasons.

First.—The necessity for stopping or even slowing down (either at the bottom or any point on the grade) would interfere with the operation of the grade by momentum, and a stop or a slowing down might be necessary from several causes: viz.

1. A track-gang at work relaying track.
2. A bad bridge, unsafe at high speeds.
3. Bad weather and cautionary orders.
4. Cattle or other obstruction on the track.
5. Signals out of order or fog.

Second.—It is objectionable to acquire a high velocity at a sag for several reasons:

1. Broken wheels are more commonly found at the foot of long grades than elsewhere.

2. At high speeds in passing the sag, the shock upon the draw-gear is severe, and vertical curves to avoid this are impracticable. Wellington's rule requires for a $-$ 1·00 grade succeeded by a $+$ 1·00 grade, a curve extending 40 stations each side of the vertex, with a consequent raising of grade from vertex to curve of 20 vertical feet.

3. Train discipline on freight trains cannot be maintained if orders are given that a speed of 15 miles an hour shall not be exceeded except at (the most dangerous places) sags in the line.

Third.—On very long grades the difference in "velocity heads" will be distributed over so great a distance that the resulting decrease in "maximum grade" (from actual to virtual) will be comparatively small; while on very short grades it is often possible (and certainly preferable) to reduce the actual grade.

Fourth.—There is always a material advantage in keeping *any* grade somewhat lower than the maximum, because train loads are seldom constant throughout an entire division, and any special piece of steep grade may happen to occur at the place when the train must haul its heaviest load, and a grade lower than maximum would allow a slight increase of load to be carried on any train over that portion of the division, and so increase the capacity of the entire division.

For these reasons the writer would always be slow to make use of "momentum grades" (for effecting a practical reduction of the maximum) on *original* location. In revising line, however, where conditions of traffic and operation are well established, a failure to take advantage of the principle, on grades which could not be reduced, would sometimes result in largely increased cost of operating. The "momentum grades" would then be made use of, not from definite preference, but as the lesser of two evils, because a failure to so use them would result in something worse, small train loads over an entire division the other parts of which would allow satisfactory and economical working.

The writer urges that in no case should the "momentum grade" be accepted as a matter of course, but it should in each case be taken into consideration by the engineer, who can then accept or reject it in view of the special circumstances existing, exercising that "engineering judgment" which in this, as in all engineering questions, considers the practical bearings in connection with the mathematical conclusions, and in view of evidence of various sorts, determines what is the wise procedure.

(*c*.) The "virtual profile," however, is capable of more extended usefulness than has thus far been shown. On any profile, as in Fig. 3, a "virtual grade" line AC may be drawn with an inclination which marks the maximum grade at which a locomotive can haul its train at uniform speed (or which represents the pull which it is intended it shall exert). The distance from the actual profile to this grade line then shows, by the "velocity head," the speed at any given point.

In the figure, for instance at 0, assume velocity $=18$ miles, then at

20, vel. head $=46{\cdot}50-10{\cdot}00=36{\cdot}50$ and vel. $=32{\cdot}1$ miles.

35, vel. head $=54{\cdot}00-25{\cdot}00=29{\cdot}00$ and vel. $=28{\cdot}6$ miles.

55, vel. head $=64{\cdot}00-\ 5{\cdot}00=59{\cdot}00$ and vel. $=40{\cdot}8$ miles.

(*d*.) In a similar way it is shown that another engine can haul its train on a grade of $+1{\cdot}50$ per 100 without losing velocity. In Fig. 3, the train is assumed to have stopped at Station 20, and so will leave it from a state of rest. At what point will it have acquired a speed of 24 miles an hour? Obviously, at the point where $DE=20{\cdot}46$, which is the "velocity head" for 24 miles an hour. This will be at Station $40+18{\cdot}4$.

(*e*.) Similarly in Fig. 4, a train starts from 0 with a speed of 30 miles an hour. The grade is to be a momentum grade. The locomotive can pull its train on a + 0·50 grade without change of velocity. When its speed is reduced to 10 miles an hour, it must proceed without further loss of velocity, and therefore the actual grade beyond this must not exceed + 0·50. At what point will its volocity be reduced to 10 miles an hour? Evidently at Station 56 + 80, where the difference in elevation between the actual and virtual grades is 3·55, which is the velocity head for 10 miles an hour.

(*f*.) In one special direction the principle of "virtual grades" can be made use of to considerable advantage, and this is in original location at stopping points. In may cases it may be predicted that all or nearly all trains will stop at each way-station. The operation of the railroad will be facilitated and economy result if the track at the station be placed at a higher elevation than would be proper if no station was there. For example, in Fig. 5, instead of the straight line for a uniform grade of + 0·50 from *A* to *E*, make the grade *BC* = + 1·00 and *CD* level. From the figure it can be seen somewhat more effectively than without it that the speed of a train from either direction will be checked as it approaches *C* for a stop, and thus less expenditure of braking force will be necessary. In leaving *C* in either direction, not only will the grades allow easier starting of the train, but velocity will be acquired much more rapidly than if the profile had followed the dotted line from *B* to *D*. The statement has been made that it was at one time proposed for the elevated railroads of New York City to place the stations at the summit of grades from either side. The idea failed of adoption probably because such an arrangement would require more exercise of leg muscle on the part of the patrons of the road in mounting to the stations.

There are many possible applications of this principle of "virtual grades" besides those mentioned. Superintendents in making time charts could determine definitely the effect of actual grades in limiting the speed of trains, the time required for acceleration of speed in leaving stations, but it is not the purpose of this letter to exhaust either the subject or the reader.

To the writer it seems very desirable to acquire a knowledge of principles in some such fashion as above, rather than to use a formula simply by substituting values, often without understanding the principles involved. This view of the matter will furnish an excuse for the space occupied in attempting to outline one method of treating the interesting subject of "momentum grades." The writer uses the term "momentum grade" in preference to "velocity grade" which occurs in the article of Dec. 8.* Neither term is very scientific, but "momentum grade" has been more used, probably than "velocity grade." The term "virtual grade" is good, but not as suggestive.

The Railroad Gazette:—12 *Jan*. 1900.

[*See pp. 160, 161.]

APPENDIX G.

THE THEORY OF VELOCITY GRADES.

ROANOKE, VA., *Nov.* 30, 1899.

TO THE EDITOR OF THE RAILROAD GAZETTE:

There are questions connected with velocity grades, and their advantage in handling tonnage, that are tedious to answer, figuring from first principles. The following may be found convenient in such cases:

The relation between the factors that make up the situation may be expressed by the following formula:

in which
$$T = \frac{0.5\,(P_1 + P_2)\,L}{RL + 20\,GL - \frac{1}{3}(S_1^2 - S_2^2)} \quad \ldots \quad \ldots \quad \ldots \mathrm{I}.$$

T = greatest weight of train, including engine and tender, in tons of 2,000 lbs., that can be handled under the conditions given.

S_1 = initial speed in miles per hour; or the speed at which the train is moving when its centre of gravity is at the beginning of the distance L.

S_2 = final speed in miles per hour. This is the speed of the train when its centre of gravity is at the end of the distance L.

L = length of track, in stations of 100 ft. each in which the speed varies from S_1 to S_2.

P_1 = traction of the engine in pounds, at the rails, when moving at the speed S; the engine being supposed to be exerting all the traction it can at that speed.

P_2 = the same at speed S_2.

R = average resistance of friction, in pounds per ton, for the entire train.

G = rate of grade, in feet per station, which is uniform for the distance L. G is the positive for an up grade, and negative for a down grade. On a level of course G is zero.

When the tractive power of an engine at varying speeds is known, Formula I. may be used with correct results. Generally, however, this is not known, and it appears best to the writer to use in such cases an empirical expression for the traction that may be reasonably expected from an engine of given weight on driving-wheels moving at a given speed. An accurate expression must necessarily include, not only the governing dimensions of the engine, but also a combined expression for the personal equations of the engineer and fireman and and the coal used. Especially is it better to use such an emperical formula when revising grades on an engine division, than to make use of the traction of a given engine that may become obsolete in a few months. The writer suggests the following as an expression for the traction in pounds, for an engine weighing D pounds on driving-wheels:

$$P = \frac{4D}{S + 10} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \mathrm{II}.$$

In this, S is the speed at which the traction, P, can be exerted; S being over four miles per hour, and sand being used at slow speeds when necessary to increase adhesion.

In the following examples of the use of Formula I., the weight of the engine on driving-wheels, D, is taken at 150,000 lbs., and the values of P_1 and P_2 from Formula II. are substituted; amusing that $P_1 = \frac{4D}{S_1 + 10}$ etc.

For a uniform speed we have $S_1 = S_2 = S$, and

$$T = \frac{600{,}000}{(S + 10)\,(R + 20\,G)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \mathrm{III}.$$

which is an expression for the total weight of the train that can be taken up a grade of G per cent. at a uniform speed, S.

If advantage is to be taken of the momentum of the train in overcoming the grade, and we assume an initial speed S_1 of 25 miles per hour reducing to $S_2 = 5$ miles per hour, and assume the average friction as $7\frac{1}{2}$ lbs. per ton we have

$$T = \frac{1{,}428\ L}{\cdot 375\ L + G\ L - 20} \quad \dots \quad \dots \quad \dots \quad \dots \text{IV.}$$

in which T is the total weight of train that can be taken up the grade by the given engine with the given initial and final speeds of 25 and 5 miles per hour. The expression $G\,L$ is the total rise in feet.

To find whether a desired initial speed can be attained for use on a given velocity grade, say, 26 miles per hour, it must be determined whether the train can attain a final speed of 26 miles per hour at the end of the approach track to the given velocity grade. Amusing that the train can reach the approach with a speed of 6 miles per hour we have $S_1 = 6$ and $S_2 = 26$, and

$$L = \frac{426{\cdot}6\ T}{27{,}150 - R\,T - 20\ G\,T} \quad \dots \quad \dots \quad \dots \quad \dots \text{V.}$$

in which L is the distance required to get up a speed of 26 miles per hour from an initial speed of 26 miles per hour. If the grade is falling the last sign in the denominator is positive.

If it is desired to find the steepest grade by which a given train can ascend a given height, with an initial speed of 25 miles per hour and a final speed of 5, we have, calling the total rise H,

$$L = \frac{T\,(H - 20)}{1{,}428 - \cdot 375\ T} \quad \dots \quad \dots \quad \dots \quad \dots \quad .. \text{VI.}$$

This gives the minimum length of the grade L, from which, with the total rise, the maximum rate of grade can be calculated.

If a train starts up a grade of indefinite length with a given speed, there is a point at which the momentum is expended, and from there on the engine must overcome the remainder of the grade by traction only, unassisted by momentum, in order to reach the summit. To locate this point for any given grade make $S =$ (say) 5 in Equation III, and substitute the resulting value of T in Equation IV. Then if R is taken at $7\frac{1}{2}$ lbs. per ton we have after transposing

$$L = \frac{186.5}{2\frac{2}{3}\ G + 1} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \text{VII.}$$

This gives the maximum length of velocity grades of given rate G. Longer grades come under the Equation III. If some other value of S had been assumed in Equation III. the maximum length of velocity grade would be altered; but in any case the length is such that just as many total tons can be taken up by taking a run at the grade of 25 miles per hour reducing to 5, as by a steady pull at the speed assumed. In this case, however, an assumed uniform speed of eleven miles per hour cannot be exceeded, as greater assumed speeds result in a negative value for L which is of no practical utility.

Formulas I. and II. may be combined and written

$$T = \frac{3\ D\ L\left(\frac{1}{S_1 + 10} + \frac{1}{S_2 + 10}\right)}{1{\cdot}5\ R\ D + 30\ G\ L - (S_1{}^2 - S_2{}^2)} \quad \dots \quad \dots \text{VIII.}$$

This formula contains seven variables, any six of which being known the seventh can be found. Formula II. appears to fairly represent traction within the limits of variation met with in practice, and may be brought in line with any given actual figures by slight alteration of the constants. Of course any expression for traction in pounds may be substituted for P_1 and P_2 in Formula I.; and the more correct the expression the more correct will be the results.

CHAS. C. WENTWORTH,
Bridge Engineer Norfolk & Western Railway.

8 *December*, 1899.

To the Editor of the Railroad Gazette:

In the article on "The Theory of Velocity Grades" in your issue of Dec. 8, p. 837, there is given an empirical formula $P = \frac{4\,D}{S + 10}$ where P = traction of engine at rails in pounds, D = wt. of engine on drivers in pounds, and S = speed in miles per hour. The assumption made here is at variance with the ordinary practice of considering that the adhesion is the measure of the traction and that the adhesion is independent of the velocity. In the brake experiments made by the Galton (in connection with Westinghouse) the coefficient of adhesion of car wheels was found to be independent of the velocity of the car. From the experiments of Prof. Goss on the effect of incorrect counterbalancing of locomotive wheels, it seems possible that for locomotive wheels some decrease of coefficient may result from an increase of speed. At moderate speeds, the experiments indicate that the effect could hardly be important, and it is not readily conceivable that the coefficient of adhesion should from this cause vary from $\frac{1}{4}$ at 6 miles an hour to $\frac{1}{7}$ at 18 miles an hour. It is difficult to understand how the "personal equation of the engineer and fireman, and the coal used," can seriously affect this in connection with velocity grades.

The personal equation of the Superintendent of Motive Power ought to be expected to enter into the matter to an extent to make the personal equation of the engineer and fireman small enough to be properly neglected. It is considered good locomotive practice to make the boiler power and the cylinder power of a locomotive at least equal to the adhesion. Where this has been done, the engineer and fireman who regularly stall their train without slipping the drivers will become the victims of the personal equation of the Superintendent of Motive Power, and cease to be disturbing factors in the operation of the velocity grade.

The practice of the Master Mechanics' Association appears in 1887 to be to take the coefficient of adhesion for freight trains at $\frac{1}{4{\cdot}25}$. The formula $P = \frac{4\,D}{S + 10}$ gives for a speed of 18 miles an hour a coefficient of $\frac{1}{7}$. With a locomotive of 45 tons on drivers, the traction in one case is 21,176 lbs. and in the other 12,857, a difference of 8,319 lbs. If train resistance on the level be taken as 6 lbs. per ton, then on a grade of 1.00 per 100 the total resistance will be 26 lbs. per ton and 8,319 lbs. measures the resistance of about 320 tons. If it should cost (for a conservative estimate) only 1 mill extra per ton-mile to haul these 320 tons on other trains, this would amount to 32 cents more per train mile; on a division of 100 miles there would be a difference of \$32 per train. Rather than allow a leak of this amount in the operation of the road, the most urgent measures could be resorted to both in the design of locomotive and in checking the work of engineer and fireman. An inspector on each train to prevent this waste would prove a source of rare economy. Before the empirical formula $P = \frac{4\,D}{S + 10}$ is accepted, data in considerable detail should be offered to demonstrate its substantial accuracy, as it appears to be opposed to ordinary practice, and unreasonable in its results.

Another point of far less importance, is in formala I, where $\frac{1}{30}$ is used as a coefficient before $(S_1{}^2 - S_2{}^2)$. This coefficient appears to neglect to take into account the rotative energy of the car wheels, and is thus in error by $4\frac{1}{5}$ to $6\frac{1}{7}$ per cent. depending upon whether the practice of the Baldwin Locomotive Works is adopted or the figures of Wellington's Economic Theory of Railway Location are taken. The difference would not be important for freight train speeds. A coefficient of $\frac{1}{7}$ would nearly agree with both Wellington and the Baldwin Locomotive Works.

KICKER.

15 *December* 1899.

ROANOKE, VA., *Dec.* 26, 1899.

TO THE EDITOR OF THE RAILROAD GAZETTE:

The following in reply to "Kicker" and Mr. Raymond in the matter of velocity grades:

Formula VII. of November 30 is correctly generalized by Mr. Raymond as

$$L = \frac{4}{3}\,\frac{(S_1 + 10\,(S_1 + S_2)}{R + 20\,G}$$

the considerations used by the writer as leading up to Formula VII. being as follows:

A velocity grade is one on which the momentum of the train is utilized; the effect being to reduce the speed from S_1 (initial) to S_2 (final).

Formula III. gives the least weight of train that can pull down the speed eventually to the final assumed rate, S_2, on the given grade, G. A greater weight of train than this will reduce the length of the velocity grade. A less weight of train will not pull the speed down to S_2.

The weight of train given by Formula III. is therefore the one that fixes the length of the maximum velocity grade. Formula VI. gives, when transposed, the length of velocity grade for a given weight of train. The combination of the two, Formula VII, expressed generally, is therefore the equation sought.

With regard to the reasonableness of the expression

$$P = \frac{4\,D}{S + 10}$$

as fairly representing traction: as "Kicker" has mentioned Wellington's Railway Location, the writer would refer him to page 532 (edition of 1887) where it is stated as a result of experiment that between speeds of 17 and 23 miles an hour the traction was found to be "nearly $\frac{1}{7}$" of the weight on driving-wheels. Also to page 519, where the traction at starting, say 6 miles an hour, and at 50 miles an hour, are found to agree closely with the formula.

At 20 miles an hour an engine of 150,000 lbs on drivers would have to indicate by cylinder diagrams appreciably in excess of 1,067 horse power, in order to agree with the formula. This is the effective out-put of power, delivered at the rails, by the formula at that speed. That this is usually or continuously exceeded by most engines of such weight is not apparent to the writer.

Further, it is best, in his opinion, not to use too high an expression for traction in planning velocity grades. Then, if a car can be added to the train when the work is done, it is far better than to have to side-track one, as absolute accuracy is out of the question.

Also, the method of determining the average traction by adding the tractions at the extremes of speed and dividing by two, gives an average traction which is in excess of the actual average. This was assumed to offset the effect of the rotative energy of the car wheels and both facts were disregarded in the general expression in consequence. Both errors, if they may be so called, become zero at the same point; namely, at a uniform speed.

The effect of the use of the formula would not be to reduce the weight of trains, as "Kicker" imagines, but it would result in an appreciation of what actually occurs; namely, that at hard pulls the speed is decreased until the traction increases sufficiently to meet the necessities of the case.

The negative value of L, mentioned Nov. 30,* resulted from combining $S_1 = 25$ and $S_2 = 5$ with $S = 11$; which, as before mentioned, is of no practical utility.

CHAS. C. WENTWORTH.

5 *January* 1900.

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* Page 160.

TO THE EDITOR OF THE RAILROAD GAZETTE:

" With regard to the reasonableness of the Formula $P = \frac{4\,D}{S\,+\,10}$," Mr Wentworth, in your issue of Jan. 5, 1900, page 1, refers " to page 532 of Wellington's Railway Location, where it is stated as a result of experiment that between speeds of 17 and 23 miles per hour the traction was found to be nearly $\frac{1}{7}$ of the weight on driving wheels." Mr. Wentworth here fails to draw a distinction between the total traction an engine is capable of exerting (its total adhesion) and the actual pull which the locomotive happened to exert in a trial where no attempt was made to determine the total force of adhesion between wheel and rail.

The symbol P which enters into several formulas on page 837, Dec. 8, 1899, is defined by Mr. Wentworth as "traction of the engine in pounds at the rails . . . the engine being supposed to be exerting all the traction it can at that speed." Wellington's words, page 532, are " actual average pull (nearly $\frac{1}{7}$ load on drivers)." An " average " pull is clearly less than the maximum exerted and evidently would not tax the adhesion.

As a matter of fact, the more complete records of the test (Journal of the Franklin Institute, April, May, 1879) show that no measure of the adhesion was taken or attempted, that the records of pull (traction) were indicator cards, that the engine was operated with an average cut-off of about $^1/_2$, so that it could not have been expected that the limit of adhesion would be reached by the engine running in that way. The indicator cards giving the maximum result for pressure of steam showed pressures far in excess of the average for the run. The cards indicating these maximum pressures showed a tractive force of about $\frac{1}{4 \cdot 25}$ of the load on the drivers, and there was no evideace given to show that even this taxed the adhesion.

Mr. Wentworth also refers " to Wellington, page 519, where the traction at starting, say six miles an hour, are found to agree closely with the formula." Wellington's words are: " Mr. Dudley found the traction at starting to be 11,000 to 12,000 lbs. for the first 100 to 200 feet, falling to 2,800 to 3,000 lbs. at 50 miles per hour." the traction here referred to is not adhesion, but simply the actual pull of the locomotive, and Dudley's experiments simply illustrate the fact already well known that it takes more force to start a train than to keep it in motion.

The only evidence, than, that Mr. Wentworth offers in support of his formula for the maximum traction possible at any speed, turns out, when analyzed, to have no bearing on that point. The experiments to which he referred to, on the other hand, furnish evidence that at moderate speeds of 17 to 23 in one case, a pull was exerted, equal to about $\frac{1}{4 \cdot 25}$ of the weight on drivers (and more than $\frac{1}{6}$ in several cases). The adhesion possible was doubtless definitely in excess of the traction exerted in either of these cases.

Allow me to state a correction two lines above my name, on page 855, Dec. 15, 1899. A coefficient of $\frac{7}{10}$ (not $\frac{1}{7}$) instead of $\frac{2}{3}$ is what I intended to write.

KICKER.

26 *January* 1900.

TROY, N. Y., *January.* 16.

TO THE EDITOR OF THE RAILROAD GAZETTE:

Mr. Wentworth's explanation of the velocity grade formulas is not altogether satisfactory to me ; nor does Professor Allen's statement of the question very much better it. I realize that space in the " Railroad Gaztte" is to valuable too be occupied with long arguments, but this subject of momentum gardes, at which a few years ago almost every one laughed, is now coming to be a very important subject, and the possible length of a momentum or velocity

grade is one of the most important considerations question of momentum grades, let us look at Mr. Wentworth's tractive force formula. It is $T = \frac{4\ D}{S\ +\ 10}$ in which T is the tractive force of the locomotive in pounds, D the weight on the drivers, also in pounds, and S is the speed in miles per hour. Assuming that this is a correct form for a tractive force formula, and needs only to be changed in its constants to be correct, and with the nnderstanding that it is not intended to fit any particular locomotive, let us see how it fits each of several locomotives whose tractive force at various speeds has been determined by Mr. E. M. Herr, and presented in a paper before the Western Railway Club. We will assume the coefficients of D to be good, and will determine what values of the denominator constant are required to make the observed results agree with the formula.

		Speed in Miles per Hour.							
Engine.		5	10	15	20	25	30	35	40
Class.	Description.	Values of derominator in $T = \frac{4\ D}{S\ +\ X}$ for various speeds.							
D_2 ...	Simple mogul	S + 11·8	S + 9·3	S + 9·2	S + 11·0	S + 13·6	...	...	...
F_1 ...	Simple consolidation	S + 10·5	S + 8·4	S + 10·8	S + 15·1	S + 22·8	...	...	...
R ...	10-wheel compound	S + 12·2	S + 9·4	S + 7·4	S + 6·5	S + 7·2	S + 8·5	S + 9·9	S + 11·2
P ...	10-wheel simple		S + 6·6	S + 3·9	S + 2·2	S + 2·1	S + 2·8	S + 3·2	S + 3·4
X ...	Compound Mastodon	S + 8·3	S + 6·7	S + 5·8	S + 6·7	S + 8·8	S + 12·0	...	...

This table shows, as was to be expected, no particular value for the denominator constant that will fairly average the various locomotives or that will represent what might be called an ideal engine. The fact is that the tractive force of the engine above, say, 10 miles per hour is not at all governed by the weight on the drivers, but by the horse power capacity of the boiler and cylinders, and it may be readily* shown that the cylinder tractive force for a constant horse power capacity is given by the simple expression $T = \frac{375\ I\ H\ P}{S}$ where S is the speed in miles per hour, and T is the tractive force in pounds. Now an ideally designed freight engine would seem to be one that could exert this full power down to some minimum speed of, say, 10 miles per hour, at which point its tractive force will be limited by the weight on the drivers. That is, the horse power and weight on drivers should be so proportioned that there shall be ample power to utilize the full adhesive tractive at low speed, and that constant horse power shall be maintained at all velocities above the prescibed minimum, to the extreme required by the service for which the engine is designed. It is a noticeable fact that freight engines of approximately the same horse power differ widely in weight on drivers. This should not be except they be designed for quite different service. The formula $T = \frac{375\ I\ H\ P}{S}$ ought to hold good down to determined minimum speed, and it would seem that 10 miles per hour is not too fast for this low limit. The realization of this formula down to this speed is nearly reached by some engines. The formula fits most engines through a considerable range of high velocities, and is in disagreement only at the lower speeds below 15 or 20 miles per hour. Of course not all this indicated horse power gets to the rails, and very much less than this gets behind the tender.

Now as to the velocity grade formulas. The first one would, I think, be improved by using the mean tractive force of the locomotive for the limiting speeds S_1 and S_2, and making the usual 6 per cent. allowance for wheel energy. This formula would then stand

$$T = \frac{P_m\ L}{L\ (R + 20\ G) - 7/10\ (S_1{}^2 - S_2{}^2)}$$

This formula simply says that the work done by the engine (= the average pull times the distance, $100L$, pulled) must be equal to the work to be done on the train (= the average rolling resistance in pounds per ton, all multiplied by the weight of the train in tons

* [See p. 168.]

and by the distance, $100L$) less the work done on the train by the surrender of velocity (= the weight of the train times the difference in velocity head for the initial and final speeds). The well known grade resistance formula is $r = 20G$, where r is the resistance in pounds per ton and G is the grade in rate per cent. The formula for velocity head, which is the distance through which a body must fall to gain the given velocity, with a 6 per cent, addition for energy of rotating wheels, is $h = 0{\cdot}0355S^2$, where S is the speed in miles per hour. The algebraic operations necessary to put the first velocity grade formula into simple shape are responsible for the particular values of the constants. Now for the purpose of analysis it will be well to transpose this formula and assuming a given load, solve for the length of grade of rate G, that can be handled by a given engine starting at a speed of S_1 and being slowed down to a speed of S_2. I believe the problem more often arises in this way than otherwise. The transposed formula is

$$L = \frac{7/10\,(S_1{}^2 - S_2{}^2)}{R + 20\;G - \frac{P_m}{T}}$$

This formula gives essentially the same results as the method suggested by Mr. McHenry, Chief Engineer of the Northern Pacific Railway, who gives this rule, $L = \frac{d}{g - g_1}$ in which d is the difference in velocity head for the initial and final velocities, g is the grade in question, and g_1 the virtual grade for the engine for the given load and mean tractive force for the given speed limits. The two formulas are exactly interchangeable, for $\frac{P_m}{T} = R + 20g_1$.

These formulas indicate that the length of the momentum grade may be obtained geometrically by laying off on a piece of profile paper at the foot of the given grade a vertical line equal to the difference in velocity head and drawing from its upper end a grade line equal to the virtual grade for the given load and mean tractive force and resistance, and noting where the virtual and actual grades intersect. The point of intersection fixes the limit of the momentum grade. This is the method of Prof. Allen in his paper in the *Railroad Gazette* of the 12th inst., except that Prof. Allen uses the virtual grade for the minimum speed rather than for the mean tractive force. This method gives lengths of grade probably much too great. Just what the virtual grade corresponding to the varying tractive force and resistance is cannot be easily told. It is doubtless a curve beginning with the grade corresponding to the initial velocity, and ending with that corresponding to the final velocity, and from some drawings I have made I believe it will almost always meet the given grade at a point in advance of that given by the mean tractive force formula. I should not be surprised if the omission of the coefficient of the revised Wentworth formula would give very near the truth for many grades. This formula would then be $L = \frac{S_1{}^2 - S_2{}^2}{R + 20\,G - \frac{P_m}{T}}$. It would be interesting to know how closely experiments on the Northern Pacific or elsewhere agree with the McHenry fomula. There are so many accidentals in train motion that can not be considered in theoretical formulas, that it is not at all unlikely that the formula for average conditions may most nearly represent what can be actually done.

But now as to Mr. Wentworth's seventh formula. It seems to me that he has not found the length of momentum grade of given rate that can be worked by a given engine, because he uses a weight that the engine can handle on the grade itself at the inferior speed limit, hence the grade is not what is called a momentum grade at all. The formula still gives the point at which the stored energy is expended, but the train goes on indefinitely, and there is no limit to such a grade. This will be readily seen if, instead of assuming the load for the minimum speed, the load for the mean tractive force be assumed. The result of substitution in the formula will be L = infinity. With the heavier load due to the slower speed, L becomes finite, but is of no service in showing how long the grade may be. If the load assumed be

that for the ruling grade of the road at the low speed, the result is perfectly rational and serviceable. But no grade of less rate than the ruling grade can be said to be a momentum grade. Such a grade is operated by stored energy only for the purpose of maintaining higher average speed, and in velocity problems we may discuss the point on any grade at which the train will get down to a uniform velocity, but this has nothing to do with the length of such grade that can be, as a matter of possibility, operated as a momentum grade, unless the grade be steeper than that for which the engine is loaded for the minimum speed.

WM. G. RAYMOND.

26 *Jan.* 1900.

To the Editor of the Railroad Gazette:

Confession is good for the soul. What Mr. C. C. Wentworth's references to Wellington failed to do, the results of Mr. Herr have accomplished. "Kicker" acknowledges that he has been wrong in maintaining that traction is independent of speed. Will it be allowable, however, to suggest a question as to the propriety of applying Mr. Herr's figures, or Mr. Wentworth's formula for traction to the case of "velocity grades"? Mr. Herr's statement is that "these curves are believed to represent about the maximum horse-power the different engines are capable of sustaining continuously under service conditions." Isn't it true that an engine, like a horse, can for a short time exert distinctly more than the normal traction? Do not locomotive engineers make preparation for a stiff grade? Don't they well fill their boilers with water and get the water hot in approaching a grade so that no cold water need be introduced for a time? Don't they force their fires and heat their boilers for a brief time, hotter than they will stand for a long period? Don't they in this way secure at the foot of the grade and over the most or all of a short grade, a traction much greater than Mr. Herr's figures show for traction "sustained continuously"? The great gains on velocity grades are made where the grades are short; on long grades the height saved is distributed over a long distance and the gain is less proportionally. If the engineer can thus secure increased traction from his engine at the higher velocities on a velocity grade, then Mr. Wentworth's formulas would require modification in considerable degree. It is even possible (though not probable) that the traction would become nearly constant. The writer would suggest that some additional data definitely bearing on this point would be of considerable value. Will it be out of place for "Kicker" to remark that after having "pitched into" Mr. Wentworth he believes that the latter deserves thanks for being the means of bringing out pretty thoroughly the fact of varying traction, which many civil engineers certainly have not appreciated.

In answer to a private criticism that there is quite a discrepancy between the experiments quoted by "Kicker" and the results of Mr. Herr, it may be stated that the discrepancy is apparent rather than real. Mr. Herr's results were for continuous performance. The result giving a coefficient of traction of about $^1/_4$ at about 20 miles per hour was based on a shorter performance and possibly represented only one or two indicator cards where the average of these experiments gave a coefficient of traction of about $^1/_7$. In view of Mr. Herr's results, it would appear that the engine may have been running nearly or fully up to its boiler capacity, although it was not so stated by the author of the paper. The experiments and the results were quoted by "Kicker" in good faith and furnish an example of how easy it is to use facts with perfect honesty to support a position which is really unsound.

"KICKER."

23 *February* 1900.

TRACTIVE FORCE FORMULAS.

To the Editor of the Railroad Gazette:

In the discussion of Passenger Train Speeds in your issue of Feb. 16, page 104, there is given one way of deducing the formula $T = \frac{375\ I.\ H.\ P.}{S}$ Another method may be of interest, as it calls attention to some of the first principles on which such formulas are based.

The expression usually given for the cylinder tractive force T is, $T = \frac{d^2\ L\ P}{D}$ (1) in which d is the diameter of the piston in inches, L the piston stroke in feet, P the cylinder mean effective pressure in pounds per square inch, and D the diameter of the drivers in feet.

The tractive force may, however, be expressed in different terms than those given in equation (1). Thus, if A is the area of the piston in square inches L the length of stroke, and P the same as used in equation (1), then will $P\ A\ L$ be the ft.-lbs. in one stroke; consequently $4\ P\ A\ L$ will equal the work done for each double stroke for two cylinders. The circumference of the drivers is $\pi\ D$, which, multiplied by the tractive force at the rails, must equal the work done in the cylinders, or, $4\ P\ A\ L = T\ \pi\ D$.

Therefore

$$T = \frac{4\ P\ A\ L}{\pi\ D} \quad \text{.........(2)}$$

In the familiar equation for indicated horse-power, (for both ends of both cylinders), $I.\ H.\ P. = 4\ \frac{P\ L\ A\ N}{33{,}000}$. P, L and A have the same meaning as these letters in equation (1), and N is the number of strokes in a single cylinder. Transposing,

$$P\ A\ L = \frac{33{,}000\ I.\ H.\ P.}{4\ N} \quad \text{.........(3)}$$

Substituting the values of $P\ A\ L$ of equation (3) in equation (2),

$$T = \frac{33{,}000\ I.\ H.\ P.}{\pi\ D\ N} \quad \text{.........(4)}$$

In which $\pi\ D\ N =$ speed in feet per minute. If $S =$ speed in miles per hour and $V =$ speed in feet per minute, $V = S \times 5{,}280 \div 60$, whence

$$\pi\ D\ N = \frac{S \times 5{,}280}{60} \quad \text{.........(5)}$$

Substituting these values in equation (4) and reducting, we obtain

$$T = \frac{375\ I.\ H.\ P.}{S}$$

W.

23 *February* 1900.

To the Editor of the Railroad Gazette:

Your recent discussions on "The Theory of Velocity Grades" seem to have come to a somewhat untimely end. "Kicker's" argumentative forces have apparently been unable to withstand the fire of Mr. Herr's data in the hands of Mr. Wentworth. Still, "K." has left a considerable stock of animation, but only a little ammunition. His only hope now of proving that traction is independent of speed is to bring to the front such an array of figures to prove his point that even the results thus far printed will be overshadowed by weightier arguments based on observed facts. That this can be done is highly improbable, but barely possible. But "Kicker" does not desire to even appear to defend a position that is not sound, and frankly admits that Mr. Wentworth has brought out pretty thoroughly the fact of varying traction—the most important truth established in the controversy.

I write to urge the further discussion of the important point raised by "Kicker" on page 113; which, in effect, is that Mr. Herr's results were for traction "sustained continuously," and are not typical of conditions on velocity grades where the engine is worked pretty hard for a short time. This seems to be the turning point of the discussion in its present stage.

It may not here be out of place to suggest that Mr. Wentworth give an explanation of how his tractive force expression—which has so far withstood all attacks—was obtained. He has indicated how it agrees with modern practice, and has apparently established his claim made in his first article of Dec. 8 last, that an empirical expression for the traction that may be reasonably expected from an engine of given weight on driving-wheels moving at a given speed, can be safely used in such formulas as Mr. Wentworth established.

He well said in his first letter that an accurate expression for tractive force must necessarily include not only the governing dimensions of the engine, but also a combined expression for the personal equations of the engineman and the fireman, and the coal used. Further, it may be said that, could such a theoretically correct expression be obtained for a given engine and under given conditions, it would be useless when the conditions changed. Still, it is well to note that the theoretical expression first used in the discussion by Prof. Raymond, $T = 375\ I.\ H.\ P. \div S$, has stood some severe tests and still it stands.

In fairness to Professor Allen, I think it should be said that his article of Jan. 12 on "Momentum Grades" did not enter into the question of the proper amount of traction or of train resistance at various speeds. Since any tractive force formula could be used with his treatment, it seems to me he has been dragged unceremoniously into a controversy in which he properly has no part.

Prof. Raymond added much to the discussion when he generalized Wentworth's seventh formula (*Railroad Gazette*, December 8 last), which gives the length of track (L), in stations of 100 ft. each, in which the speed varies from S_1 to S_2, putting it in the form: $L = \frac{4}{3}\frac{(S_1 + 10)(S_1 + S_2)}{R + 20\ G.}$, which does not contain the weight on the drivers, nor indicates any negative value for L.

As for as I can make out, Mr. Wentworth has assured us that it is easier and safer to use the formulas which he has established than to solve problems in velocity grades by reasoning each time from the first principles. If the discussion, already very profitable, could conclude with further explanation of the new tractive force formula and an answer to "Kicker's" query, we might get a step closer to the correct theory or theories of velocity grades and make the whole subject a little more practical.

A. W.

23 *March* 1900.

APPENDIX H.

THE AIR RESISTANCE OF TRAINS.

All that is definitely known about train resistance can be summed up in a few general statements. The formulæ which have been derived from specific tests are unreliable, and have been found to be so widely in error for many conditions of service. that it is largely a matter of opinion which formula is nearest correct. A serious objection to the best known formulæ is that an attempt has always been made to include in a single expression all the factors which enter into the problem. There seems, however, to be a tendency to discard many of the theories formerly accepted, and to try to obtain some general basis from which equations can be derived which will cover specific cases; it is evident that this is just the reverse of the usual methods of investigating the resistance of trains.

The total resistance of a train is made up of so many factors that it is now generally acknowledged that they cannot to advantage be studied collectively. Thus there is the effect due to grades, curves, accelerations in speed, the rolling friction of the wheel on the rail, the flange friction on straight track, journal friction and the resistance of the atmosphere. In any road test all these enter to affect the final results so that data obtained from service tests of necessity must apply to only a particular set of conditions, and even fairly accurate conclusions for general application cannot be drawn. Before a useful analysis can be made of the resistance of a train, each factor entering into the final result must be considered separately under such conditions that all the factors can be controlled at will. This is a problem similar to those which remained unsolved until the plan was devised for mounting a locomotive on a testing plant, and while we are not prepared to admit that the laboratory offers facilities for determining all the required information concerning train resistance, or that such results can be directly applied to practice without modification, yet it would appear that laboratory research may eventually offer the best general solution for portions at least, and possibly for the whole of the problem.

The effect of grades and changes in acceleration can be accurately calculated, and Prof. Denton has very thoroughly investigated the friction of car journal bearings.* Prof. Goss has collected much important data regarding the internal friction of locomotives, not yet made public, and has also made an extended study of the resistance to the motion of trains offered by the atmosphere, the results of which are given in another portion of this issue.

The method used by Prof. Goss in his study of the atmospheric resistance to trains is novel and his deductions go so far toward explaining the contradictory evidence from former tests that we have reprinted the greater portion of his paper.† For those readers, however, who have neither the time nor inclination to read so long an article, a statement of the important points may not be out of place.

Briefly stated, the apparatus consisted of small models $\frac{1}{32}$ the size of a standard box car, placed near the centre of a long wooden conduit, as this portion was found by experiment to have a nearly uniform flow of air, when a rotary fan connected to one end of the conduit was in operation. Each car was attached to a dynamometer which registered the force tending to produce longitudinal displacement, while the velocity of the air was measured by suitable gauges and controlled by regulating the speed of the fan. The observations consisted in measuring the velocity of the passing air current and noting the readings of the dynamometers of the several cars. The atmospheric conditions of the tests corresponded to those of a train moving in still air, and no effort was made to determine the effects resulting from oblique or other winds. Various arrangments of the models were tried ranging from a single model to a train of twenty-five models and the velocity of the air was varied from about 25 to 105 miles an hour. The conclusions drawn from the results of these experiments are:

* Translations of the Am. Soc. M. E., Vol. XII.

† *Railroad Gazette*: 20 May, 1900.

1. The force with which the air current acts upon each element of the train, or upon the train as a whole, increases as the square of the velocity.

2. The effect upon a single model, standing alone, measured in terms of the pressure per unit area of cross-section is approximately 0·5 the pressure per unit area as shown by the gage recording the pressure of the air current.

3. The effect upon the different models composing a train varies with different positions in the train; it is most pronounced upon the first model, the last model coming next in order, then all the intermediate models expecting the second, and least of all is the effect of the air upon the second model.

4. The relative effect upon the different portions of a train is approximately the same for all velocities.

5. The ratio of the effect upon each of the several models composing a train, measured in pressure per unit area of cross-section, compared with the pressure per unit area of the air current as shown by the gauge is approximately: for the first model 0·4; for the last model 0·1; for any intermediate model between the second and last 0·04; and for the second model 0·032.

It is then assumed that had the models been full-sized cars under similar conditions, the results would have varied according to the extent of the exposed surfaces, and from the data obtained from the experiments with model trains, equations are evolved, expressing the relation between the speed of locomotives, passenger trains and freight trains, and the resistance offered by still air to their progress. Twelve equations are thus presented for different combinations, which equations take into account both the cross-section and the length of the train and distinguish between the head resistance and the frictional resistance of the intermediate cars. So far as we know this is the first time equations of this kind have been brought out. The results of the application of these formulæ are given in tabular form, which tables show the tractive-power and horse-power required to overcome the atmospheric resistance, for speeds of from 10 to 100 miles an hour of a locomotive running alone and at the head of a train, and also of trains varying from 100 to 2,000 ft. long. These values are much lower than those obtained from the application of several well-known formulæ, which formulæ are now considered by those best informed to give results too high.

The scheme of these experiments is ingenious, and the results are presented in such a way that they can be readily adapted to almost any general case, provided it is correct to assume that the relative atmospheric effects on similar bodies are directly proportional to their respective surfaces. As the final expressions depend largely for their values upon the relation that exists between the effects upon small and large similar bodies, it would seem very desirable to establish this relation by trial, at least within the limits of the apparatus. If, for instance, the relation between the effects of models, even $\frac{1}{64}$, $\frac{1}{32}$ and $\frac{1}{16}$ the size of the standard car were found to be directly proportional to the area of surfaces exposed, one would feel more safe in applying the experimental results to actual conditions; doubtless this relation could be determined for a wider range of sizes in other ways.

However, the equations deduced by Prof. Goss are more rational and satisfactory han any heretofore presented, and will probably be found satisfactory for practical purposes.

[*Railroad Gazette*:—20 *May* 1898.]

TRAIN RESISTANCE.

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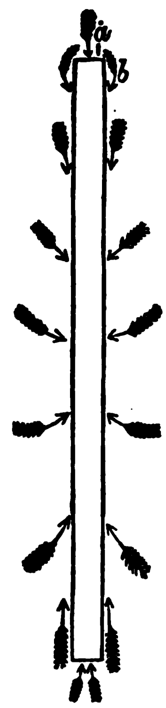
Prof. Dudley's experiments have demonstrated that the resistance per ton **decreases with the length and weight of the train at high speeds.** From the engraving, which represents a long train in motion, this will be readily understood. The atmosphere being the principal resistance at high speeds, is met by the engine's front, as per arrow, *a*, assuming a comparatively quiet state of the air, and is deflected as per arrows, *b*, and continues to



catch in the window depressions and between the cars, until on a long train, it is believed at and near the back end, a belt of air is drawn along the sides with the train by suction, hence **the addition of cars to the back end of a long train, if the speed is high, will not increase the resistance in anything like the degree that the adding of a car to a train of two or three cars would;** because the air at the back end of the train is already put in motion by the cars at and near the front end, and follows with the train to some extent, and thus the resistance of the last cars is largely made up of journal and flange friction, while the resistance of the leading cars is composed of these factors, and, in addition, that of the atmosphere.

It is a well known fact that American cars haul **easier per ton** than foreign or European cars, from the fact that the American truck is free to adjust itself to the curves, while the European cars have their wheels set in rigid jaws, which are incapable of swivelling to accommodate the curves, etc. It is of course true that European cars are shorter and track curves longer, still, with disadvantages of our roads in these respects, the swivelling truck much more than puts them on an equality for easy draught. A leading English paper, and an authority on railroad matters, recently estimated the resistance per ton at a speed of 50 miles per hour for American cars in a train of 460 tons to be 25 pounds per ton—probably judging from English cars, while it has been already shown by Prof. Dudley's experiments, that the resistance is as low as 10 pounds per ton. This fact is, it appears, recognized in England, for in Colburn's "Locomotive Engineering" * we find:

"But the great practical improvement of the day, at least so far as English rolling-stock is concerned, is to be the substitution of radial axles and axle boxes. . . . The practice of the United States, in the universal adoption of bogies under the engine tenders, carriages, and wagons is well worthy of analysis. . . . It must also be affirmed that by means of the bogie the tractive resistance of engines and trains is notably less than that of parallel axled stock.

The Scientific American Supplement: 6 October, 1883.

[*Zerah Colburn was, however, an American, not Englishman.—TR.]

APPENDIX I.

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EXAMPLE OF EQUIVALENT LENGTH.

* * * * * * *

Train Speed.—The average speed of fast passenger trains between Lucerne and Chiasso, has hitherto been 40 kilometres (25 miles)' including stoppages: the maximum between stations, 50 kilometres (31 miles per hour, which, with the new express engines lately put upon the line, will be increased to 60 kilometres (37·5 miles) as average speed, and to 90 kilometres 56·4 miles) per hour on flat sections. Having regard to the heavy grades of 2·6 and 2·7 per cent., these speeds are highly creditable, and much better than those on the connecting lines from Lucerne to Bâle, and from Chiasso to Milan, worked by the Swiss Central and the Italian Mediterranean Company respectively. In order to further accelerate the express service from Bâle to Milan and *vice versâ*, the St. Gothard Company contemplates running its new express engines over the whole distance between these two terminals, about 380 kilometres or 240 miles.

It is noteworthy that the speed on the St. Gothard Railway is also in excess of that on the other great Alpine railways, which have, moreover, easier maximum gradients, except the Arlberg line, as is shown by the following Table:

—	Maximum Gradient.	Average Speed per Hour.		—
		Kilometres.	Miles.	
	per cent.			
St. Gothard	2·7	{ 40	25	Old engines.
		{ 60	37·5	New ,,
Mont Cenis	2·5	35	22	
Arlberg	3·0	34	21	
Brenner	2·5	32	20	
Semmering	2·5	35	22	

Equivalent Length.—In order to realise what train speeds of 40 and 50 kilometres, or 25 and 31 miles mean, on such a railway as the St. Gothard, it is necessary to determine the equivalent length of the whole line, viz., *the length which the actual line, with its gradients and curves, represents on the straight and level.* This equivalent length may, in this case, be determined according to Ghega's formula:

$$EL = L + G\left(1 + \frac{100}{g}\right) + 0.75\,\frac{A}{360}.$$

where L is the sum of all level sections (57 kilometres); G the sum of all grade sections (208 kilometres); g the mean gradient (1 in 83), and A the sum of all curve angles (15,624 deg.). The result is as follows:

$$57 + 208\left(1 + \frac{100}{83}\right) + 0{\cdot}75\,\frac{15{,}624}{360} = 546{\cdot}6 \text{ kilometres, or 342 miles.}$$

This result is strikingly confirmed if we determine the **coefficient of equivalent length** in another way, viz., from the mechanical work on the average gradient, at average speed, divided by the same on the level at maximum speed. As before stated, the average train load on the St. Gothard Railway is 480 tons, the mean grade 1·2 per cent. (1 in 83), or 12 metres per kilometre, the average speed 8·5 metres per second, or 30 kilometres per hour, whilst the maximum speed on the flat sections is 50 kilometres per hour, or 14 metres per second. Taking the coefficient of traction at 5 kilogrammes (11 lb.) per ton, we have the coefficient of equivalent length:

$$\frac{480 \text{ tons } (12 + 5) \times 8{\cdot}5}{480 \text{ tons } (\,0 + 5) \times 14} = \frac{68{,}510}{33{,}600} = 2{\cdot}04$$

Hence the equivalent length of the whole line:

265 kilometres × 2·04 = 540 kilometres or 338 miles,
or virtually the same as the length determined by Ghega's formula.

The coefficient of equivalent length being thus 2, it follows that the speed on the St. Gothard Railway of 40 and 50 kilometres is equal to 80 and 100 kilometres, or 50 and 60 miles per hour on the straight and level, and that an average run per engine of, say, 60 miles per day on the St. Gothard Railway is equal to 120 miles on ordinary lines with easy grades and flat curves.

In the same way, the total train and engine mileage on the St. Gothard Railway corresponds to double that mileage on the straight and level, and hence, inversely, the working expenditure of 3·32 fr. per train kilometre or 4s. 2d. per train-mile, is equal to half, viz., respectively 1·66 fr. and 2s. 1d. on the straight and level.

Engineering:—18 *January*, 1895.

APPENDIX J.

RAILWAY PRACTICE IN NEW SOUTH WALES.

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There are no fixed rules as to limiting gradients and curves, which vary according to the nature of the country to be traversed, and the direction and amount of the traffic to be dealt with. For short branch lines to partly agricultural districts, actual or possible, 1 in 40 to 1 in 60 gradients, and 10-chain curves, where unavoidable except by heavy works, are not unusual, and on these 1 in 60 against the loaded up traffic, and 1 in 40 against the lighter traffic from the sea-port, represent about equal gross traction. The great bulk of the up traffic of the colony, except for coal, which is dealt with by the old lines nearer the coast, is of low specific gravity, such as wool, live-stock, hay and chaff, skins, &c., and as the population of districts producing these commodities is necessarily scanty, and the supplies to them form the bulk of the return down loading, the proportion between the up and down grading remains much the same as in the heavier grain-carrying lines with their larger population. Special attention has therefore to be given to this balancing of resistances, where the ruling gradient is required, so as to ensure that the full engine power necessary to take the partially loaded trains one way will be utilized in returning with the fully loaded trains in the reverse direction. On main connections, or deviations of old main lines, 1 in 80 against the up traffic, and 1 in 55 *vice versâ*, are generally aimed at, with 15-chain curves, while in the longer western lines across the plains, where traffic is most economically taken in long but infrequent trains, 1 in 100 is generally obtainable as a surface line. In these latter especially, but generally in all but the suburban lines, the number of passengers is comparatively so insignificant that goods traffic is the guide in these matters.

The tendency is, on main lines in New South Wales, notwithstanding the great reduction of train-mileage by the introduction of some of the most powerful locomotives in existence, **to reduce it** still further **by cutting down the gradients of the old lines,** and projecting easy grading on the new lines. The same tendency, partially for a different reason, is noticeable in the case of proposed short branches, where comparatively easy grading is required to utilize, in their service, the weaker old engines which are being gradually cast from the heavy traffic of the main lines. The frequent employment of curves as sharp as 10 chains radius to economize construction as against greater wear and tear to rails and rolling-stock, principally in tires, is a question which has properly received much consideration. It is really all a matter of extent of traffic. Many of the branch lines of New South Wales have no greater traffic, nor is there any likelihood of its increasing much for many years, than that represented by 1,000 to 2,000 train-miles per open mile per annum. When it is considered that the total cost of rolling-stock repair and renewals is only about 8*d.* per train-mile, or say £45 per open mile per annum on an average branch, and only **a small fraction** of this is affected even if the whole length be sharply curved, it is easy to see that sharp curvature, to save a very moderate amount of construction, is amply justified on such lines. The wear of rails is also insignificant in these cases. It is a very different matter where main lines or connections are with their much larger traffic.

All curves of lesser radius than 20 chains have **transition** or easing, **curves** connecting them with the adjoining straight. Vertical curves are also used at summits and sags in the section. This eases the strain on the drawbars and buffer springs, and diminishes the wear of rolling-stock generally.

Care is taken also that unnecessary losses of level in long ascents are not incurred, and the cost of working them is set against the saving in construction which might be gained by their adoption. 0·40*d.* per 1,000 foot-tons is a fair average extra cost of working such ascents in New South Wales, this representing roughly the extra amount of fuel and water

used in rising over a given height, with an ordinary goods train, in addition to that expended on a level line of the same length, extra wear and tear to rolling-stock being also included.

Another precaution adopted is that the ruling gradient and the sharpest curve on any line, or division of a line between engine-stations, is avoided through long tunnels, as otherwise, through the loss of adhesion, the severity of the ruling gradints would be practically increased. Level benches in long gradients are also introduced.

The cost of running train-mileage, that is to say, the wages of trains' crew, the cost of fuel, water, and stores, the wear and tear of rolling-stock, and maintenance of road only, but excluding general superintendance and station expenses, which are not generally affected is an item which has constantly to be considered in testing the value of alternate projects. This is estimated to vary in the colony, according to local circumstances, between 2*s.* and 2*s.* 6*d.* per train-mile.

*   *   *   *   *   *   *

Since 1890, the survey work executed amounts to about 5,450 miles, and estimates have been made, in the same period, to the extent of about £38,000,000; but the extent of the latter work has diminished, relatively, the labour in connection with it, as innumerable **examples and constants are tabulated and are available for successive estimates**; and the practice of dealing with so large a number, and of such great variety, has so trained those concerned with them, that almost **by the mere inspection of a section, an estimate might be made**, fairly approximately, to within £200 or £300 per mile.

A large staff of engineers, surveyors, and draughtsmen are employed at this subdivision of the railway construction branch, averaging from thirty to sixty in number from time to time, according to the extent of the operations. Formerly, surveyors alone were employed at this work in the field, and a separate engineering staff was allotted to construction work proper, but for some years past, though the officers are still under separate subordinate control, the staff is interchangeable, with great advantage to the railway surveys, **as no qualification is so valuable, in the execution of such work, as practical experience on construction.**

A. O. BURGE,
"Railway Construction in N. S. W."
—Mins. Procs., I. C. Engrs. Vol. CXXXII.

APPENDIX K.

THE RAILROAD SPIRAL.

In the *Railroad Gazette* for June 30, 1893, p. 490, there was acknowledged the receipt of a pamphlet on "The Transition Curve," by Mr. E. S. M. Lovelace, M. Can. Soc. C. E., and the suggestion was made that while the curve seemed simple and practicable, the limits of the tables were not well chosen. Mr. Lovelace has found this to be true, and that his formulas were rather too complex for field use. He has therefore simplified the formulas and extended the tables and has presented his new discussion to the Canadian Society of Civil Engineers, in a paper read Dec. 7, 1899.

The curve Mr. Lovelace originally proposed was the lemniscata, which was independently worked out for use as a transition curve by Mr. Charles H. Tutton (see *Railroad Gazette*, Aug. 4, 1893). Mr. Lovelace has made certain modifications of his formulas which greatly simplify them and make them entirely practical for field use, and he calls his new curve the modified lemniscata. But in this modification he has simply reached the true railroad spiral and has produced formulas essentially the same as those of Prof. Talbot and others.

The appearance of this paper suggests that **a review of the condition of the transition curve problem** may not be out of place.

Besides the method of three centre curves in use on the Pennsylvania, and possibly other roads, there are two general types of transition curves in use on American railroads.

(1) The compound transition curve which is a curve composed of short arcs, usually equal, of circular curves of ever decreasing radius from tangent to curve. These usually begin with an arc of a 0° 30′ curve or of a 1° 0′ curve, and increase by 0° 30′ or 1° 0′ or other angular unit till the degree of the main central curve is reached. The arcs making up the transition curve, equal throughout any one curve, are from 10 feet long as may be desired or required by circumstances.

(2) The spiral, which is a curve whose radius begins with infinity and varies inversely as the length of the curve, so much of the curve being used as will bring the radius down to that of the main curve to be used. The curve may be made long or short by varying the rate at which the radius shall change. The spiral does not admit of simple mathematical discussion or easy procise location, but does admit of simple approximate demonstration and easy approximate location, and the approximations are wholly within the usual limits of precision in railroad work.

The compound transition curve does admit of precise location and simple mathematical discussion, but is not so flexible as the spiral and requires the use of more or less extensive tables for field location, while the approximate formulas for the spiral are so simple as to need no tables at all, or at most two brief ones that can be copied on the fly-leaf of an ordinary field book.

Mr. Searles' "Railroad Spiral" is a compound transition curve, and is used on several roads. The Southern Pacific Company uses a similar curve, issuing its own tables, computed for arcs of 30 feet.

The late A. M. Wellington put into practical shape for use, the Froude curve mentioned by Rankine, and published simple tables in the *Railroad Gazette* many years ago. Later he elaborated his discussion in the series of articles that were to have constituted a field-book. These were published in the *Engineering News*. Mr. Wellington's deductious were good so long as the angle consumed by the transition curve was small, and some of his

formulas are good in any case, and it is an interesting fact that almost of all the later demonstrators of the transition curve, although going at the problem in different ways eventually by their approximations get down to the fundamental Wellington formula of $L = 1.\ 86\sqrt{\frac{O}{D}}$ or others essentially similar.

The necessity for the transiticn curve is now admitted by practically all intelligent engineers, but the reason for its use is stated in several ways. One says the curve is to lessen the shock incident to the sudden change of motion from tangent to curve; another that it is necessary for the proper gradual elevation of the outer rail; another admitting either or both these considerations claims as one of the greatest advantages of such curves the ready means they provide for better fitting the ground in location, since the tangents may be located to to good advantage and the curves offsetted as may seem best, the transition curve covering the gap. This last consideration is likely to lead to carelessness in location in the use of too large offsets, necessitating too long transition curves. In exemplifying his formulas Mr. Lovelace uses the large offsets of 27 feet with a 14° curve, necessitating a transition curve of over 500 feet. It goes almost without saying that such long curves of constantly changing radius must be nuisances to the track men. In the relining of old track no such offsets are possible, since the track must be kept on the roadbed.

The considerations which should govern in choosing a transition curve are: (1) The rate of change of direction of motion should be as rapid as is consistent with smooth riding as determined by the passenger and the track man who watches the tendency of his track to get out of line. (2) The rate of obtaining the full difference of elevation of the rails should be as rapid as is consistent with smooth riding, should be uniform, and the difference of elevation should at any point be that due to the speed and the radius at the point. This last consideration holds for the tangent, where, the radius being infinite, the difference in elevation is zero. Since smooth riding probably means a constant rate of change of elevation per unit of time, the rate of change to be assumed will theoretically vary with the speed, but practically two or three rates will suffice.

All of these considerations point to a curve that shall begin with an infinite radius or zero degree and shall vary in radius inversely as its length, and in degree directly as its length. Such is the curve of Wellington, of Holbrook, of Talbot, Crandall, Raymond and others, and practically such is the curve of those who start with some other form, as the cubic parabola or lemniscata, and making certain approximations for simplicity end with true spiral formulas. Mr. Lovelace's formulas for what he calls the modified lemniscata are essentially the same as those of Prof. Talbot, among which are the fundamentrl formulas of Mr. Wellington.

Most of the modern field-books contain discussions of the transition curve, which has been so simplified that there seems to be no need for new curves. Henck, Nagle, Godwin and Frost, and perhaps others, all treat of the spiral in some form, but the most concise statement of "how to lay it out" is perhaps found in Prof. Allen's new and admirable text-book on curves and earthwork. It is probably not too much to say that Prof. Talbot's little monograph. "The Railway Transition Spiral," leaves nothing to be desired in the exposition of the theory and practice of transition curves from a practical standpoint. As has been said, Prof. Allen's statement of the work in the field is model of conciseness, Prof. Crandall shows how to write right-of-way descriptions, and Henck and Prof. Raymond show how to compute ordinates for rail-bending; but all these can be formulated by anyone who has read understandingly Prof. Talbot's book. And yet it must be said that the formulas of Mr. Lovelace though not so comprehensive as those of Prof. Talbot, are nevertheless, so far as they go, fully as simple. Their simplicity is largely due to the use of the length of the spiral in feet rather than in stations. They are as follows, R being the radius of the main curve, L the length in feet of the spiral, O the offset between tangent and offsetted main curve, δ the deflection angle for the whole spiral from tangent to $P.\ C.\ C.$, Δ the central angle consumed by the spiral, t

the tangent distance from tangent point of offsetted main curve to the P. C. of the spiral, and C the long chord of the spiral:

$O = \frac{L^2}{24R}$ common to several discussions.

$\delta = \frac{L}{6R} = \frac{4O}{L} = \sqrt{\frac{2O}{3R}}$ also common to several discussions.

δ in minutes $= \frac{LD}{10}$ where $D = \frac{5730}{R}$.

$$\left.\begin{aligned} t &= \frac{L}{2} - \frac{5}{2}\,\frac{O^2}{L} \\ C &= L - \frac{32}{5}\,\frac{O^2}{L} \end{aligned}\right\} \text{The subtractive term is useful only with large O's.}$$

Deflection from P. C. to a point on spiral $3l$ feet distant $= \delta\,\frac{l^2}{L^2} = \delta^1$.

Deflection from P. C. to a point on spiral $2l$ feet distant $= 3\delta$.

Deflection from P. C. to a point on spiral $3l$ feet distant $= 9\delta^1$, etc., etc.

Mr. Lovelace does not give formulas for deflection angles from other points than the point of spiral. The deflection at the P. C. C. from the long chord of the spiral to the common tangent is 2δ or $\frac{2}{3}\Delta$, and the deflection from the common tangent at the P. C. C. to any point on the spiral l feet distant is the deflection for the main curve for l feet less the deflection δ^1 for the spiral from the point of spiral to a point l feet distant, which means that the spiral departs from the main curve just as it does from the tangent.

The tangent distance from intersection point to point of spiral is best given by Prof. Talbot's formula, I being the I of Henck and Δ of Searles.

$T = t + (R + O) \tan \frac{1}{2} I.$

The tangent distance to the point opposite the P. C. of the offsetted curve is $T - t$, or simply $(R + O) \tan \frac{1}{2} I$.

Mr. Lovelace does not discuss spirals between the two branches of a compound curve, as do most writers. The simplest statement for this spiral is that it is in length and offset required between the two branches of the compound curve the same as the spiral computed for a curve whose degree is the difference of degrees of the two branches, and the spiral departs from these curves just as from a tangent and its simple curve.

The central angle consumed will be the average degree of the two branches, multiplied by the length of the spiral in stations.

The railroad spiral is, with the help of these simple formulas and directions, given in any one of several papers, little more difficult to run in than an ordinary simple curve, and the expressions given are certainly easily within the comprehension of any ordinary transit man. It will doubtless be but a short time before the use of the spiral will be universal.

Railroad Gazette:—9 *February* 1900.

HOLBROOK'S SPIRAL CURVES.

BY E. HOLBROOK.*

About 21 years ago the writer found occasion to use transition curves on railroads, and found that the problem had not been worked out. This problem was to make a transition from circle to tangent such that the superelevation of the outer rail should at all points be proportionate to the centrifugal force. This could best be accomplished by passing fro circle to tangent by a curve with a uniform rate of transition; the superelevation of the out rail would then begin at the point of spiral and increase uniformly, reaching the maximu

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at the point of circle and remaining constant till the decreasing spiral was reached at the other end of the circle. This would give the simplest method for the trackman's use.

The nature of such a spiral is expressed by the equation

$$R\,L = A,$$

when R is the radius of curvature of the spiral at any point; L the length of the spiral measured from its origin, and A a constant which determines the rate of transition.

Having worked out the properties of such a spiral in connection with its relation to the adjacent tangent and circle and put it into use, the writer distributed among various assistant engineers of the "Pan Handle road," copies of the demonstration and tables, and from them it went to other roads. It was published briefly in "The Railroad Gazette" in December, 1880. The first engineer besides the writer to use these formulas extensively was Geo. W. Kittridge, now Chief Engineer of the "Big Four," who put spiral ends on many of the curves of the "Pan Handle," both east and west of Steubenville, O.

Since that time volumes have been written on the subject and many tables arranged, with countless variations, to suit the ideas of engineers as to how best to lay out this curve. They are all, however, but different aspects or parts what was contained in the original work, and have been made with one or more of three objects in view, viz.: to devise a scheme where tables could be dispensed with; to overcome a fancied want of elasticity, or to escape a little higher mathematics which enter into the demonstration but not into the application.

By the help of the following figure and table the actual work in the field is briefly explained:

Let $O\,A$ and $A\,D$ be two tangents, intersecting at the point A, making the angle I.

$O\,B$ is the spiral connecting the circle $H\,B\,C$ with the tangent $O\,A$, having a length $O\,B$ equal L.

Δ is the total curvature of the spiral.

d is the deflection angle to be turned off from the tangent to locate any point, as B.

To locate the point O for any given degree of curve and intersection angle I, we have: $O\,A = T = A\,F + F\,E + E\,O = R \text{ tang } \tfrac{1}{2}\,I + X_0 \text{ tang } \tfrac{1}{2}\,I + Y_0$, or $(R + X_0) \text{ tang } \tfrac{1}{2}\,I + Y_0$.

If we wish to lay out the spiral with a rate of transition of one degree in 60 ft. enter the Table I at the line beginning with the required degree and take out the value of R, to which add by inspection the value of X_0 found on the same line and multiply the sum by tang $\frac{1}{2}$ I, as in ordinary circular curves; to this product add the value of Y_0 found on the line, and you will have the length of tangent for locating O. Set the transit at O and turn off from the tangent the deflections found in the table for various lengths of spiral for locating the spiral until the required degree is reached, as shown by the figures in the first column. Then set up at B, backsighting on O, and turn off twice the total deflection of the spiral, and the line of sight will then be on the common tangent of spiral and circle. The circle can then be produced in the usual manner.

Putting in true transition curves adds practically nothing to the work except as they make the curves longer. They can be put in as quickly by one who is familiar with them as any approximate method that is not rank fudging. There are no offsets to make, no parallel tangents, no curve to run that is not used and no stake to drive that is not a centre stake, and stakes may be driven at any station and as few or as many as desired.

As to the elasticity of this method, 20 years' experience has demonstrated that a single rate of transition should be used on new work, and it is very rare that more than two

are required on old work. If we use two rates, viz.: one degree in thirty feet, and one degree in sixty feet., we can relocate curves up to 6° without getting the stakes outside of the rails; but if some other rate is desired, say one degree in twenty feet, inspect the table. Table No. I. has a rate of one degree in sixty feet, and No. II. has one degree in thirty feet. Compare the value of d on corresponding lines of the two tables and note that one is just double the other, or vary as the rate of transition, so we can lay out the desired spiral by adding 50 % to the values of d found in Table No. II. Note that the values of X_0, on lines beginning with the same degree, are as 4 to 1, or vary inversely as the squares of the rates of transition, which enables us to find the required value of X_0 with very little work. Observe that Y_0 may be obtained by subtracting a small quantity from ½ of L; and observe that this quantity varies in the tables as the cubes of the rates of transition, which enables us to find Y_0 in case the spiral is so long that it cannot be seen by inspection.

Many have objected to the use of the true transition curve because its demonstration requires some higher mathematics. The objection is no better founded than an objection to a table of sines and cosines, because they require the same. A wide experience has demonstrated that young engineers with only a rudimentary knowledge of trigonometry can grasp the general idea and learn to apply it as easily as they can learn to solve a triangle.

At the time that the original demonstration was made the writer did not attempt to ascertain the relation directly between Δ and d, and did not observe for a time that to the limit of the table, $d = \frac{\Delta}{3}$ and consequently that the angle at B turned off to get on the common tangent was twice d, that is $\Delta - d = 2d$. This ratio does not hold for large values of Δ, which goes on increasing indefinitely, while d never reaches 65°.

No case has yet occured to the writer where he could not stake out the whole spiral at one setting; but if the case arises the transit can be moved along the tangent and the remaining deflection easily calculated by use of the values of X and Y found in the tables. Usually, a table like the one given below supplies all that is required; Δ being obtained is required by multiplying d by 3.

(1° = 60 ft.)

Degs.	L.	X.	Y.	Y_0	d.
0° 20′	20	0·01	20·00	10·00	0° 00·7′
0° 40′	40	0·03	40·00	20·00	0° 02·7′
1° 00′	60	0·10	60·00	30·00	0° 06·0′
1° 20′	80	0·25	80·00	40·00	0° 10·7′
1° 40′	100	0·48	100·00	50·00	0° 16·7′
1° 50′	110	0·64	110·00	55·00	0° 20·2′
2° 00′	120	0·84	120·00	60·00	0° 21·0′
2° 10′	130	1·06	129·99	65·00	0° 28·2′
2° 20′	140	1·33	139·99	70·00	0° 32·7′
2° 30′	150	1·64	149·98	75·00	0° 37·5′
2° 40′	160	1·99	159·98	80·00	0° 42·7′
2° 50′	170	2·38	169·97	85·00	0° 48·2′
3° 00′	180	2·83	179·96	90·00	0° 54·0′
3° 10′	190	3·33	189·95	95·00	1° 00·1′
3° 20′	200	3·89	199·93	99·99	1° 06·7′
3° 30′	210	4·49	209·91	104·99	1° 13·5′
3° 40′	220	5·16	219·89	109·98	1° 20·7′
3° 50′	230	5·90	229·86	114·98	1° 28·2′
4° 00′	240	6·70	239·83	119·97	1° 36·0′
4° 10′	250	7·58	249·79	124·97	1° 44·2′

If it is desired to use a subchord, or a degree of curvature between those given in the tables, d for the subchord can easily be found by remembering that d increases as the square of the length, X_0 and Y_0 change so slowly that the former may be obtained by second differences and the latter from first differences, as found in the table. For curves of small total curvature it is sometimes best to make the whole curve a transition. In such case the total curvature of each spiral will be $\Delta = \frac{1}{2} I$, and the length of the tangent will be $y + x$ tang $\frac{1}{2} I$. If the rate of transition in the table does not give an external distance to suit the

location, find the ratio of the one given in the table to the one desired and increase or decrease the length of the tangents and the chords used in staking out accordingly, since all such curves between tangents of a given intersection angle are similar.

For the external distance E we have

$$E = x \sec \tfrac{1}{2} I.$$

It is sometimes necessary to put in a circular curve with transition that will have a given external distance, I being given. Find the degree of a simple curve having the required external distance, then

Let

$E' =$ the required external distance,

$D =$ the degree of a simple curve with external E,

$D' =$ the degree of curve with transition and external E',

$$D' = D \left\{ \frac{E + X_0 \sec \frac{1}{2} I}{E} \right\},$$

where X_0 corresponds to D'; but X_0 changes so slowly and the difference between D' and D is not usually over 10 minutes, so the value of X_0 can be estimated sufficiently close to get the value of D' at the first approximation.

If it is desired to find E for any given value of R and I we have

$$E = R \operatorname{tang} \tfrac{1}{2} I \operatorname{tang} \tfrac{1}{2} I + X_0 \sec \tfrac{1}{2} I.$$

A large number of special cases of compound and reverse curves with transitions will be found worked out in the appendix to the Ohio State Railway Commission's report for 1884. However, keeping in mind that in all cases the circle is removed a distance X_0 from the tangent, and in compound curves the two circles stand in the same relation to each other that the tangent and circles do in the ordinary case, no difficulty will be experienced.

TABLE I.—(1° = 60 ft.)

(Spiral increasing 0° 01′ per ft.)

Degrees.	Radius, ft.	L.	Δ	X.	Y.	X_0.	Y_0.	d.
0° 20′	17,188·75	20	0° 02′	0·01	20·00	0·00	10·00	0° 00·7′
0° 40′	8,594·37	40	0° 08′	0·03	40·00	0·01	20·00	0° 02·7′
1° 00′	5,729·60	60	0° 18′	0·10	60·00	0·03	30·00	0° 06·0′
1° 20′	4,297·15	80	0° 32′	0·25	80·00	0·07	40·00	0° 10·7′
1° 40′	3,437·75	100	0° 50′	0·48	100·00	0·11	50·00	0° 16·7′
1° 50′	3,125·21	110	1° 00½′	0·64	110·00	0·16	55·00	0° 20·2′
2° 00′	2,864·80	120	1° 12′	0·84	120·00	0·21	60·00	0° 24·0′
2° 10′	2,644·41	130	1° 24½′	1·06	129·99	0·27	65·00	0° 28·2′
2° 20′	2,455·52	140	1° 38′	1·33	139·99	0·32	70·00	0° 32·7′
2° 30′	2,291·82	150	1° 52½′	1·64	149·98	0·42	75·00	0° 37·5′
2° 40′	2,148·57	160	2° 08′	1·99	159·98	0·52	80·00	0° 42·7′
2° 50′	2,022·20	170	2° 24½′	2·38	169·97	0·60	85·00	0° 48·2′
3° 00′	1,909·85	180	2° 42′	2·83	179·96	0·71	90·00	0° 54·0′
3° 10′	1,809·20	190	3° 00½′	3·33	189·95	0·84	95·00	1° 00·1′
3° 20′	1,718·80	200	3° 20′	3·89	199·93	0·99	99·99	1° 06·7′
3° 30′	1,637·01	210	3° 40½′	4·49	209·91	1·13	104·99	1° 13·5′
3° 40′	1,562·60	220	4° 02′	5·16	219·89	1·28	109·98	1° 20·7′
3° 50′	1,494·66	230	4° 24½′	5·90	229·86	1·48	114·98	1° 28·2′
4° 00′	1,432·39	240	4° 48,	6·70	239·83	1·67	119·97	1° 36·0′
4° 10′	1,375·09	250	5° 12½′	7·58	249·79	1·90	124·97	1° 44·2′
4° 20′	1,322·20	260	5° 38′	8·51	259·75	2·13	129·97	1° 52·7′
4° 30′	1,273·20	270	6° 04½′	9·53	269·70	2·38	134·97	2° 01·5′
4° 40′	1,227·76	280	6° 32′	10·63	279·64	2·66	139·96	2° 10·7′
4° 50′	1,185·40	290	7° 00½′	11·81	289·57	2·96	144·94	2° 20·1′
5° 00′	1,145·91	300	7° 30′	13·07	299·49	3·26	149·92	2° 30·0′
5° 10′	1,108·95	310	8° 00½′	14·42	309·40	3·61	154·91	2° 40·2′
5° 20′	1,074·28	320	8° 32′	15·86	319·29	3·97	159·89	2° 50·7′
5° 30′	1,041·73	330	9° 04½′	17·39	329·17	4·34	164·87	3° 01·5′
5° 40′	1,011·09	340	9° 38′	19·02	339·04	4·76	169·85	3° 12·7′
5° 50′	982·21	350	10° 12½′	20·74	348·89	5·19	174·82	3° 24·2′
6° 00′	954·93	360	10′ 48′	22·56	358·72	5·65	179·79	3° 36·0

TABLE II.—($1° = 30$ ft.)

(Spiral increasing 0° 02′ per ft.)

Degrees.	Radius, ft.	L.	Δ	X.	Y.	X_0.	Y_0.	d.
0° 20′	17,188·75	10	0° 01′	0·00	10·00	0·00	5·00	0° 00·3′
0° 40′	8,594·37	20	0° 04′	0·01	20·00	0·00	10·00	0° 01·3′
1° 00′	5,729·60	30	0° 09′	0·03	30·00	0·01	15·00	0° 03·0′
1° 20′	4,297·15	40	0° 16′	0·06	40·00	0·02	20·00	0° 05·3′
1° 40′	3,437·75	50	0° 25′	0·12	50·00	0·03	25·00	0° 08·3′
2° 00′	2,864·80	60	0° 36′	0·21	60·00	0·05	30·00	0° 12·0′
2° 20′	2,455·53	70	0° 49′	0·33	70·00	0·08	35·00	0° 16·3′
2° 40′	2,148·59	80	1° 04′	0·49	80·00	0·13	40·00	0° 21·3′
3° 00′	1,909·86	90	1° 21′	0·70	90·00	0·17	45·00	0° 27·0′
3° 20′	1,718·89	100	1° 40′	0·97	99·99	0·24	50·00	0° 33·3′
3° 40′	1,562·81	110	2° 01′	1·29	109·99	0·31	55·00	0° 40·3′
4° 00′	1,432·39	120	2° 24′	1·67	119·98	0·41	60·00	0° 48·0′
4° 20′	1,322·21	130	2° 49′	2·12	129·97	0·52	65·00	0° 56·3′
4° 40′	1,227·76	140	3° 16′	2·65	139·95	0·63	70·00	1° 05·3′
5° 00′	1,145·91	150	3° 45′	3·26	149·94	0·81	74·99	1° 15·0′
5° 20′	1,074·29	160	4° 16′	3·96	159·91	0·98	79·98	1° 25·3′
5° 40′	1,011·10	170	4° 49′	4·75	169·88	1·18	84·98	1° 36·3′
6° 00′	954·93	180	5° 24′	5·64	179·84	1·40	89·97	1° 48·0′
6° 20′	904·67	190	6° 01′	6·63	189·79	1·65	94·97	2° 00·3′
6° 40′	859·44	200	6° 40′	7·74	199·73	1·93	99·96	2° 13·3′
7° 00′	818·52	210	7° 21′	8·95	209·66	2·28	104·94	2° 27·0′
7° 20′	781·30	220	8° 04′	10·30	219·57	2·57	109·93	2° 41·3′
7° 40′	747·38	230	8° 49′	11·76	229·46	2·93	114·92	2° 56·3′
8° 00′	716·20	240	9° 36′	13·37	239·33	3·34	119·89	3° 12·0′

For the benefit of those who may care to look into the mathematical properties of the spiral, I append the following:

(1) $$R\,L = A.$$

For a transition at the rate of 1° in 60 ft., we have

$$A = 343{,}775\ .\ R = \frac{343{,}775}{L}.$$

Since the curvature at any point is twice the average curvature between the point and the origin we have the total curvature.

(2) $$\Delta = \frac{L}{2} \times \frac{L}{100} = \frac{L^2}{200} \text{ (in minutes);}$$

or

$\Delta = 0{\cdot}0000014545\ L^2$ (in terms of arc radius unity).

Let

$$0{\cdot}0000014545 = a, \text{ then } \Delta = a\,L^2.$$

By inspection of figures we see

(3) $$\frac{d\,y}{d\,L} = \cos\Delta = \cos a\,L^2,\ \therefore$$

$$\therefore\ Y = \int_0^L \cos a\,L^2\,d\,L = L - \frac{12\,a^2\,L^5}{1.2.3.4.5.} + \frac{1680\,a^4\,L^9}{1.2.3.4.5.6.7.8.9.}$$

$$+\ \&c.$$

(4) $$\frac{d\,x}{d\,L} = \sin \qquad = \sin a\,L^2,$$

$$X = \int_0^L \sin a\,L^2\,d\,L = \frac{2\,a\,L^3}{1.2.3.} + \frac{1200\,a^3\,L^7}{1.2.3.4.5.6.7.} + \&c.$$

or putting in the value of a and reducing we have

(5) $Y = L - 0{\cdot}000000000000211557\, L^5 + \text{\&c.}$

(6) $X = 0{\cdot}0000004848\, L^3 + \text{\&c.}$

(7) $X_0 = X - R$ versin Δ

(8) $Y_0 = Y - R$ sine Δ

(9) $\tan d = \dfrac{X}{Y}$, from which value of d can be computed or dividing equation (6) by equation (5), substituting in equation for value of $\tan \Delta$, and reducing we have

(10) $$d = \frac{L^2}{600} - \text{\&c.};$$

but

(11) $$\Delta = \frac{L^2}{200},$$

within the range of practice we may use

(12) $$d = \frac{\Delta}{3}.$$

Likewise

$$X_0 = \frac{X}{4} - \text{\&c.}$$

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